

2017 Specialist Mathematics Trial Exam 2 Solutions

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SECTION A – Multiple-choice questions

1	2	3	4	5	6	7	8	9	10
D	E	A	C	D	A	E	D	D	A

11	12	13	14	15	16	17	18	19	20
E	E	B	E	A	B	C	D	A	D

Q1 $y = x + \frac{10100}{(x-101)(x-100)}$

D

Q2 $\sin^{-1}(ax) + \frac{\pi}{2} = \frac{\pi}{2} - b, x = \frac{\sin(-b)}{a} = -\frac{\sin b}{a}$

E

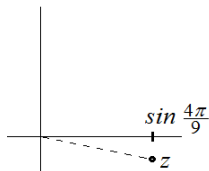
Q3 $\cot u = x + 1, \sec^2 u = y + 2, 1 + \tan^2 u = \sec^2 u$

$\therefore 1 + \frac{1}{(x+1)^2} = y + 2, \therefore (x+1)^2(y+1) = 1$

A

Q4

C



Q5 $z = (a+b)\left(\sqrt{2}\text{cis}\left(\frac{3\pi}{4}\right) + 2\right) = (a+b)\sqrt{2}\text{cis}\left(\frac{\pi}{4}\right)$

$z^n = (a+b)^n(\sqrt{2})^n \text{cis}\left(\frac{n\pi}{4}\right) \in R$ when n is a multiple of 4.

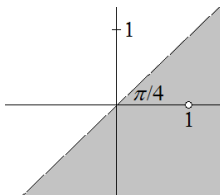
D

Q6 Symmetry: $-\text{cosec}\left(\frac{x}{a} + \pi\right) = \text{cosec}\left(\frac{x}{a}\right)$

A

Q7 $\left|\frac{z-i}{z-1}\right| > 1, |z-i| > |z-1|$ for $z \neq 1, \therefore$ the shaded region shown below.

E



Q8 $x = \sin\left(\frac{1}{2}\right)$ or $x = \sqrt{3}$, but $\sin^{-1}x - \frac{1}{2}$ is undefined for $x = \sqrt{3}$

D

Q9 $x = \cos t - \sin t$ and $y = -\frac{1}{2}\cos 2t$

$\frac{dx}{dt} = -\sin t - \cos t, \frac{dy}{dt} = \sin 2t$

When $t = \frac{5\pi}{4}, \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\sin 2t}{-\sin t - \cos t} = \frac{1}{\sqrt{2}}$

D



Q10 $x_0 = 1, y_0 = 0, f' = 0.5$

$x_1 = 1.1, y_1 \approx 0 + 0.1 \times 0.5 \approx 0.05, f' \approx 0.4545$

$x_2 = 1.2, y_2 \approx 0.05 + 0.1 \times 0.4545 \approx 0.095$

A

Q11

E

Q12 $(3\tilde{i} - \tilde{j})(-\alpha\tilde{i} + 3\alpha\tilde{j} - \tilde{k}) = -6\alpha \neq 0$ when $\alpha \neq 0$

E

Q13 Vector resolute parallel to $\tilde{b}$ is:
 $(3\tilde{i} - 2\tilde{j} + \tilde{k}) - (\tilde{i} - 2\tilde{j} + 3\tilde{k}) = 2\tilde{i} - 2\tilde{k}$

B

Q14

E

Q15 $f(|x|) = \begin{cases} f(-x) & \text{for } x < 0 \\ f(x) & \text{for } x \geq 0 \end{cases}$

$\therefore \int_{-1}^1 f(|x|)dx = 2 \times \int_0^1 f(x)dx = -1$

A

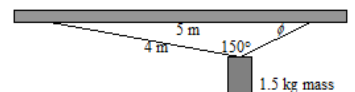
Q16 $f(0) = \int_{\frac{\pi}{4}}^0 \cot^2\left(\frac{\pi}{4} + x\right)dx + 2 \approx 1.8$

B

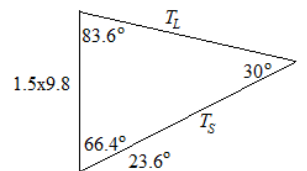
Q17

C

$\frac{\sin \phi}{4} = \frac{\sin 150^\circ}{5}, \phi \approx 23.6^\circ$



$\frac{\sin 66.4^\circ}{T_L} = \frac{\sin 83.6^\circ}{T_S}, \therefore \frac{T_S}{T_L} \approx 1.084$



Q18 $\text{Var}(X) = 0.050^2 = 0.0025, \text{Var}(Y) = 3^2 \times 0.0025 = 0.0225$
 $\text{Var}(X_1) + \text{Var}(X_2) + \text{Var}(Y_1) + \text{Var}(Y_2) = 2 \times \text{Var}(X) + 2 \times \text{Var}(Y)$
 $= 2 \times 0.0025 + 2 \times 0.0225 = 0.05$
 $\therefore \text{sd}(X_1 + X_2 + Y_1 + Y_2) = \sqrt{0.05} \approx 0.224$

D

Q19 $E(\bar{X}) \approx \mu = 168.0, \text{sd}(\bar{X}) = \frac{9}{\sqrt{9}} = 3$

$\Pr(\bar{X} = 175.0) = \Pr(174.5 < \bar{X} < 175.5) \approx 0.0089$

A

Q20

D



SECTION B

Q1a $\angle PTS = 180^\circ - 60^\circ = 120^\circ$

Q1b $\angle POS = 2 \times 60^\circ = 120^\circ$

Q1c $\overrightarrow{PS} = \vec{c} - \vec{a}$

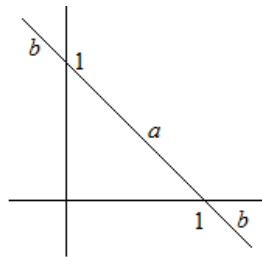
Q1d $|\overrightarrow{PS}|^2 = (\vec{c} - \vec{a})(\vec{c} - \vec{a}) = \vec{c} \cdot \vec{c} - 2\vec{c} \cdot \vec{a} + \vec{a} \cdot \vec{a}$
 $= c^2 - 2|\vec{c}||\vec{a}|\cos 120^\circ + a^2 = b^2 + 2ab\cos 60^\circ + a^2 = a^2 + ab + b^2$

Q1e Let $\overrightarrow{OP} = \vec{p}$ and $\overrightarrow{OS} = \vec{s}$, $|\overrightarrow{OP}| = p$ and $|\overrightarrow{OS}| = s$
 $|\overrightarrow{PS}|^2 = |\vec{s} - \vec{p}|^2 = (\vec{s} - \vec{p})(\vec{s} - \vec{p}) = \vec{s} \cdot \vec{s} - 2\vec{s} \cdot \vec{p} + \vec{p} \cdot \vec{p}$
 $= s^2 - 2sp\cos 120^\circ + p^2 = r^2 + r^2 + r^2 = 3r^2$, $\therefore a^2 + ab + b^2 = 3r^2$

Q2a Midpoint between $z=1$ and $z=i$, i.e. $z = \frac{1}{2}(1+i)$.

Q2b

Length of longest chord $= a + 2b = 2$



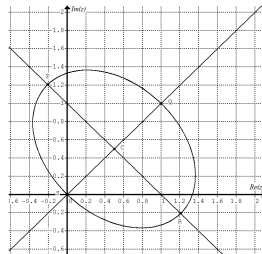
Q2c

$P: z = \left(\frac{1}{2} - \frac{1}{\sqrt{2}}\right) + \left(\frac{1}{2} + \frac{1}{\sqrt{2}}\right)i$

$Q: z = 1 + i$

$R: z = \left(\frac{1}{2} + \frac{1}{\sqrt{2}}\right) + \left(\frac{1}{2} - \frac{1}{\sqrt{2}}\right)i$

$S: z = 0$



Q2d $|(x-1) + yi| = 2 - |x + (y-1)i|$

Square both sides and simplify to $2|x + (y-1)i| = 2 + (x-y)$

Square both sides and simplify to $3(x+y)^2 = 4(xy+y)$

Q2e $\frac{d}{dx} 3(x+y)^2 = \frac{d}{dx} 4(xy+y)$

$6(x+y)\left(1 + \frac{dy}{dx}\right) = 4\left(1 + (x+1)\frac{dy}{dx} + y\right)$, $y = 2 - 3x$ when $\frac{dy}{dx} = 0$

Q2f $3(x+y)^2 = 4(xy+y)$

Let $y = 2 - 3x$, $3(x+2-3x)^2 = 4(x+x(2-3x)+2-3x)$

$\therefore 6x^2 - 6x + 1 = 0$, $x = \frac{3 \pm \sqrt{3}}{6}$

$\text{Re}(z) = \frac{3 + \sqrt{3}}{6}$ when $\text{Im}(z)$ is a minimum value.

Q3a $\vec{v} = \int_0^t -9.8\tilde{j} dt + 20\cos\theta\tilde{i} + 20\sin\theta\tilde{j}$

$\therefore \vec{v} = 20\cos\theta\tilde{i} + (20\sin\theta - 9.8t)\tilde{j}$

$\vec{r} = (20\cos\theta)t\tilde{i} + ((20\sin\theta)t - 4.9t^2)\tilde{j}$

Q3b $x = (20\cos\theta)t$, $y = (20\sin\theta)t - 4.9t^2$

Eliminate t , $y = (\tan\theta)x - \frac{0.01225x^2}{\cos^2\theta}$

Q3c Let $x=10$ and $y=8$ ($y > 8$ to pass through the ring)

$\theta^\circ = 46.7^\circ$ or 81.9° (46.673 or 81.986)

Let $x=10$ and $y=10$ ($y < 10$ to pass through the ring)

$\theta^\circ = 53.3^\circ$ or 81.7° (53.342 or 81.658)

$\therefore \theta \in [46.7, 53.3]$ or $\theta \in [81.7, 81.9]$

Q3d The distance is longest when the projection angle is 81.7° .

$y = (\tan 81.658^\circ)x - \frac{0.01225x^2}{\cos^2 81.658^\circ}$, $\therefore y = 6.8197x - 0.5820x^2$

$\frac{dy}{dx} = 6.8197 - 1.1640x$

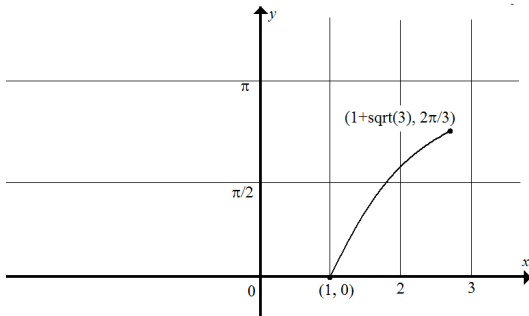
Distance $= \int_0^{10} \sqrt{1 + (6.8197 - 1.1640x)^2} dx \approx 32.488 \approx 32.5$ m

Q3e Time taken: Consider the $\tilde{i}$ component,

$\Delta t = \frac{10}{20\cos 81.658^\circ} \approx 3.446$ s, average speed $\approx \frac{32.488}{3.446} \approx 9.4$ m s⁻¹



Q4a



Q4b $y = 2 \tan^{-1}(x-1)$, $x = 1 + \tan \frac{y}{2}$

$$V = \int_0^{\frac{2\pi}{3}} \pi x^2 dy = \int_0^{\frac{2\pi}{3}} \pi \left(1 + \tan \frac{y}{2}\right)^2 dy \text{ m}^3$$

Q4c $V = \int_0^h \pi \left(1 + \tan \frac{y}{2}\right)^2 dy = 2\pi \left(\tan \frac{h}{2} - 2 \log_e \left(\cos \frac{h}{2}\right)\right) \text{ m}^3$

Q4d

$$V_{\max} = \int_0^{\frac{2\pi}{3}} \pi \left(1 + \tan \frac{y}{2}\right)^2 dy \approx 19.593 \text{ m}^3$$

and given $\frac{dh}{dt} = 0.05 \text{ m min}^{-1}$

$$V = 2\pi \left(\tan \frac{h}{2} - 2 \log_e \left(\cos \frac{h}{2}\right)\right), \therefore \frac{dV}{dh} = 2\pi \left(\frac{1}{2} \sec^2 \frac{h}{2} - \tan \frac{h}{2}\right)$$

Let $2\pi \left(\tan \frac{h}{2} - 2 \log_e \left(\cos \frac{h}{2}\right)\right) = \frac{1}{2} \times 19.593$, $\therefore h \approx 1.5015 \text{ m}$

When $h \approx 1.5015$, $\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt} \approx 0.59 \text{ m}^3 \text{ per minute}$

Q4e Time taken $= \frac{\frac{2\pi}{3}}{0.05} = \frac{40\pi}{3} \text{ minutes}$

Q5a $a = \frac{F_{\text{net}}}{m} = \frac{12000 - 240 \times 10 - 160 \times 10}{2000 + 2000} = 2 \text{ m s}^{-2} \text{ forward}$

Q5b $T - 160 \times 10 = 2000 \times 2$, $T = 5600 \text{ N}$

Q5c $400v = 12000$, $v = 30 \text{ m s}^{-1}$

Q5d $4000 \times 30 = 120000 \text{ kg m s}^{-1}$

Q5ei $a = \frac{12000 - 240v - 160v}{4000}$, $\therefore a = 3 - \frac{v}{10}$, $\therefore v \frac{dv}{dx} = 3 - \frac{v}{10}$
 $\therefore \frac{dv}{dx} = \frac{3}{v} - \frac{1}{10}$

Q5eii $\frac{dv}{dx} = \frac{30-v}{10v}$, $\frac{dx}{dv} = \frac{10v}{30-v}$

Distance $= \int_0^{10} \frac{10v}{30-v} dv = \int_0^{10} \left(\frac{300}{30-v} - 10\right) dv = 300 \log_e \left(\frac{3}{2}\right) - 100 \text{ m}$

Q6a Mean of $\bar{X} = E(\bar{X}) \approx \mu = 25.0 \text{ mm}$

Q6b $\text{sd}(\bar{X}) = \frac{\sigma}{\sqrt{n}} = \frac{0.2}{\sqrt{5}} = 0.089 \text{ mm}$

Q6c $\Pr(24.8 < \bar{X} < 25.2) \approx 0.975$

Q6d $\mu = 25.1 \text{ mm}$

Q6e $\left(25.1 - 1.96 \times \frac{0.2}{\sqrt{5}}, 25.1 + 1.96 \times \frac{0.2}{\sqrt{5}}\right)$,
 i.e. $(24.925, 25.275) \text{ in mm}$

Q6eii $95\% \times 20 = 19$

Please inform mathline@itute.com re conceptual and/or mathematical errors