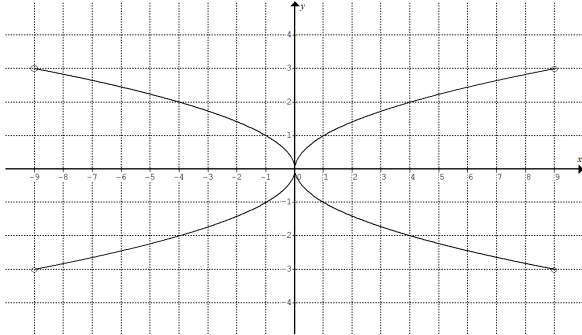


2017 Specialist Mathematics Trial Exam 1 Solutions

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Q1a



Q1b $|y| = \sqrt{|x|}$, $y^2 = |x|$, $y^2 = \sqrt{x^2}$,

$$2y \frac{dy}{dx} = \frac{x}{\sqrt{x^2}} = \frac{x}{|x|}, \frac{dy}{dx} = \frac{x}{2y|x|} = \pm \frac{1}{2\sqrt{|x|}}$$

Q1c $(-9, 0) \cup (0, 9)$

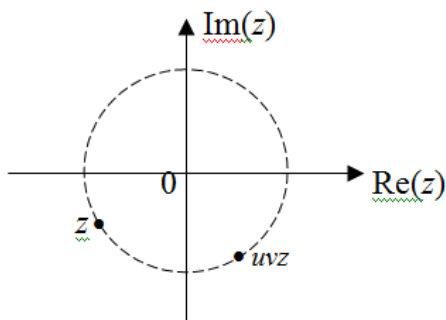
Q2a $\tilde{a} + \tilde{b} + \tilde{c} + \tilde{d} = \tilde{0}$, $\therefore \tilde{d} = -(\tilde{a} + \tilde{b} + \tilde{c})$

Q2b $\overrightarrow{PQ} = \frac{1}{2}(\tilde{a} + \tilde{b}) = \frac{1}{2}\overrightarrow{AC}$, $\overrightarrow{SR} = \frac{1}{2}(-\tilde{c} - \tilde{d}) = \frac{1}{2}\overrightarrow{AC}$
 $\therefore \overrightarrow{PQ} = \overrightarrow{SR}$, $\therefore PQRS$ is a parallelogram.

Q3a $v = \frac{\bar{z}}{1-i}$, $\bar{v} = \frac{z}{1+i}$, $u\bar{v} = (1+i)z \times \frac{z}{1+i} = z^2$

Q3b $uv = (1+i)z \times \frac{\bar{z}}{1-i} = \frac{(1+i)z\bar{z}}{1-i} = \frac{(1+i)^2 |z|^2}{2} = i$

Q3c $uvz = iz$, uvz is the image of z after an anticlockwise rotation by 90° about O .

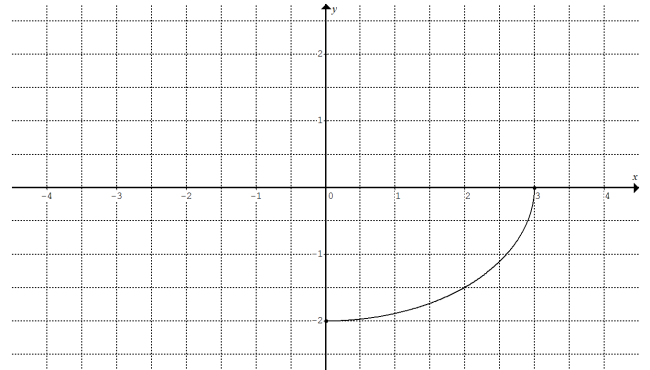


Q4a $x = \sqrt{3} \sin 2t + \cos 2t$, $\dot{x} = 2\sqrt{3} \cos 2t - 2 \sin 2t$
 $\ddot{x} = -4\sqrt{3} \sin 2t - 4 \cos 2t$, maximum speed occurs when $\ddot{x} = 0$
 $4\sqrt{3} \sin 2t = -4 \cos 2t$, $\tan 2t = -\frac{1}{\sqrt{3}}$, $2t = \frac{5\pi}{6}$, $t = \frac{5\pi}{12}$

Q4b Max. speed = $\left| \dot{x} \left(\frac{5\pi}{12} \right) \right| = \left| 2\sqrt{3} \cos \frac{5\pi}{6} - 2 \sin \frac{5\pi}{6} \right| = |-3-2| = 5$



Q5a $x = 3 \sin 2t$, $y = -2 \cos 2t$, $\frac{x^2}{9} + \frac{y^2}{4} = 1$, $0 \leq t \leq \frac{\pi}{4}$
 $0 \leq x \leq 3$, $-2 \leq y \leq 0$

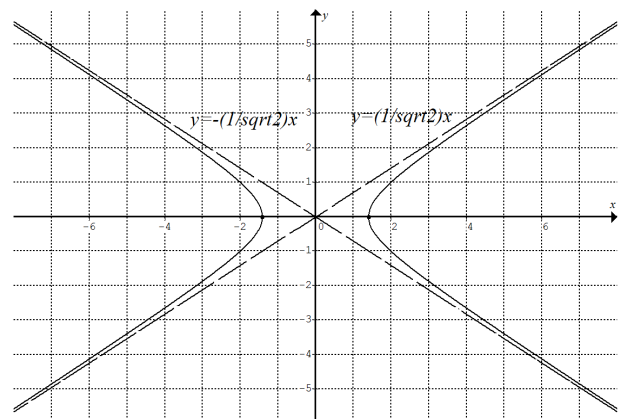


Q5b Arc length = $\frac{1}{4} \times 2\pi \times 1 \times 3 \times 2 = 3\pi$ metres

Q6a $\frac{dy}{dx} - \frac{x}{2y} = 0$, $\int 2y dy = \int x dx$, $y^2 = \frac{x^2}{2} + c$

$(2, 1)$ is on the curve, $\therefore c = -1$ and $y^2 = \frac{x^2}{2} - 1$ or $\frac{x^2}{2} - y^2 = 1$

Q6b Hyperbola: x -intercepts are $(-\sqrt{2}, 0)$ and $(\sqrt{2}, 0)$, asymptotes are $y = \pm \frac{1}{\sqrt{2}}x$



Q7a $\mu = \frac{3}{5} \times 32 + \frac{2}{5} \times 29 = 30.8$, $E(\bar{X}) = \mu = 30.8$

Q7b $\text{Var} = \left(\frac{3}{5} \right)^2 \times 8^2 + \left(\frac{2}{5} \right)^2 \times 10^2 = 39.04$, $\sigma = \sqrt{39.04} \approx 6.2482$
 $\text{sd}(\bar{X}) = \frac{\sigma}{\sqrt{n}} = \frac{6.2482}{\sqrt{10}} \approx 1.98$



Q8a All 6 roots lie on the unit circle centred at O , their arguments are separated by $\frac{\pi}{3}$.

Given $z = -1$ is a root, then $z = 1$ is also a root.

The others are: $z = \text{cis}\left(\pm \frac{\pi}{3}\right) = \cos\left(\pm \frac{\pi}{3}\right) + i \sin\left(\pm \frac{\pi}{3}\right) = \frac{1}{2} \pm \frac{\sqrt{3}}{2}i$

and $z = \text{cis}\left(\pm \frac{2\pi}{3}\right) = \cos\left(\pm \frac{2\pi}{3}\right) + i \sin\left(\pm \frac{2\pi}{3}\right) = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$

Q8b $z - 2i = \pm 1, \frac{1}{2} \pm \frac{\sqrt{3}}{2}i, -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$

$z = \pm 1 + 2i, \frac{1}{2} \pm \left(2 + \frac{\sqrt{3}}{2}\right)i, -\frac{1}{2} \pm \left(2 + \frac{\sqrt{3}}{2}\right)i$

Q9a $\tilde{s} = -2\tilde{p} + 3\tilde{q} - \tilde{r}$

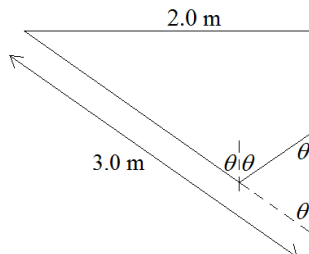
$= -2(2\tilde{i} + \tilde{j} + 2\tilde{k}) + 3(-\tilde{i} - 2\tilde{j} + 2\tilde{k}) - (2\tilde{i} - 2\tilde{j} - \tilde{k})$
 $= -9\tilde{i} - 6\tilde{j} + 3\tilde{k}$

Q9b Let $l(2\tilde{i} + \tilde{j} + 2\tilde{k}) + m(-\tilde{i} - 2\tilde{j} + 2\tilde{k}) + n(2\tilde{i} - 2\tilde{j} - \tilde{k}) = \tilde{0}$

$\therefore 2l - m + 2n = 0, l - 2m - 2n = 0$ and $2l + 2m - n = 0$

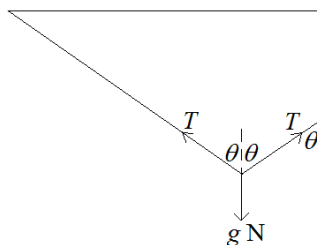
$\therefore l = m = n = 0, \therefore \tilde{p}, \tilde{q}$ and $\tilde{r}$ are linearly independent vectors.

Q10a



Refer to the above diagram, $\sin \theta = \frac{2}{3}$.

Q10b



Refer to the above diagram, $2T \cos \theta = g$

$2T \sqrt{1 - \sin^2 \theta} = g, 2T \sqrt{1 - \left(\frac{2}{3}\right)^2} = g, T = \frac{3\sqrt{5}}{10} g \text{ N}$

Please inform mathline@itute.com re conceptual and/or mathematical errors