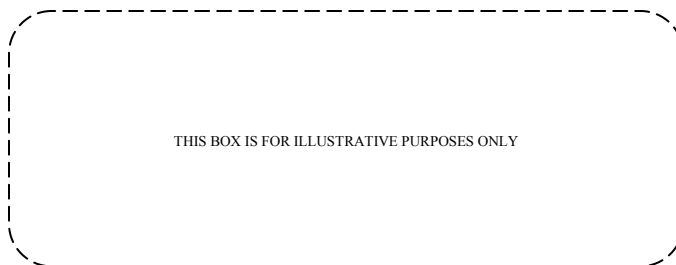




2016 Trial Examination



THIS BOX IS FOR ILLUSTRATIVE PURPOSES ONLY

STUDENT
NUMBER

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Letter

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SPECIALIST MATHEMATICS

Units 3 and 4 – Written examination 1

Reading time: 15 minutes

Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper and/or white out liquid/tape.
- No calculator is permitted in this examination.

Materials supplied

- Question and answer book of 11 pages.

Instructions

- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.

Unless otherwise specified, an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are not drawn to scale.

Take the acceleration due to gravity to have magnitude: $g \text{ m/s}^2$, where $g = 9.8$.

Question 1 (4 marks)

In a toy factory a sample of 121 toy cars are tested. The cars are moving on a straight line track. It is found that the dragging force produced by the battery is four times the mass of the car in grams. The resistance force is three times the speed of the car in metres per minute.

The mass of the cars are normally distributed with a mean 200 grams and standard deviation of 3 grams. The speed is also normally distributed with a mean 80 metres per minute and a standard deviation of 4 metres. Assume that the mass and speed are independent.

- a.** Find the expected value and the standard deviation of the resultant force.

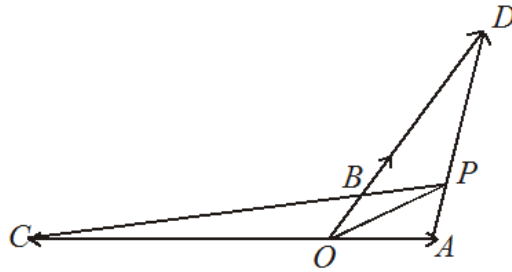
3 marks

- b.** Find the standard deviation of the mean of the resultant force for the sample.

1 mark

TURN OVER

Question 2 (3 marks)

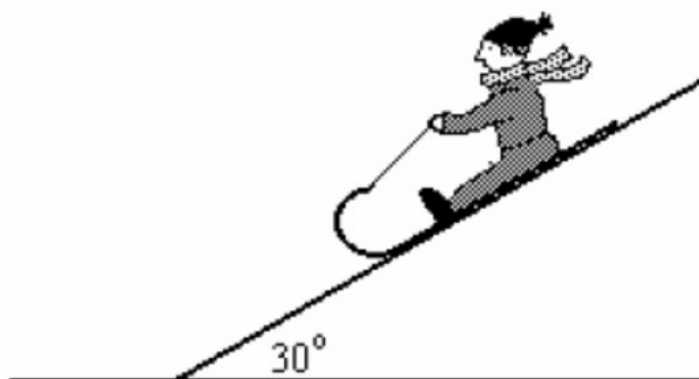


Let O, A and B be three non-collinear points on a plane.

$\vec{OA} = \vec{a}$, $\vec{OB} = \vec{b}$, $\vec{OC} = -3\vec{a}$ and $\vec{OD} = 4\vec{b}$. P is the intersection of the lines AD and CB.

If $\vec{OP} = m\vec{OA} + n\vec{OB}$, find the values of m and n .

Question 3 (3 marks)



A boy on a toboggan slides down a frictionless snow-covered slope from rest. The boy and the toboggan together have mass of m kg and the slope is at an angle of 30° to the horizontal. Find the distance travelled by the boy in the first $\frac{10}{\sqrt{g}}$ seconds.

TURN OVER

$$\underset{\sim}{r}(t) = \left(\frac{\sqrt{3}}{8}t^2 + 1\right)\underset{\sim}{i} + \left(\frac{1}{3}t^3 - \frac{7t}{2}\right)\underset{\sim}{j} + \left(\frac{\sqrt{3}}{4}t^2 - 1\right)\underset{\sim}{k}$$
[illegible]

Question 5 (6 marks)

Given that $z = -3$ and $z = -2 + i$ are solutions of the equation:

$$z^4 + 5z^3 + az^2 + bz + c = 0$$

where $a, b, c \in \mathbb{R}$. Find:

a. The other real root of the equation;

4 marks

b. The values of a, b and c .

2 marks

TURN OVER

Question 6 (4 marks)

A tank has the shape of an inverted cone with a height of 8m. The radius of the top is 4m.

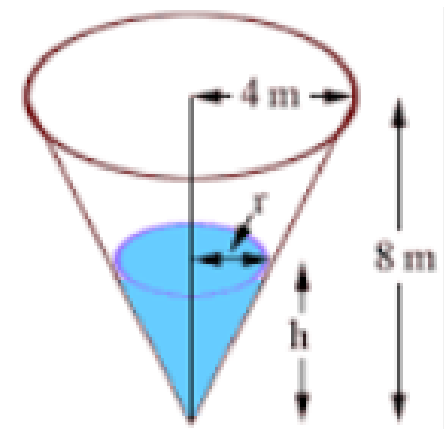
The tank is initially fully filled with water.

The water flows out from a tap at a 2 metres height of the

tank with a rate of $\frac{\pi(h^2-1)}{8}$ litres per hour,

where h is the depth of the water.

Find the time it takes in hours when $h = 8\text{ m}$ for the water to be $h = 2\text{ m}$.

[illegible]

Question 7 (6 marks)

Let $f(x) = \frac{e^x + e^{-x}}{2}$.

- a.** Find the arc length of the curve represented by $y = f(x)$ for the interval $[-1, 1]$.

3 marks

- b.** Find the volume of the solid revolution formed by rotating the region bounded by $y = f(x)$, the x-axis, the lines $x = -1$ and $x = 1$.

3 marks

TURN OVER

Question 8 (6 marks)

a. Find: $\{x: \arctan(x^2) < \arctan(2x + 15), x \in R\}$.

3 marks

b. Solve $\sqrt{3} \cos(2x) + \sin(2x) = \sqrt{2}, 0 \leq x \leq \pi$.

3 marks

$$\frac{dy}{dx} = \frac{1+y^2}{(2+e^x)y}, x, y \in [0, \infty),$$
[illegible]

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