

Trial Examination 2016

VCE Specialist Mathematics Units 3&4

Written Examination 1

Question and Answer Booklet

Reading time: 15 minutes

Writing time: 1 hour

Student's Name: _____

Teacher's Name: _____

Structure of Booklet

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
9	9	40

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.

Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied

Question and answer booklet of 8 pages.

Formula sheet.

Working space is provided throughout the booklet.

Instructions

Write your **name** and your **teacher's name** in the space provided above on this page.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2016 VCE Specialist Mathematics Units 3&4 Written Examination 1.

Neap Trial Exams are licensed to be photocopied or placed on the school intranet and used only within the confines of the school purchasing them, for the purpose of examining that school's students only. They may not be otherwise reproduced or distributed. The copyright of Neap Trial Exams remains with Neap. No Neap Trial Exam or any part thereof is to be issued or passed on by any person to any party inclusive of other schools, non-practising teachers, coaching colleges, tutors, parents, students, publishing agencies or websites without the express written consent of Neap.

Instructions

Answer **all** questions in the spaces provided.

Unless otherwise specified, an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ m/s}^2$, where $g = 9.8$.

Question 1 (2 marks)

Find the value of $\int_0^{\frac{\pi}{4}} \frac{\sec^2(x)}{1 + \tan(x)} dx$.

Question 2 (3 marks)

The position vector of a particle of mass 0.25 kg at time t seconds, $t \geq 0$, is given by

$$\underline{r}(t) = (t^4 - 2t^2)\underline{i} + (4t^3 - t^4)\underline{j}.$$

- a. Find an expression for the momentum $\underline{p}$ of the particle at time t .

1 mark

- b. A force $\underline{F}$ acts on the particle at time t .

Find when $\underline{F}$ acts in the direction of unit vector $\underline{j}$.

2 marks

Question 3 (3 marks)

Solve the equation $\cos\left(\frac{x}{2}\right) = \sin\left(\frac{x}{4}\right)$, $0 \leq x \leq 4\pi$.

Question 4 (6 marks)

Consider $P(z) = z^3 + z^2 + bz + 12$, $b \in R$, $z \in C$.

- a. Given that $P(1 - \sqrt{3}i) = 0$, solve the equation $P(z) = 0$.

3 marks

- b. Find the smallest positive integer, k , such that $(1 - \sqrt{3}i)^k \in R$ and, for this value of k , state the value of $(1 - \sqrt{3}i)^k$.

3 marks

Question 5 (4 marks)

The study scores of a large population of Victorian Specialist Mathematics students have a mean of 30 and a standard deviation of 7. A random sample of 100 students is to be selected from the population. The sample mean is denoted by $\bar{X}$.

- a. State the mean of $\bar{X}$. 1 mark

- b. Find the standard deviation of $\bar{X}$. 1 mark

- c. The sample size is now increased by a factor of 4.
Determine how the mean of $\bar{X}$ and the standard deviation of $\bar{X}$ would change as a result of this increase in sample size. 2 marks

Question 6 (3 marks)

If $y = \cos^{-1}\left(\frac{2}{x}\right)$, $x > 2$, show that $\frac{dy}{dx} = \frac{2}{x\sqrt{x^2 - 4}}$.

Question 7 (9 marks)

The acceleration of a particle moving in the x - y plane is $-g\mathbf{j}$. At time $t = 0$, the particle leaves the point with position $\mathbf{r}(0) = h\mathbf{j}$ with velocity $\dot{\mathbf{r}}(0) = V\cos(\theta)\mathbf{i} + V\sin(\theta)\mathbf{j}$.

- a. Show that the particle's position vector at time t is given by

$$\mathbf{r}(t) = V\cos(\theta)t\mathbf{i} + \left(V\sin(\theta)t - \frac{1}{2}gt^2 + h\right)\mathbf{j}.$$

4 marks

- b. Show that the particle's path is given by $y = h + \tan(\theta)x - \frac{g\sec^2(\theta)}{2V^2}x^2$.

2 marks

When a projectile is fired horizontally with a speed of U m/s from the top of a cliff of height h metres above sea level, the projectile hits a stationary target in the water. In addition, if the projectile is fired from the same position with a speed of U m/s, but with an angle of elevation of $\tan^{-1}(3)$, the projectile also hits the target.

- c. Find, in terms of U and g , an expression for the horizontal distance, x metres, travelled by the projectile.

3 marks

Question 8 (3 marks)

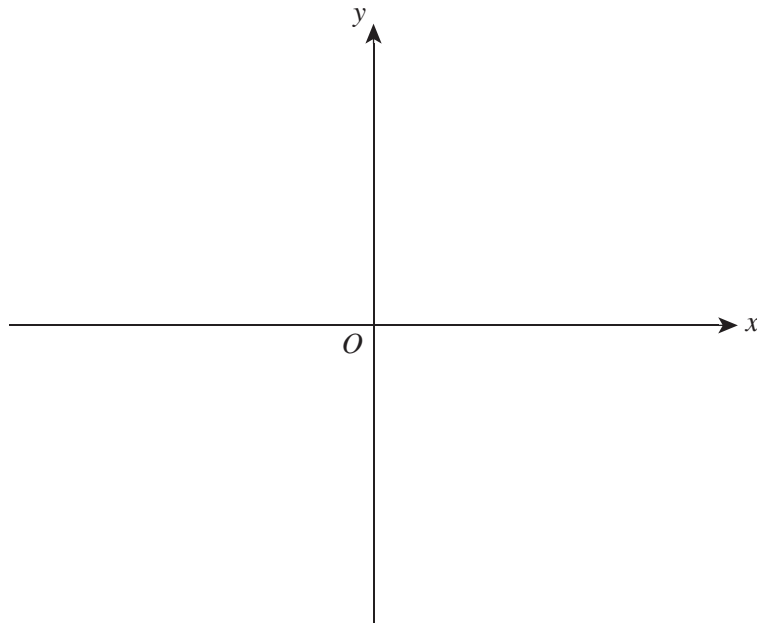
Show that the vectors $\vec{a} = \vec{i} + 2\vec{j}$, $\vec{b} = -\vec{i} + 5\vec{k}$ and $\vec{c} = \vec{i} + 6\vec{j} + 10\vec{k}$ are linearly dependent.

Question 9 (7 marks)

The curve C has equation $y = \frac{x^2}{x-2}$.

- a. On the set of axes below, sketch C , labelling the equations of any asymptotes and the coordinates of any axial intercepts and turning points.

4 marks



- b. Hence, find the set of values for p such that the equation $x^2 = p(x^2 - 4)$ has no real roots. 3 marks

END OF QUESTION AND ANSWER BOOKLET