

The Mathematical Association of Victoria

Trial Exam 2016

SPECIALIST MATHEMATICS

Written Examination 1

STUDENT NAME _____

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of Book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 15 pages
- Formula sheet
- Working space is provided throughout the book.

Instructions

- Write your **name** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Question 2 (5 marks)

The vector $\vec{a} = -\vec{i} - 2\sqrt{3}\vec{j} - \sqrt{2}\vec{k}$, where $\vec{i}$, $\vec{j}$ and $\vec{k}$ are unit vectors in the positive directions of the x , y and z axes respectively.

- a.** Find the vector of length $\sqrt{45}$ in the direction of $\vec{a}$. 1 mark

- b.** The vector $\vec{b} = 2\vec{i} - m\vec{j} - 3\sqrt{2}\vec{k}$ where $m \in \mathbb{R}$. Find all possible values of m if:

- i.** $|\vec{b}| = 5$. 2 marks

ii. $\vec{b}$ is perpendicular to $\vec{a}$.

2 marks

TURN OVER

Solve $z^3 - (2-i)z^2 = z - 2 + i$ where $z \in \mathbb{C}$.

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page. There are no margins, text, or other markings on the paper.

Solve $\frac{dy}{dx} = \frac{y-1}{x+1}$ for y , given that $y = \frac{1}{2}$ when $x = -2$.

[illegible]

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Question 5 (4 marks)

The velocity $v \text{ ms}^{-1}$ of a body moving in a straight line in terms of its position $x \text{ m}$ is given by the relation $2v^2 - 3vx - 2x^2 = 1$ for $x \geq 0$.

Find the acceleration of the body when $x = \frac{1}{5}$ and $v > 0$.

[illegible]

Question 6 (5 marks)

The position of a particle at time t seconds is given by $\vec{r} = t^2 \vec{i} + \frac{4t^2}{5} \sqrt{t} \vec{j}$, $t \geq 0$, where components are measured in metres.

- a. Find the velocity of the particle when it has an acceleration of $2\vec{i} + 2\vec{j} \text{ ms}^{-2}$.

2 marks

TURN OVER

3 marks

[illegible]

Question 7 (5 marks)

Galaxian Highscore and Max Hardcase regularly run a distance of 5 km as part of a fitness program. The time it takes Galaxian to run 5 km is normally distributed with a mean of 31 minutes and a standard deviation of 3 minutes. The time it takes Max to run 5 km is normally distributed with a mean of 33 minutes and a standard deviation of 4 minutes. The times run by Galaxian and Max are independent.

Let the random variable X represent the time it takes Galaxian to run 5 km and let the random variable Y represent the time it takes Max to run 5 km.

- a. Find the standard deviation of the difference between the times it takes Max and Galaxian to run 5 km.

2 marks

Max Hardcase decides to try and decrease his running times by also doing some high intensity interval training. Max finds that for his next 25 runs over 5 km, the mean time is 31 minutes. Assume that the standard deviation of 4 minutes is unchanged.

- b. i. State appropriate null and alternative hypotheses for the running time T of Max in this situation.

1 mark

- ii. The p -value for this test is given by the expression $\Pr(Z \leq a \mid H_0)$, where Z has the standard normal distribution.

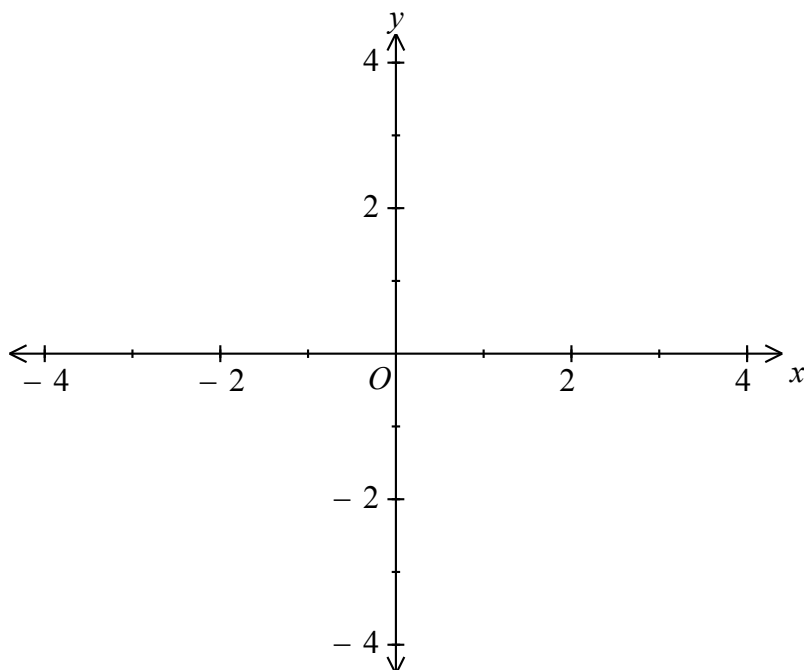
Find the value of a and **hence** determine whether or not the null hypothesis should be rejected at the 0.05 level of significance.

2 marks

TURN OVER

- b. Sketch a graph of $y = f(x)$. Label all asymptotes with their equation and all stationary points with their coordinates.

3 marks

**Working space**

- c.** Hence find the implied domain of $g(f(x))$.

3 marks

[illegible]

- d.** Find the implied range of $g(f(x))$.

2 marks

[illegible]

END OF QUESTION AND ANSWER BOOK