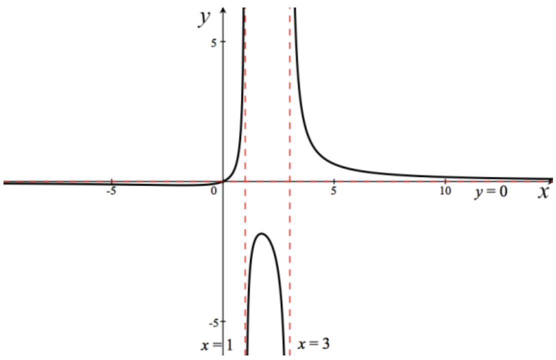
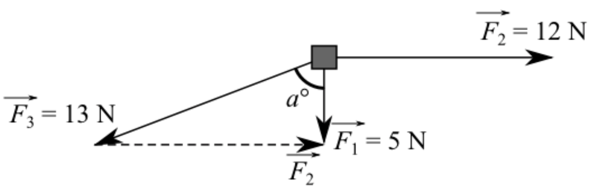
SPECIALIST MATHEMATICS Units 3 & 4

Written examination 2 Solutions

SECTION 1

Question	Answer	Solution
1	C	$f(x)=\dfrac{x}{x^2-4x+3}$ $=\dfrac{x}{(x-1)(x-3)}$ 
2	D	Let $u=\sqrt{x-1}$ $\Rightarrow x-1=u^2$ $x=u^2+1$ $dx=2u\,du$ $\int_2^3 x\sqrt{x-1}\,dx=\int_1^{\sqrt{2}}(u^2+1)\times u\times 2u\,du$ $=\int_1^{\sqrt{2}}2u^2(u^2+1)\,du$ The terminals of the integral become: When $x = 2$, $u = 1$. When $x = 3$, $u = \sqrt{2}$.
3	B	$V=\pi\int_0^1\left(x\sqrt{x}\right)^2dx$ $=\dfrac{\pi}{4}$
4	C	The three forces are in equilibrium, $\vec{F_1}+\vec{F_2}+\vec{F_3}=0$.  Use the converse of Pythagoras theorem to show that the triangle formed by the three forces is a right-angled triangle. $\left\|\vec{F_1}\right\|^2+\left\|\vec{F_2}\right\|^2=\left\|\vec{F_3}\right\|^2$ $5^2+12^2=13^2$ $25+144=169$

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$169 = 169 \Rightarrow$ The angle between $\overrightarrow{F}_1$ and $\overrightarrow{F}_2$ is 90°.

Use the cosine rule to calculate angle $a°$.

$$\cos(a°) = \frac{\left|\overrightarrow{F}_1\right|^2 + \left|\overrightarrow{F}_3\right|^2 - \left|\overrightarrow{F}_2\right|^2}{2\left|\overrightarrow{F}_1\right|\left|\overrightarrow{F}_3\right|}$$

$$= \frac{25 + 169 - 144}{2 \times 5 \times 13}$$

$$= \frac{5}{13}$$

$$a° = \cos^{-1}\left(\frac{5}{13}\right)$$

$$= 67.4°$$

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5

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E

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$$\frac{dy}{dx} = \frac{x}{y}$$

$$xdx = ydy$$

$$\int x\,dx = \int y\,dy$$

$$\frac{x^2}{2} + c_1 = \frac{y^2}{2} + c_2$$

$x^2 = y^2 + c$, where $c = 2(c_2 - c_1)$

When $x = 0$, $y = 0 \Rightarrow c = 0$ and $y^2 = x^2$.

$\Rightarrow$ first statement is true

There are no restrictions on the domain, therefore x can take negative values also.

$\Rightarrow$ second statement is false

There are no asymptotes for $y = |x|$.

$\Rightarrow$ third statement is false

To have a stationary point at $x = 0$, $\frac{dy}{dx} = \frac{x}{y} = 0$. However, when $x = 0$, $y = 0$, therefore the derivative function is undefined at $x = 0$.

$\Rightarrow$ fourth statement is false

$$\frac{d^2y}{dx^2} = \frac{y - y'x}{y^2}$$

$$= \frac{y - \frac{x^2}{y}}{y^2}$$

$$= \frac{y^2 - x^2}{y^3}$$

$= 0 \ \forall x \neq 0$ since $y^2 = x^2$

The second derivative is undefined at $x = 0$.

$\Rightarrow$ the fifth statement is false

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6	**A**	Two vectors are linearly dependent if they have the same direction. $\boldsymbol{a} = k\boldsymbol{b},\ k \in R$ $m\boldsymbol{i} + n\boldsymbol{j} = kp\boldsymbol{i} + kq\boldsymbol{j}$ $m = kp$ and $n = kq$ $\frac{m}{p} = \frac{n}{q} = k$
7	**B**	$E(3Y - 2X) = 25$ becomes $3E(Y) - 2E(X) = 25$ $E(4X - Y) = 18$ becomes $4E(X) - E(Y) = 18$ Solve the system of two simultaneous equations for $E(X)$ and $E(Y)$. $E(X) = 7.9,\ E(Y) = 13.6$ Substitute the values for $E(X)$ and $E(Y)$ into $E(-5X + Y)$. $E(-5X+Y) = -5E(X) + E(Y)$ $= -5 \times 7.9 + 13.6$ $= -25.9$
8	**D**	The maximal domain of $f(x)$ is determined by the intersection between the domains of the two terms of the function. $\frac{x}{1-x} \geq 0$ when $x \in [0,1)$ $4x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ $x \in \left(-\frac{\pi}{8}, \frac{\pi}{8}\right)$ The intersection of the two intervals is $\left[0, \frac{\pi}{8}\right)$.
9	**C**	$\cos(x) + \sin(x) = a,\ x \in [0, 2\pi]$ Multiply the equation by $\frac{\sqrt{2}}{2}$. $\frac{1}{\sqrt{2}}\cos(x) + \frac{1}{\sqrt{2}}\sin(x) = \frac{1}{\sqrt{2}}a$ $\sin\left(x + \frac{\pi}{4}\right) = \frac{a}{\sqrt{2}} \in [-1,1]$ $-1 \leq \frac{a}{\sqrt{2}} \leq 1$ ... multiply the equation by $\sqrt{2}$ $-\sqrt{2} \leq a \leq \sqrt{2}$ $a \in \left[-\sqrt{2}, \sqrt{2}\right]$

10	**E**	$\bar{x}=\dfrac{103.5+108.3}{2}=105.9$ The z score for the 95% confidence interval is z = 1.96. $\bar{x}+z\times\dfrac{s}{\sqrt{n}}=105.9+1.96\times\dfrac{s}{\sqrt{80}}$ $\Rightarrow\ 105.9+1.96\times\dfrac{s}{\sqrt{80}}=108.3$ $s=10.95217$ $s=11.0$ mm
11	**C**	Let $u=\dfrac{1}{5}x^2+\dfrac{4}{5}x$ $\dfrac{du}{dx}=\dfrac{2}{5}x+\dfrac{4}{5}\ \Rightarrow\ du=\left(\dfrac{2}{5}x+\dfrac{4}{5}\right)dx$ $du=\dfrac{2}{5}(x+2)dx$ $\dfrac{5}{2}du=(x+2)dx$ When x = 0, u = 0. When x = 1, u = 1. $\int_0^1 f\left(\dfrac{1}{5}x^2+\dfrac{4}{5}x\right)(x+2)dx=\int_0^1 f(u)\dfrac{5}{2}du$ $=\dfrac{5}{2}\int_0^1 f(u)du$ $=\dfrac{5}{2}A$
12	**C**	The period of the product function is a multiple of the periods of the two functions. $\text{Period}_{f(x)}=\dfrac{2\pi}{a}$ $\text{Period}_{g(x)}=\dfrac{\pi}{c}$ **Option A** ac cannot be a multiple of the period of $f(x)$ because $ac\div\dfrac{2\pi}{a}=\dfrac{a^2c}{2\pi}$. $\dfrac{ac^2}{2\pi}$ cannot be an integer regardless of the values of a and c. **Option B** Similarly, $2ac$ cannot be a multiple of the period of $f(x)$ because $2ac\div\dfrac{2\pi}{a}=\dfrac{a^2c}{\pi}$ is not an integer.

		Option C $\left.\begin{array}{l} 2\pi \div \dfrac{2\pi}{a} = a \\ 2\pi \div \dfrac{\pi}{c} = 2c \end{array}\right\} \Rightarrow 2\pi$ could be the period of $h(x) \Rightarrow 2\pi$ could be the period of $h(x)$ **Option D** $\dfrac{2\pi}{ac}$ cannot be a multiple of the period of $f(x)$ because $2ac \div \dfrac{2\pi}{ac} = \dfrac{a^2c^2}{\pi}$ is not an integer. **Option E** $\dfrac{\pi}{ac}$ cannot be a multiple of the period of $f(x)$ because $2ac \div \dfrac{\pi}{ac} = \dfrac{2a^2c^2}{\pi}$ is not an integer.
13	**E**	Determine the coordinates of the point of intersection between the two functions, to the right of the y – axis. $f(x) = g(x)$ $e^x - x = 3$ $x = 1.5052415$ The volume generated by the rotation about the x – axis is given by the formula $V = \pi\int_a^b \left[g(x) - f(x)\right]^2 dx$ $= \pi\int_0^{1.505} \left(3 - e^x + x\right)^2 dx$ $= 11.14$
14	**A**	If the particle is in equilibrium, the $\boldsymbol{F_1} + \boldsymbol{F_2} + \boldsymbol{F_3} = 0$ $2\boldsymbol{i} + 3\boldsymbol{j} - \boldsymbol{i} - 4\boldsymbol{j} + \boldsymbol{F_3} = 0$ $\boldsymbol{F_3} = -\boldsymbol{i} + \boldsymbol{j}$ $\lvert \boldsymbol{F_3} \rvert = \sqrt{2}$
15	**B**	A confidence interval is given by $\left(\bar{x} - z\dfrac{s}{\sqrt{n}},\ \bar{x} + z\dfrac{s}{\sqrt{n}}\right)$ $\dfrac{z}{\sqrt{200}} = 0.1386$ $z = 0.1386 \times \sqrt{200}$ $= 1.96$ $z = 1.96$ corresponds to a 95% confidence interval

16	**B**	$z=\left(\sqrt{3}-i\right)\left(1+i\right)\left(-2i\right)$ $=2\text{cis}\left(-\frac{\pi}{6}\right)\times\sqrt{2}\text{cis}\left(\frac{\pi}{4}\right)\times 2\text{cis}\left(-\frac{\pi}{2}\right)$ $=4\sqrt{2}\text{cis}\left(-\frac{\pi}{6}+\frac{\pi}{4}-\frac{\pi}{2}\right)$ $=4\sqrt{2}\text{cis}\left(-\frac{5\pi}{12}\right)$
17	**D**	**Option A** $\boldsymbol{u}\cdot\boldsymbol{v}$ $=(2a\boldsymbol{i}+b\boldsymbol{j})\cdot(b\boldsymbol{i}+a\boldsymbol{j})$ $=2ab+ab$ $=3ab$ … true **Option B** Vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ are are collinear therefore there is at least one real number $k\neq 0$ such that $\boldsymbol{v}=k\boldsymbol{u}$. $\boldsymbol{v}=k\boldsymbol{u}\Rightarrow 2a\boldsymbol{i}+b\boldsymbol{j}=k(b\boldsymbol{i}+a\boldsymbol{j})$ Equate corresponding coefficients. $b=2ka$ … [1] $a=kb$ … [2] From equation [2], $k=\frac{a}{b}$. Substitute the expression for a into equation [1]. $b=2\times\frac{a}{b}\times a\Rightarrow b^2=2a^2$ ... true **Option C** From $b^2=2a^2, \frac{b^2}{a^2}=2$ $\Rightarrow \frac{b}{a}=\pm\sqrt{2}$ … true **Option D** $\lvert\boldsymbol{u}\rvert=\sqrt{(2a)^2+b^2}$ $=\sqrt{4a^2+b^2}$ $\lvert\boldsymbol{v}\rvert=\sqrt{a^2+b^2}$ $\lvert\boldsymbol{u}\rvert\neq\lvert\boldsymbol{v}\rvert$ … incorrect option **Option E** $\lvert\boldsymbol{v}\rvert=\sqrt{a^2+b^2}$ … true

18	**E**	**Option A** $z = \text{cis}(\theta) = \cos(\theta) + i\sin(\theta)$ $\bar{z} = \cos(\theta) - i\sin(\theta)$ $= \cos(-\theta) + i\sin(-\theta)$ $= \text{cis}(-\theta)$ ... true **Option B** $z^2 = [\text{cis}(\theta)]^2$ $= \text{cis}(2\theta)$ … true **Option C** $\lvert \bar{z} \rvert = \sqrt{\cos^2(\theta) + \sin^2(\theta)}$ $= 1$ ... true **Option D** $\dfrac{z}{\bar{z}} = \dfrac{\text{cis}(\theta)}{\text{cis}(-\theta)}$ $= \text{cis}(\theta + \theta)$ $= \text{cis}(2\theta)$ ... true **Option E** $z\bar{z} = \text{cis}(\theta)\text{cis}(-\theta)$ $= \text{cis}(\theta - \theta)$ or $z\bar{z} = \left(\lvert z \rvert\right)^2$ … false $= \text{cis}(0)$ $\quad = 1$ $= 1$
19	**B**	$a = 2v^2 - v$, where $a = \dfrac{dv}{dt} = 2v^2 - v$ $\dfrac{dv}{2v^2 - v} = dt$ Using partial fractions, $\dfrac{1}{2v^2 - v} = \dfrac{1}{v(2v - 1)}$ $= \dfrac{2}{2v - 1} - \dfrac{1}{v}$ $\int_1^v \left(\dfrac{2}{2v - 1} - \dfrac{1}{v} \right) dv = \int_0^t dt$ $\left[\log_e(2v - 1) - \log_e(v) \right]_1^v = t$ $\log_e\left(\dfrac{2v - 1}{v} \right) = t$ $\dfrac{2v - 1}{v} = e^t$ $2v - 1 = ve^t$ $2v - ve^t = 1$ $v = \dfrac{1}{2 - e^t}$

20	**C**	$F = ma \Rightarrow a = \dfrac{16}{m}$ m/s^2 $s = \dfrac{1}{2}(u + v)$ $12 = \dfrac{1}{2}(u + 20)$ $\Rightarrow u = 4$ m/s $v = u + at$ $20 = 4 + \dfrac{16}{m} \times 4$ $\Rightarrow m = 4$ kg
21	**C**	The new random variable is (100% + 30%) of X plus an extra \$2. (100 + 30) % = 130% = 1.3 Therefore, the random variable for the new charges is $1.3X + 2$.
22	**B**	$z_1 = 1 \Rightarrow 1 - a + b = 0$ $\Rightarrow a = 1 + b \ldots [1]$ $z_2 = 1 - i \Rightarrow (1 - i)^4 - a(1 - i) + b = 0$ $\Rightarrow -4 - a + ai + b = 0 \ldots [2]$ Substitute [1] into [2]. $-4 - (1 + b) + (1 + b)i + b = 0$ $-4 - 1 - b + i + bi + b = 0$ $-5 + i + bi = 0$ $bi = 5 - i$ $b = 5i + 1$ $a = 1 + b$ $= 2 + 5i$

SECTION 2

Question 1 (12 marks)

a.

$a=\frac{1}{6},\ b=\frac{1}{3}$ **1A**

b.

$E(x)=1\times\frac{1}{6}+2\times\frac{1}{3}+3\times\frac{1}{2}$ **1M**

$E(x)=\frac{7}{3}$ **1A**

c.

$\text{Var}(X)=E\left(X^2\right)-E\left(X\right)^2$ **1M**

$\text{Var}(X)=1^1\times\frac{1}{6}+2^2\times\frac{1}{3}+3^2\times\frac{1}{2}-\left(\frac{7}{3}\right)^2$

$=\frac{36}{6}-\frac{49}{9}$ **1A**

$=\frac{5}{9}$

d.

y	2	3	4	5	6
$\Pr(Y=y)$	$\frac{1}{36}$	$\frac{1}{9}$	$\frac{5}{18}$	$\frac{1}{3}$	$\frac{1}{4}$

1A

$\Pr(Y=2)=\frac{1}{6}\times\frac{1}{6}$ $\qquad$ $\Pr(Y=4)=\frac{1}{3}\times\frac{1}{3}+\frac{1}{6}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{6}$ **1M**

$=\frac{1}{36}$ $\qquad$ $=\frac{5}{18}$

e.

$E(Y)=2\times\frac{1}{36}+3\times\frac{1}{9}+4\times\frac{5}{18}+5\times\frac{1}{3}+6\times\frac{1}{4}$

$=\frac{14}{3}$ **1A**

$E\left(Y^2\right)=4\times\frac{1}{36}+9\times\frac{1}{9}+16\times\frac{5}{18}+25\times\frac{1}{3}+36\times\frac{1}{4}$

$=\frac{1}{9}+1+\frac{40}{9}+\frac{25}{3}+9$ **1A**

$=\frac{206}{9}$

$$E(Y)^2 + E(Y) - \frac{E(Y^2)}{2} = \left(\frac{14}{3}\right)^2 + \frac{14}{3} - \frac{1}{2} \times \frac{206}{9}$$

$$= \frac{196}{9} + \frac{14}{3} - \frac{103}{9}$$

$$= \frac{93}{9} + \frac{42}{9}$$ **1A**

$$= \frac{135}{9}$$

$$= 15 \text{ ... as required}$$

f.

Median occurs at 0.5. **1M**

$$\frac{1}{4} = 0.25$$

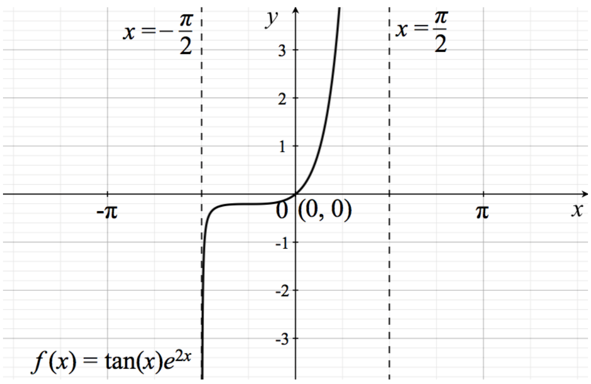
$$\frac{1}{3} + \frac{1}{4} = \frac{7}{12} = 0.58$$

The median is 5. **1A**

Alternatively, $\frac{1}{36} + \frac{1}{9} + \frac{5}{18} = 0.42$ and $\frac{1}{36} + \frac{1}{9} + \frac{5}{18} + \frac{1}{3} = 0.75$.

Question 2 (12 marks)

a.



1A

The x and y intercepts are both at (0, 0).

Two vertical asymptotes: $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$ **1A**

b.

$$f(x) = \tan(x)e^{2x}$$

$$f'(x) = \frac{1}{\cos^2(x)}e^{2x} + \tan(x) \times 2e^{2x}$$

$$= \frac{1}{\cos^2(x)}e^{2x} + 2\tan(x)e^{2x}$$

$$= \frac{1}{\cos^2(x)}e^{2x} + 2f(x)$$

1A

$f''(x) = -2\cos^{-3}(x)(-\sin(x))e^{2x} + \dfrac{1}{\cos^2(x)} \times 2e^{2x} + 2f'(x)$ **1A**

$f''(x) = 2\tan(x)\sec^2(x)e^{2x} + 2\sec^2(x)e^{2x} + 2f'(x)$

$f''(x) = 2f'(x) + 2\sec^2(x)[f(x) + e^{2x}]$ … as required **1A**

c.

Points of inflection occur when $\dfrac{d^2y}{dx^2} = 0$. **1A**

The point of inflection has coordinates (–0.785, –0.208).

To one decimal place, (–0.8, –0.2). **1A**

d.

When $x = \dfrac{\pi}{4}$, $y = f\left(\dfrac{\pi}{4}\right)$ **1A**

$$= \tan\left(\frac{\pi}{4}\right)e^{2\times\frac{\pi}{4}}$$

$$= e^{\frac{\pi}{2}}$$

$\left.\dfrac{dy}{dx}\right|_{x=\frac{\pi}{4}} = 4e^{\frac{\pi}{2}}$ **1A**

e.

The equation of the tangent at $x = \dfrac{\pi}{4}$ is

$y - e^{\frac{\pi}{2}} = 4e^{\frac{\pi}{2}}\left(x - \dfrac{\pi}{4}\right)$

$y = e^{\frac{\pi}{2}} + 4e^{\frac{\pi}{2}}x - \dfrac{\pi}{4}4e^{\frac{\pi}{2}}$ **1M**

$y = 4e^{\frac{\pi}{2}}x + e^{\frac{\pi}{2}}(1 - \pi)$

$m = 4e^{\frac{\pi}{2}},\ c = e^{\frac{\pi}{2}}(1 - \pi)$ **2A**

Question 3 (12 marks)

a.

$|z + 4i - 1| = |x + iy + 4i - 1|$

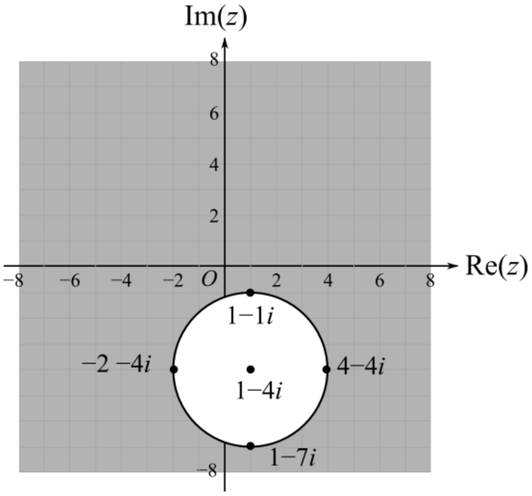
$= \sqrt{(x-1)^2 + (y+4)^2}$ **1M**

$\sqrt{(x-1)^2 + (y+4)^2} = 3$ ... square both sides of the equation

$(x-1)^2 + (y+4)^2 = 9$

The set of complex numbers P represents a circle with radius 3 and centre (1, –4). **1A**

b.



1A + 1A

The shaded region satisfies the conditions given including the boundary of the circle.

c.

The solutions of the equation $|z + 2 + 4i| = |z - 4 + 4i|$ represent the locus of points at the same distance from $-2 - 4i$ and $4 - 4i$.

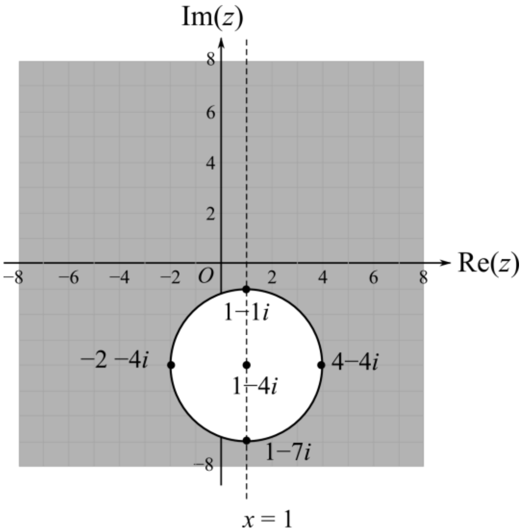
The locus of points is the median bisector of the line segment passing through $(-2, -4)$ and $(4, -4)$ with equation $x = 1$. **1A**

The points needed are in fact the solutions of the system of equations

$$\begin{cases} x = 1 \\ (x-1)^2 + (y+4)^2 = 9 \end{cases} \Rightarrow \begin{cases} x = 1,\ y = -7 \\ x = 1,\ y = -1 \end{cases}$$

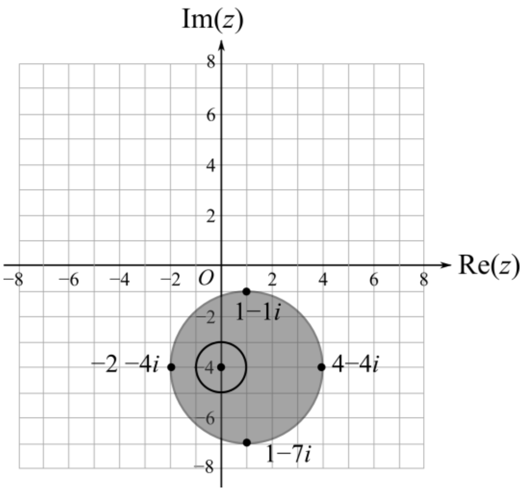
$z_1 = 1 - 7i$

$z_2 = 1 - i$



1A

d.



1A

e. i.

Solve the system of simultaneous equations

$$\begin{cases}(x-1)^2+(y+4)^2=9 \;...\; [1]\\(x-0)^2+(y+4)^2=c \;...\; [2]\end{cases}$$ … subtract equation [2] from [1] **1M**

$$(x-1)^2-x^2=9-c$$

$$x^2-2x+1-x^2=9-c$$

$$-2x=8-c$$ **1A**

$$x=\frac{c}{2}-4$$

Substitute the value of *x* into equation [2].

$$\left(\frac{c}{2}-4\right)^2+(y+4)^2=c$$

$$(y+4)^2=c-\frac{c^2}{4}+4c-16$$

$$(y+4)^2=5c-\frac{c^2}{4}-16$$

$$y+4=\pm\sqrt{5c-\frac{c^2}{4}-16}$$

$$y=-4\pm\sqrt{5c-\frac{c^2}{4}-16}$$

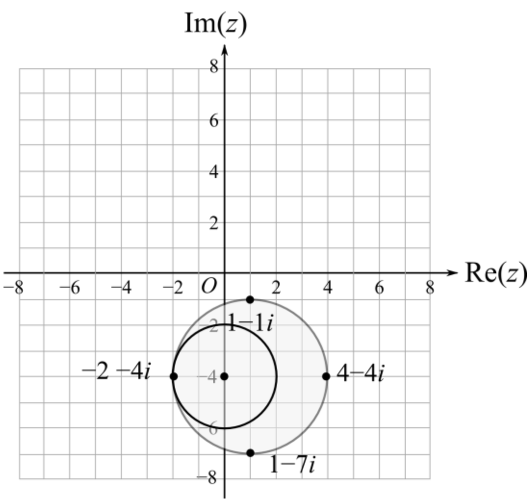
The second circle has to be inside the first circle. Therefore, $y=-4+\sqrt{5c-\frac{c^2}{4}-16}$ is the expression that satisfies this condition. **1A**

e. ii.

For the largest value of *c*, the two circles must only have one point in common. The centre of the inside circle is (0, –4), therefore its maximum radius is 2.

$c = 2^2 = 4$. **1A**

e. iii.



1A

Question 4 (9 marks)

a.

The object moves under constant acceleration.

$v = u + at$, where $u = 0$, $t = 10$ s and $a = 3$ m/s^2.

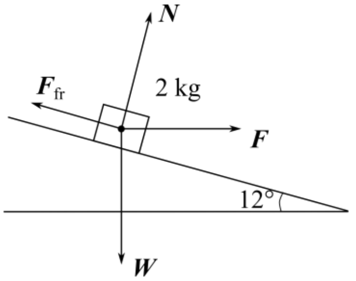
$v = 0 + 3 \times 10 = 30$ m/s **1A**

b.

$$s = \frac{1}{2}(u+v)t$$
$$= \frac{1}{2} \times 30 \times 10$$
$$= 150 \text{ m}$$

1A

c.



2A

d.

The object is moving under constant deceleration (negative acceleration) to come to a stop.

$u = 30\cos(12°)$ is the velocity at the end of the horizontal surface and the horizontal component at the beginning of the inclined plane.

$v = 0$ (at the end of the motion on the inclined plane)

$v^2 = u^2 + 2as$

$0 = [30\cos(12°)]^2 + 2a \times 25$

$a = -17.22$ ms^{-2} **1M + 1A**

e.

The horizontal components: $2a = F\cos(12°) + 2g\cos(78°) - F_{\text{fr}}$ **1A**

The vertical components: $N + F\sin(12°) = 2g\cos(12°)$ **1A**

$$\begin{cases} 2a = F\cos(12°) + 2g\cos(78°) - F_{\text{fr}} \\ N + F\sin(12°) = 2g\cos(12°) \end{cases}$$

$$\begin{cases} 2 \times (-17.22) = 6\cos(12°) + 2g\cos(78°) - \mu\left(2g\cos(12°) - 6\sin(12°)\right) \\ N = 2g\cos(12°) - 6\sin(12°) \end{cases}$$

$$\mu = \frac{-2 \times (-17.22) + 6\cos(12°) + 2g\cos(78°)}{2g\cos(12°) - 6\sin(12°)}$$ **1A**

$$= 2.48$$

Question 5 (13 marks)

a.

At $t = 0$, $\boldsymbol{r}(0) = 0$.

$$\begin{cases} 3\sin(0) - \sqrt{3}\cos(0) + a = 0 \\ -\sqrt{3}\sin(0) - 3\cos(0) + b = 0 \end{cases}$$ **1M**

$$\begin{cases} -\sqrt{3} + a = 0 \\ -3 + b = 0 \end{cases}$$

$a = \sqrt{3}$ and $b = 3$ **1A**

b.

$$\left|\boldsymbol{r}(t)\right| = \left|\left[3\sin(t) - \sqrt{3}\cos(t) + \sqrt{3}\right]\boldsymbol{i} + \left[-\sqrt{3}\sin(t) - 3\cos(t) + 3\right]\boldsymbol{j}\right|$$

$$= \sqrt{\left[3\sin(t) - \sqrt{3}\cos(t) + \sqrt{3}\right]^2 + \left[-\sqrt{3}\sin(t) - 3\cos(t) + 3\right]^2}$$ **1M**

$$= \sqrt{-24\cos(t) + 14}$$

$$\sqrt{-24\cos(t) + 14} = 6$$

$$-24\cos(t) + 14 = 36$$

$$-24\cos(t) = 12$$

$$\cos(t) = -\frac{1}{2} \Rightarrow t = \left\{\frac{2p}{3}, \frac{4p}{3}\right\}$$

$t_1 = 2.09$ s and $t_2 = 4.19$ s **1A**

c.

$$\boldsymbol{r}(t) = \left[3\sin(t) - \sqrt{3}\cos(t) + \sqrt{3}\right]\boldsymbol{i} + \left[-\sqrt{3}\sin(t) - 3\cos(t) + 3\right]\boldsymbol{j}$$

$$\dot{\boldsymbol{r}}(t) = \left[3\cos(t) + \sqrt{3}\sin(t)\right]\boldsymbol{i} + \left[-\sqrt{3}\cos(t) + 3\sin(t)\right]\boldsymbol{j}$$ **1A**

$$\ddot{\boldsymbol{r}}(t) = \left[-3\sin(t) + \sqrt{3}\cos(t)\right]\boldsymbol{i} + \left[\sqrt{3}\sin(t) + 3\cos(t)\right]\boldsymbol{j}$$

$$\left|\ddot{\boldsymbol{r}}(t)\right| = \sqrt{\left[-3\sin(t) + \sqrt{3}\cos(t)\right]^2 + \left[\sqrt{3}\sin(t) + 3\cos(t)\right]^2}$$

$$= \sqrt{12\sin^2(t) + 12\cos^2(t)}$$ **1A**

$= \sqrt{12}$ ... as required

d.

$$\dot{\boldsymbol{r}}(t) = \left[3\cos(t) + \sqrt{3}\sin(t)\right]\boldsymbol{i} + \left[-\sqrt{3}\cos(t) + 3\sin(t)\right]\boldsymbol{j}$$

$$m = \sqrt{3^2 + \sqrt{3}^2}$$

$$= \sqrt{12}$$ **1A**

$$= 2\sqrt{3}$$

$a = \tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$ and $b = \tan^{-1}\left(-\frac{3}{\sqrt{3}}\right)$ **1A**

$= 0.52$ $\quad\quad = -1.05$

$\dot{\boldsymbol{r}}(t) = \left[2\sqrt{3}\cos(t+0.52)\right]\boldsymbol{i} - \left[2\sqrt{3}\cos(t-1.05)\right]\boldsymbol{j}$ **1A**

e.

$$\begin{cases} x = 3\sin(t) - \sqrt{3}\cos(t) + \sqrt{3} \\ y = -\sqrt{3}\sin(t) - 3\cos(t) + 3 \end{cases}$$

$$\begin{cases} x - \sqrt{3} = 3\sin(t) - \sqrt{3}\cos(t) \text{ ... multiply by } \sqrt{3} \\ y - 3 = -\sqrt{3}\sin(t) - 3\cos(t) \end{cases}$$

$$\begin{cases} \sqrt{3}x - 3 = 3\sqrt{3}\sin(t) - 3\cos(t) \text{ ... [1]} \\ y - 3 = -\sqrt{3}\sin(t) - 3\cos(t) \text{ ... [2]} \end{cases} \text{ ... subtract [1] - [2]}$$ **1M**

$\sqrt{3}x - 3 - y + 3 = 3\sqrt{3}\sin(t) + \sqrt{3}\sin(t)$

$\sin(t) = \frac{\sqrt{3}x - y}{4\sqrt{3}}$ ... rationalise the denominator

$\sin(t) = \frac{3x - \sqrt{3}y}{12}$ ... [3] **1A**

Substitute [3] in [2].

$y - 3 = -\sqrt{3} \times \frac{3x - \sqrt{3}y}{12} - 3\cos(t)$

$3\cos(t) = 3 - y - \sqrt{3} \times \frac{3x - \sqrt{3}y}{12}$

$\cos(t) = 1 - \frac{y}{3} - \frac{\sqrt{3}x - y}{12}$

$\cos(t) = \frac{12 - 4y - \sqrt{3}x + y}{12}$

$\cos(t) = \frac{12 - 3y - \sqrt{3}x}{12}$ ... [4] **1A**

$\sin^2(t) + \cos^2(t) = 1$

$$\left(\frac{3x - \sqrt{3}y}{12}\right)^2 + \left(\frac{12 - 3y - \sqrt{3}x}{12}\right)^2 = 1$$

$12x^2 + 12y^2 - 12\sqrt{3}xy - 72y - 24\sqrt{3}x = 0$

$x^2 + y^2 - \sqrt{3}xy - 6y - 2\sqrt{3}x = 0$ **1A**