

Year 2016

VCE

Specialist Mathematics

Trial Examination 1



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Victorian Certificate of Education 2016

STUDENT NUMBER

Figures
Words

Letter

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SPECIALIST MATHEMATICS

Trial Written Examination 1

Reading time: 15 minutes

Total writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 18 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.

Unless otherwise specified an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ m/s}^2$, where $g = 9.8$.

Question 1 (3 marks)

Resolve the vector $4\mathbf{i} - 2\mathbf{j} - 2\mathbf{k}$ into two vector components, one which is parallel to the vector $3\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}$ and one which is perpendicular to it.

Question 2 (3 marks)

Let $f(z) = z^4 - 6z^3 + 17z^2 - 30z + 60$, where $z \in \mathbb{C}$.

- a.** Given that $z = \sqrt{5} \operatorname{cis}\left(-\frac{\pi}{2}\right)$ is a solution of $f(z) = 0$, find a quadratic factor of $f(z)$. 1 mark

- b.** Hence find all the roots of $f(z) = 0$. 2 marks

Question 4 (5 marks)

A bowl is formed when the region bounded by the graph of $y = x^2 - 16$, the x -axis, the y -axis and the line $y = 20$, is rotated about the y -axis. Units are in centimetres.

- a.** Find the capacity of the bowl. 2 marks

- b.** If the bowl is being filled at a rate of $2 \text{ cm}^3/\text{sec}$, find the rate at which the height of the fluid in the bowl is rising at, when the height is 14 cm. 3 marks

Question 5 (5 marks)

a. Show that $\int \frac{\sin(2x)}{\cos^2(2x)} dx = \frac{1}{2} \sec(2x) + c$ 1 mark

A particle moves along a curve so that at time $t \geq 0$ its velocity vector is given by $\underline{v}(t) = 4 \tan(2t) \sec(2t) \underline{i} + 6 \sec^2(2t) \underline{j}$.

b.i. Given that $\underline{r}(0) = 3\underline{i} + 2\underline{j}$ find the position vector $\underline{r}(t)$. 2 marks

- ii. The particle moves along the curve from $t = 0$ to $t = \frac{\pi}{5}$.

This distance can be expressed in terms of t as the definite integral

$$\int_0^{\frac{\pi}{5}} p \sec(2t) \sqrt{q \tan^2(2t) + r} dt . \text{ Determine the values of } p, q \text{ and } r .$$

Do not attempt to evaluate the definite integral.

2 marks

Question 6 (4 marks)

Find the gradient of the normal to the curve defined by $y^2(4-x) = x^3$ at the point in the fourth quadrant where $x = 2$.

Question 7 (4 marks)

A teacher at a school wishes to test if the students in a certain year have above or below average intelligence. To do this he takes a random sample of 36 students and finds that their mean IQ score is 103. The mean population IQ is 100 with a standard deviation of 15. Perform a hypothesis test, at the 95% level of significance, stating the null and alternative hypothesis, calculating the test statistic and clearly stating your conclusions.

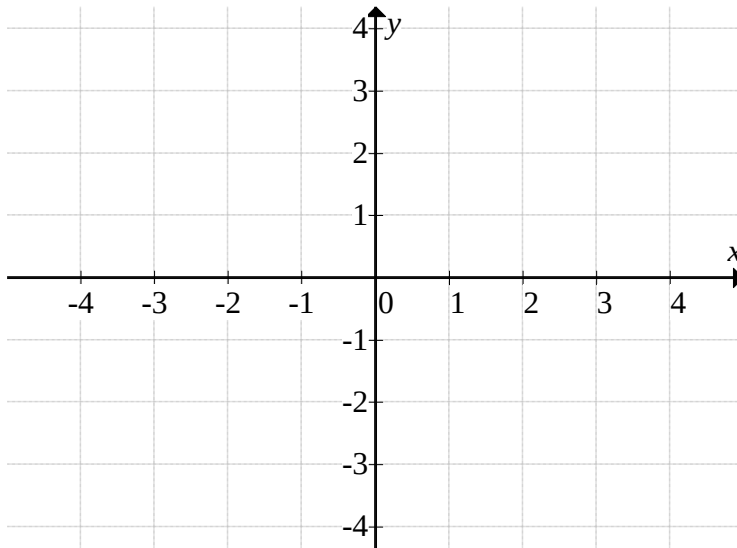
[illegible]

Question 9 (4 marks)

Consider the function $f(x) = 2\arcsin\left(\frac{x}{3}\right)$.

- a. Sketch the graph of the function on the axes below, clearly indicating the co-ordinates of the endpoints.

1 mark



- b. Find the inverse function.

1 mark

- c. Hence find the area in the first quadrant bounded by the function f and the x -axis.

2 marks

SPECIALIST MATHEMATICS

Written examination 1

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Specialist Mathematics formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder	$2\pi rh$
volume of a cylinder	$\pi r^2 h$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a pyramid	$\frac{1}{3}Ah$
area of triangle	$\frac{1}{2}bc \sin(A)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
cosine rule	$c^2 = a^2 + b^2 - 2ab \cos(C)$

Circular (trigonometric) functions

$\cos^2(x) + \sin^2(x) = 1$	
$1 + \tan^2(x) = \sec^2(x)$	$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$
$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$	$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$
$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$	$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$
$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$	$\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$
$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$	
$\sin(2x) = 2\sin(x)\cos(x)$	$\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$

Circular (trigonometric) functions - continued

Function	$\sin^{-1}$ (arcsin)	$\cos^{-1}$ (arccos)	$\tan^{-1}$ (arctan)
Domain	$[-1, 1]$	$[-1, 1]$	R
Range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (complex numbers)

$z = x + yi = r(\cos(\theta) + i \sin(\theta)) = r \operatorname{cis}(\theta)$	
$ z = \sqrt{x^2 + y^2} = r$	$-\pi < \operatorname{Arg}(z) \leq \pi$
$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$	$\frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$
$z^n = r^n \operatorname{cis}(n\theta)$ (de Moivre's theorem)	

Probability and statistics

for random variables X and Y	$E(aX + b) = aE(X) + b$ $E(aX + bY) = aE(X) + bE(Y)$ $\operatorname{Var}(aX + b) = a^2 \operatorname{Var}(X)$
for independent random variables X and Y	$\operatorname{Var}(aX + bY) = a^2 \operatorname{Var}(X) + b^2 \operatorname{Var}(Y)$
approximate confidence interval for μ	$\left(\bar{x} - z \frac{s}{\sqrt{n}}, \bar{x} + z \frac{s}{\sqrt{n}} \right)$
distribution of sample mean $\bar{X}$	mean $E(\bar{X}) = \mu$ variance $\operatorname{Var}(\bar{X}) = \frac{\sigma^2}{n}$

Vectors in two and three dimensions

$\underline{r} = x\hat{i} + y\hat{j} + z\hat{k}$
$ \underline{r} = \sqrt{x^2 + y^2 + z^2} = r$
$\dot{\underline{r}} = \frac{d\underline{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$
$\underline{r}_1 \cdot \underline{r}_2 = r_1 r_2 \cos(\theta) = x_1 x_2 + y_1 y_2 + z_1 z_2$

Mechanics

momentum	$\underline{p} = m\underline{v}$
equation of motion	$\underline{R} = m\underline{a}$

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$
$\frac{d}{dx}(e^{ax}) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$
$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e x + c$
$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$	$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$
$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$	$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$
$\frac{d}{dx}(\tan(ax)) = a \sec^2(ax)$	$\int \sec^2(ax) dx = \frac{1}{a} \tan(ax) + c$
$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c, a > 0$
$\frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^2}}$	$\int \frac{-1}{\sqrt{a^2-x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, a > 0$
$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$	$\int \frac{a}{a^2+x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$
	$\int (ax+b)^n dx = \frac{1}{a(n+1)} (ax+b)^{n+1} + c, n \neq -1$
	$\int (ax+b)^{-1} dx = \frac{1}{a} \log_e ax+b + c$
product rule	$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$
Euler's method	If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$
acceleration	$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$
arc length	$\int_{x_1}^{x_2} \sqrt{1+(f'(x))^2} dx \quad \text{or} \quad \int_{t_1}^{t_2} \sqrt{(x'(t))^2 + (y'(t))^2} dt$

END OF FORMULA SHEET