

2016 VCAA Specialist Mathematics Exam 2 Solutions

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SECTION A – Multiple-choice questions

1	2	3	4	5	6	7	8	9	10
E	B	C	E	A	C	A	D	C	B

11	12	13	14	15	16	17	18	19	20
D	C	A	B	D	D	E	A	B	E

Q1 $\operatorname{cosec}^2 t - \cot^2 t = 1, \frac{x}{3} - \frac{(y+1)^2}{16} = 1, (y+1)^2 = \frac{16(x-3)}{3}$ **E**

Q2 $-1 \leq \frac{x-a}{b} \leq 1, a-b \leq x \leq a+b$ **B**

Q3 $f(x) = x - \frac{a}{x}$ where $x \neq 0$, asymptotes are $y = x$ and $x = 0$ **C**

Q4 $z^3 + bz^2 + cz = z(z^2 + bz + c)$
Consider the quadratic: $c = \text{product of roots}$, $b = -(\text{sum of roots})$
 $\therefore c = (3-2i)(3+2i) = 13$ and $b = -6$ **E**

Q5 z is in the third quadrant. **A**

Q6 **C**

Q7 $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\cos 2t}{\cos t + \sin t} = \frac{(\cos t - \sin t)(\cos t + \sin t)}{\cos t + \sin t}$
 $= \cos t - \sin t$ **A**

Q8 $u = x^4, \frac{du}{dx} = 4x^3, \frac{1}{4} \frac{du}{dx} = x^3$
 $\int_a^b x^3 e^{2x^4} dx = \int_a^b \frac{1}{4} e^{2u} \frac{du}{dx} dx = \frac{1}{4} \int_a^{b^4} e^{2u} du$ **D**

Q9 $x = 2, y_0 = 0, \frac{dy}{dx} = f(2) = 6$

$x = 2.1, y_1 \approx 0 + 0.1 \times 6 = 0.6, \frac{dy}{dx} = f(2.1)$

$x = 2.2, y_2 \approx 0.6 + 0.1 \times f(2.1) = 1.272, \frac{dy}{dx} = f(2.2)$

$x = 2.3, y_3 \approx 1.272 + 0.1 \times f(2.2)$

Q10 **B**

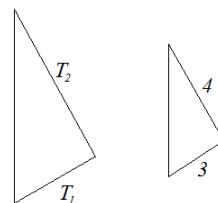
Q11 $\tilde{a}\tilde{b} = \frac{10 + \alpha^3}{\sqrt{17 + \alpha^4}} = \frac{74}{\sqrt{273}}, \alpha = 4$ **D**

Q12 $\tilde{a} - \tilde{b} = (-2+m)\tilde{i} - 2\tilde{j} + \tilde{k}$
 $(\tilde{a} - \tilde{b})\tilde{b} = ((-2+m)\tilde{i} - 2\tilde{j} + \tilde{k})(-m\tilde{i} + \tilde{j} + 2\tilde{k}) = 0$
 $-m(-2+m) - 2 + 2 = 0, \therefore m = 0 \text{ or } 2$ **C**



Q13 Net force $= \sqrt{9^2 + 12^2} = 15 = 5a, a = 3 \text{ ms}^{-2}$ **A**

Q14 $\frac{T_1}{T_2} = \frac{3}{4}$ **B**



Q15 $v = 3 - x^2, \frac{1}{2}v^2 = \frac{1}{2}(3 - x^2)^2,$
 $a = \frac{d}{dx}\left(\frac{1}{2}v^2\right) = -2x(3 - x^2) = 2x^3 - 6x, F = ma = 6x^3 - 18x$ **D**

Q16 Vertically: $u = 20 \sin 30^\circ = 10, v = -10, a = -g,$
 $t = \frac{v - u}{a} = \frac{20}{g}$

Horizontally, $u = 20 \cos 30^\circ = 10\sqrt{3}, s = ut = 10\sqrt{3} \times \frac{20}{g} = \frac{200\sqrt{3}}{g}$ **D**

Q17 $\Delta p = m(v - u) = 3(2 - (-2)) = 12 \text{ kg ms}^{-1}$ **E**

Q18 Assume that the fruits are randomly selected, $\therefore$ the masses of the fruits are independent.

Let random variable O be the mass of an orange and L be the mass of a lemon.

$E(O_1 + O_2 + O_3 + L_1 + L_2) = E(O_1) + E(O_2) + E(O_3) + E(L_1) + E(L_2)$
 $= 3 \times E(O) + 2 \times E(L) = 3 \times 204 + 2 \times 76 = 764$

$\text{Var}(O_1 + O_2 + O_3 + L_1 + L_2)$
 $= \text{Var}(O_1) + \text{Var}(O_2) + \text{Var}(O_3) + \text{Var}(L_1) + \text{Var}(L_2)$
 $= 3 \times \text{Var}(O) + 2 \times \text{Var}(L) = 3 \times 9^2 + 2 \times 3^2 = 261$

$\text{sd}(O_1 + O_2 + O_3 + L_1 + L_2) = \sqrt{261} = 3\sqrt{29}$ **A**

Q19 $\left(210 - 1.96 \times \frac{16}{\sqrt{100}}, 210 + 1.96 \times \frac{16}{\sqrt{100}}\right) \approx (206.9, 213.1)$ **B**

Q20 Normal distribution of the sample mean $\bar{X}$:

$E(\bar{X}) = \mu = 20, \text{sd}(\bar{X}) = \frac{\sigma}{\sqrt{n}} = \frac{2}{5}, \Pr(\bar{X} > 19.3) \approx 0.9599$ **E**

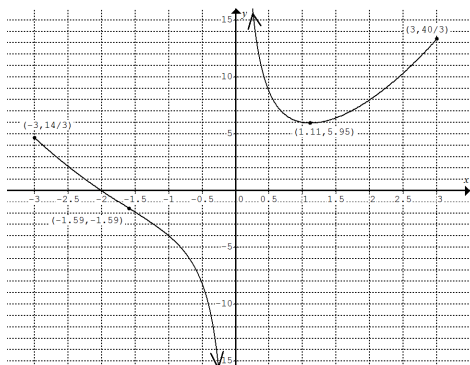


SECTION B

Q1a $y = \frac{4 + x^2 + x^3}{x}$, stationary point (1.11, 5.95) approx.

Q1b Find (x, y) such that $\frac{d^2y}{dx^2} = 0$, point of inflection
(-1.59, -1.59)

Q1c



Q1di $\int_{-3}^{-0.5} \sqrt{1 + (f'(x))^2} dx = \int_{-3}^{-0.5} \sqrt{1 + \left(\frac{2x^3 + x^2 - 4}{x^2}\right)^2} dx$

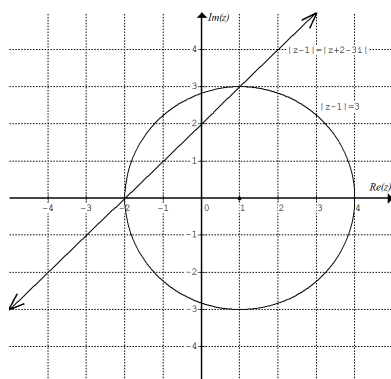
Q1dii 13.18 units

Q1e $a = \pi$, $b = f(-3) = \frac{14}{3}$, $c = f(-0.5) = -\frac{33}{4}$

Q2a Let $z = x + yi$, $|(x-1) + yi| = |(x+2) + (y-3)i|$
 $(x-1)^2 + y^2 = (x+2)^2 + (y-3)^2$, $y = x + 2$

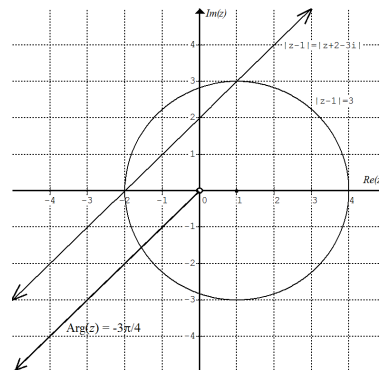
Q2b Circle $(x-1)^2 + y^2 = 9$, line $y = x + 2$
 $\therefore x = -2, 1$ and $y = 0, 3$ respectively
Points of intersection are $(-2, 0)$ and $(1, 3)$

Q2c



Q2d Area of major segment $= \frac{1}{2} \times 3 \times 3 + \frac{3}{4} \times \pi \times 3^2 = \frac{9(2 + 3\pi)}{4}$ unit²

Q2e



Q2f $\left(-1, -\frac{3}{4}\right) \cup \left(\frac{1}{4}, 1\right]$

Q3a $\frac{dx}{dt} = -\frac{x}{20+t}$, $x > 0$, $t \geq 0$

$\int \frac{1}{x} dx = \int -\frac{1}{20+t} dt$, $\log_e x = -\log_e (20+t) + c$

When $t = 0$, $x = 20$, $c = 2 \log_e 20 = \log_e 400$,

$\log_e x = \log_e 400 - \log_e (20+t)$ $\therefore x = \frac{400}{20+t}$

Q3b y kg of salt in $(100 + 10t)$ L of solution after t min

Concentration $= \frac{y}{100 + 10t}$ kg per L

Q3c Rate of inflow of salt $= \frac{1}{60} \times 20 = \frac{1}{3}$ kg per min

Rate of outflow of salt $= \frac{y}{100 + 10t} \times 10 = \frac{y}{10 + t}$ kg per min

$\therefore \frac{dy}{dt} = \frac{1}{3} - \frac{y}{10+t}$, $\therefore \frac{dy}{dt} + \frac{y}{10+t} = \frac{1}{3}$

Q3d $y = \frac{t^2 + 20t + 900}{6(10+t)}$, $\frac{y}{10+t} = \frac{t^2 + 20t + 900}{6(10+t)^2}$

$\frac{dy}{dt} = \frac{1}{6} \times \frac{(10+t)(2t+20) - (t^2 + 20t + 900)}{(10+t)^2} = \frac{t^2 + 20t - 700}{6(10+t)^2}$

$\therefore \frac{dy}{dt} + \frac{y}{10+t} = \frac{1}{3}$

$y = \frac{t^2 + 20t + 900}{6(10+t)}$, when $t = 0$, $y = \frac{900}{60} = 15$

Q3e Concentration $= \frac{y}{100 + 10t} = \frac{y}{10(10+t)} = \frac{t^2 + 20t + 900}{60(10+t)^2} = 0.095$

$\therefore t \approx 3.05$ min



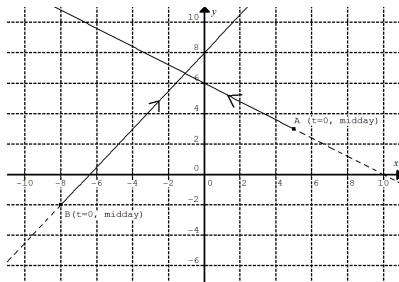
Q4a Consider the time when the ships have the same x -coordinate.

$$5(1-t) = 4(t-2), \quad t = \frac{13}{9}. \quad \text{At } t = \frac{13}{9}, \text{ } y\text{-coordinates of } A \text{ and } B \text{ are}$$

$$\text{different, } y_A = \frac{22}{3}, \quad y_B = \frac{47}{9}, \quad \therefore \text{the two ships will not collide.}$$

Q4b A: $x = 5(1-t), \quad y = 3(1+t)$, eliminate $t, \quad y = -\frac{3}{5}x + 6$

B: $x = 4(t-2), \quad y = 5t-2, \quad \therefore y = \frac{5}{4}x + 8$



Q4c Let α be the obtuse angle between the two paths, and the paths of A and B make angles θ and ϕ with the positive x -axis respectively. $\alpha = \theta - \phi$,

$$\tan \alpha = \frac{\tan \theta - \tan \phi}{1 + \tan \theta \tan \phi} = \frac{m_A - m_B}{1 + m_A m_B} = \frac{-\frac{3}{5} - \frac{5}{4}}{1 - \frac{3}{5} \times \frac{5}{4}} = -\frac{37}{5}, \quad \therefore \alpha \approx 97.7^\circ$$

Q4di $\tilde{r}_B - \tilde{r}_A = (9t-13)\tilde{i} + (2t-5)\tilde{j}$

$$|\tilde{r}_B - \tilde{r}_A|^2 = (9t-13)^2 + (2t-5)^2,$$

$$\frac{d}{dt}|\tilde{r}_B - \tilde{r}_A|^2 = 18(9t-13) + 4(2t-5) = 0, \quad t \approx 1.494$$

Q4dii When $t \approx 1.494$,

$$\text{min. distance} = |\tilde{r}_B - \tilde{r}_A| = \sqrt{(9t-13)^2 + (2t-5)^2} \approx 2.06 \text{ km}$$

Q5a $t \in [0, 5], \quad t = 0, \quad v = 0, \quad x = 0$

$$a = \frac{F}{m} = \frac{50-10t}{2} - g, \quad \frac{dv}{dt} = 25-9.8-5t, \quad \frac{dv}{dt} = \frac{76}{5} - 5t$$

Q5b $v = \frac{76}{5}t - \frac{5}{2}t^2, \quad v(5) = 13.5 \text{ ms}^{-1}$

Q5c $x = \frac{38}{5}t^2 - \frac{5}{6}t^3, \quad x(5) \approx 85.83, \quad \text{height} \approx 85.83 \text{ m}$

Q5d $u = 13.5, \quad a = -9.8, \quad v = 0, \quad s = \frac{v^2 - u^2}{2a} \approx 9.30$

$$\text{Maximum height} \approx 85.83 + 9.30 = 95.13 \text{ m}$$

Q5e After the initial 5 s, the time to reach the ground:

$$u = 13.5, \quad a = -9.8, \quad s = -85.83, \quad \therefore -85.83 = 13.5t + \frac{1}{2}(-9.8)t^2$$

$$t \approx 5.78$$

$$\text{Total time of flight} \approx 5 + 5.78 \approx 10.8 \text{ s}$$

Q6a Population: $\mu = 1.1 \text{ mg/L}, \quad \sigma = 0.16 \text{ mg/L}$

Samples: $n = 25$,

$$\text{Mean of } \bar{X} = E(\bar{X}) = \mu = 1.1, \quad \text{sd}(\bar{X}) = \frac{\sigma}{\sqrt{n}} = \frac{0.16}{5} = 0.032$$

Q6b H_0 : The mean level of pollutant remains the same.

H_1 : The mean level of pollutant has increased.

Q6ci $p \text{ value} = \Pr\left(Z \geq \frac{1.2-1.1}{0.032}\right) \approx 0.0009$

Q6cii Since $p < 0.05$, the sample supports H_1

Q6d $\Pr(\bar{X} > \bar{x}_c | \mu = 1.1) = 0.05$,

$$\Pr(\bar{X} < \bar{x}_c | \mu = 1.1) = \Pr\left(Z < \frac{\bar{x}_c - 1.1}{0.032}\right) = 0.95$$

$$\bar{x}_c \approx 1.153$$

Q6e $\Pr(\bar{X} < 1.163 | \mu = 1.2) = \Pr\left(Z < \frac{1.163-1.2}{0.032}\right) \approx 0.124$

Please inform mathline@itute.com re conceptual and/or mathematical errors