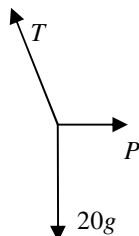


Q1a



Q1b $\sin \theta = \frac{1}{\frac{5}{3}} = \frac{3}{5}$

Q1c $T \cos \theta = 20g$, $T = \frac{20g}{\cos \theta} = \frac{20g}{\frac{4}{5}} = 25g = 245 \text{ N}$

Q2 $\sigma = \sqrt{16} = 4$, sample size $n = 25$,

sample mean $\bar{x} = \frac{2625}{25} = 105$ grams, $\therefore$ 95% confidence interval

$\approx \left(\bar{x} - 2 \frac{\sigma}{\sqrt{n}}, \bar{x} + 2 \frac{\sigma}{\sqrt{n}} \right) = \left(105 - \frac{8}{5}, 105 + \frac{8}{5} \right) = (103.4, 106.6)$

Q3 $\cos y + y \sin x = x^2$, $\frac{d}{dx}(\cos y + y \sin x) = \frac{d}{dx} x^2$

$-\sin y \frac{dy}{dx} + y \cos x + \sin x \frac{dy}{dx} = 2x$,

$(-\sin y + \sin x) \frac{dy}{dx} = 2x - y \cos x$, $\therefore \frac{dy}{dx} = \frac{2x - y \cos x}{\sin x - \sin y}$

At $\left(0, \frac{-\pi}{2} \right)$, $m_T = \frac{dy}{dx} = \frac{\frac{\pi}{2}}{1} = \frac{\pi}{2}$, $\therefore m_N = -\frac{2}{\pi}$

Equation of the normal at $\left(0, \frac{-\pi}{2} \right)$:

$y + \frac{\pi}{2} = -\frac{2}{\pi}x$, $y = -\frac{2}{\pi}x - \frac{\pi}{2}$

Q4 $x = \tan^{-1} t$, surface area $A(t) = 6(\tan^{-1} t)^2 \text{ mm}^2$,

$\frac{dA}{dt} = \frac{12 \tan^{-1} t}{1+t^2}$

When $t = 1$, $\frac{dA}{dt} = \frac{12 \tan^{-1} 1}{1+1^2} = 6 \times \frac{\pi}{4} = \frac{3\pi}{2} \text{ mm}^2/\text{day}$

Q5a $\hat{b} = \frac{1}{\sqrt{14}}(\tilde{i} - 2\tilde{j} + 3\tilde{k})$ Scalar resolute: $\tilde{a} \cdot \hat{b} = \frac{-13}{\sqrt{14}}$,

vector resolute: $(\tilde{a} \cdot \hat{b})\hat{b} = -\frac{13}{14}(\tilde{i} - 2\tilde{j} + 3\tilde{k})$

Q5b Linear dependent, let $\tilde{c} = m\tilde{a} + n\tilde{b}$

$\therefore 3m + n = 1 \dots\dots (1)$

$5m - 2n = 0 \dots\dots (2)$ and $-2m + 3n = d$

$(1) - (2): -2m + 3n = 1$, $\therefore d = 1$

Q6 $\frac{(1-\sqrt{3}i)^4}{1+\sqrt{3}i} = \frac{(2\text{cis}(-\frac{\pi}{3}))^4}{2\text{cis}\frac{\pi}{3}} = \frac{8\text{cis}(-\frac{4\pi}{3})}{\text{cis}\frac{\pi}{3}} = 8\text{cis}\left(-\frac{5\pi}{3}\right) = 8\text{cis}\frac{\pi}{3}$

$= 4 + 4\sqrt{3}i$



Q7 $\frac{dy}{dx} = x(x^2 + 2)^{\frac{1}{2}}$, arc length $= \int_0^2 \sqrt{1+x^2(x^2+2)} dx$

Q8a $\tilde{r} = (3 \sin 2t - 2)\tilde{i} + (3 - 2 \cos 2t)\tilde{j}$

$\tilde{v} = \frac{d\tilde{r}}{dt} = (6 \cos 2t)\tilde{i} + (4 \sin 2t)\tilde{j}$

speed $= |\tilde{v}| = \sqrt{(6 \cos 2t)^2 + (4 \sin 2t)^2} = \sqrt{36 \cos^2 2t + 16 \sin^2 2t}$

Q8b When $t = \frac{\pi}{12}$, speed $= 4\sqrt{\frac{5}{4}\left(\frac{\sqrt{3}}{2}\right)^2 + 1} = 4\sqrt{\frac{15}{16} + 1} = \sqrt{31} \text{ m/s}$

Q8c $\tilde{v} = (6 \cos 2t)\tilde{i} + (4 \sin 2t)\tilde{j}$,

$\tilde{a} = \frac{d\tilde{v}}{dt} = (-12 \sin 2t)\tilde{i} + (8 \cos 2t)\tilde{j}$

Net force $\tilde{F} = m\tilde{a} = (-36 \sin 2t)\tilde{i} + (24 \cos 2t)\tilde{j}$

$|\tilde{F}| = \sqrt{(-36 \sin 2t)^2 + (24 \cos 2t)^2} = 4\sqrt{81 \sin^2 2t + 36 \cos^2 2t}$

Q9 $\cos(x-y) = \cos x \cos y + \sin x \sin y = \frac{3}{5}$

$\tan x \tan y = \frac{\sin x \sin y}{\cos x \cos y} = 2$, $\therefore \sin x \sin y = 2 \cos x \cos y$

$\therefore 3 \cos x \cos y = \frac{3}{5}$, $\therefore \cos x \cos y = \frac{1}{5}$

$\cos(x+y) = \cos x \cos y - \sin x \sin y = -\cos x \cos y = -\frac{1}{5}$

Q10 $\sqrt{2-x^2} \frac{dy}{dx} = \frac{1}{2-y}$

$\int (2-y) dy = \int \frac{1}{\sqrt{2-x^2}} dx$, $\frac{(2-y)^2}{-2} = \sin^{-1}\left(\frac{x}{\sqrt{2}}\right) + c$

Given $y(1) = 0$, $\therefore -2 = \frac{\pi}{4} + c$, $c = -2 - \frac{\pi}{4}$

$\frac{(2-y)^2}{-2} = \sin^{-1}\left(\frac{x}{\sqrt{2}}\right) - 2 - \frac{\pi}{4}$, $(2-y)^2 = \frac{\pi}{2} + 4 - 2 \sin^{-1}\left(\frac{x}{\sqrt{2}}\right)$

$2-y = \sqrt{\frac{\pi}{2} + 4 - 2 \sin^{-1}\left(\frac{x}{\sqrt{2}}\right)}$ will satisfy $y(1) = 0$

$\therefore y = 2 - \sqrt{\frac{\pi}{2} + 4 - 2 \sin^{-1}\left(\frac{x}{\sqrt{2}}\right)}$

Please inform mathline@itute.com re conceptual and/or mathematical errors