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## **SPECIALIST MATHEMATICS UNITS 3 & 4**

### **TRIAL EXAMINATION 2**

**2016**

Reading Time: 15 minutes

Writing time: 2 hours

#### **Instructions to students**

This exam consists of Section A and Section B.  
Section A consists of 20 multiple-choice questions and should be answered on the detachable answer sheet which can be found on page 27 of this exam.  
Section B consists of 6 extended-answer questions.  
Section A begins on page 2 of this exam and is worth 20 marks.  
Section B begins on page 11 of this exam and is worth 60 marks.  
There is a total of 80 marks available.  
All questions in Section A and B should be answered.  
In Section B, where more than one mark is allocated to a question, appropriate working must be shown.  
An exact value is required to a question unless otherwise directed.  
Unless otherwise stated, diagrams in this exam are not drawn to scale.  
The acceleration due to gravity should be taken to have magnitude  $g \text{ m/s}^2$  where  $g = 9.8$   
Students may bring one bound reference into the exam.  
Students may bring into the exam one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory does not need to be cleared. For approved computer-based CAS, full functionality may be used.  
A formula sheet can be found on pages 24 - 26 of this exam.

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## SECTION A – Multiple-choice questions

### Question 1

The graph of  $y = \frac{x^2 - 3x - 4}{x^2 - x - 12}$  will include

- A. asymptotes at  $x = 1$  and  $x = -3$
- B. asymptotes at  $x = 4$  and  $x = -3$
- C. an asymptote at  $x = 1$  and a point of discontinuity at  $x = 2$
- D. an asymptote at  $x = 4$  and a point of discontinuity at  $x = -3$
- E. an asymptote at  $x = -3$  and a point of discontinuity at  $x = 4$

### Question 2

The domain of the function  $f(x) = \arccos(3 - x)$  is

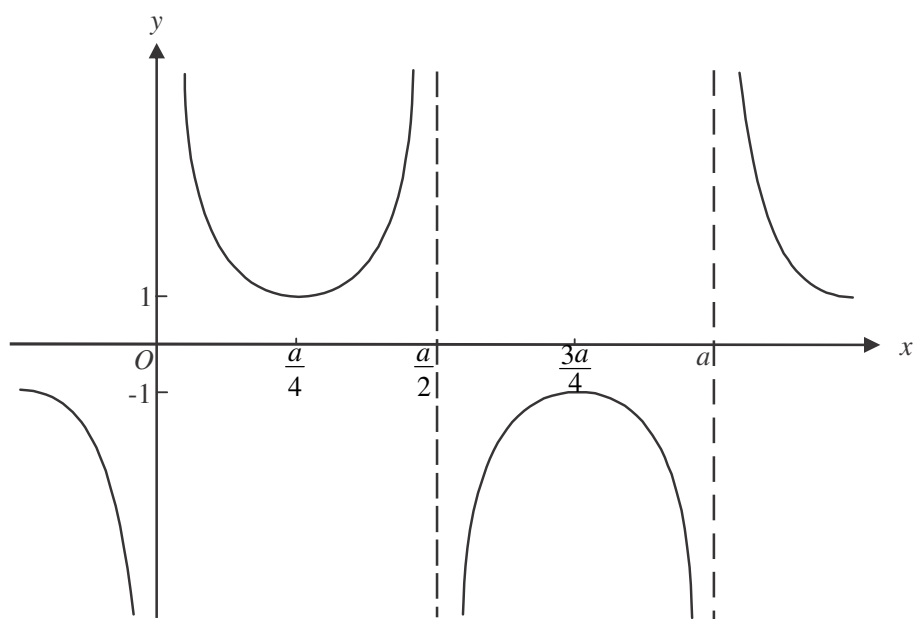
- A.  $[-1, 1]$
- B.  $[-4, 2]$
- C.  $[-2, 4]$
- D.  $[2, 4]$
- E.  $(-\infty, 2] \cup [4, \infty)$

### Question 3

The algebraic function  $\frac{2x^2 + 1}{(x^2 - 1)(x^2 + 4)}$  could be expressed as the partial fractions

- A.  $\frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x+2} + \frac{D}{x-2}$
- B.  $\frac{A}{x-1} + \frac{B}{(x+1)^2} + \frac{Cx+D}{x^2+4}$
- C.  $\frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+D}{x^2+4}$
- D.  $\frac{A}{x-1} + \frac{B}{x^2-1} + \frac{Cx+D}{x^2+4}$
- E.  $\frac{Ax^2+B}{x^2-1} + \frac{Cx+D}{x^2+4}$

### Question 4



A rule for the function shown above where  $a$  is a positive constant, could be

- A.  $y = \sec\left(\frac{2\pi}{a}\left(x - \frac{a}{4}\right)\right)$
- B.  $y = \sec\left(\frac{\pi}{a}\left(x - \frac{a}{4}\right)\right)$
- C.  $y = \sec\left(\frac{2\pi}{a}\left(x + \frac{a}{4}\right)\right)$
- D.  $y = \sec\left(\frac{\pi}{a}\left(x + \frac{a}{4}\right)\right)$
- E.  $y = \sec\left(\pi\left(x - \frac{a}{2}\right)\right)$

### Question 5

On an Argand diagram, the graphs of the relations  $|z - 2 - 2i| = 2$  and  $\text{Arg}(z) = \theta$  intersect. The maximum possible interval for values of  $\theta$  is

- A.  $-\pi < \theta \leq \pi$
- B.  $0 \leq \theta \leq \pi$
- C.  $0 < \theta < \frac{\pi}{2}$
- D.  $0 \leq \theta \leq \frac{\pi}{2}$
- E.  $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$

**Question 6**

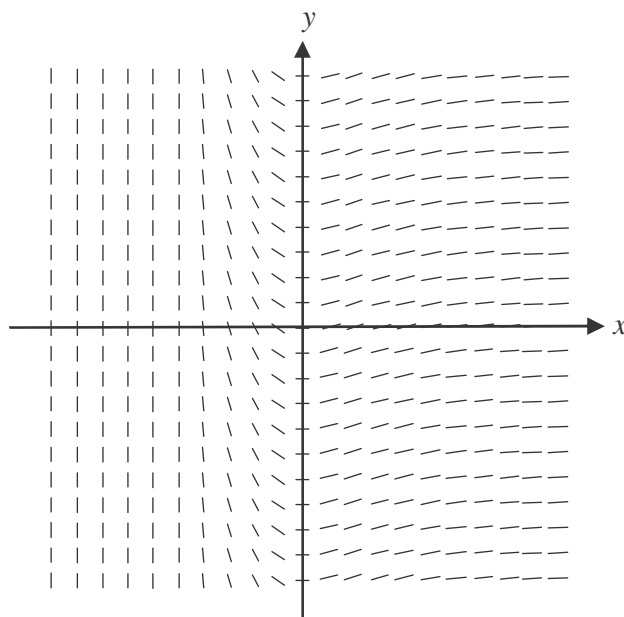
If  $z_1 = 1 - \sqrt{3}i$  and  $z_2 = (z_1)^{12}$  then

- A.  $\text{Arg}(z_2) < 0$
- B.  $\text{Im}(z_2) + \text{Re}(z_2) < 0$
- C.  $|z_2| < 0$
- D.  $\text{Re}(z_2) = 0$
- E.  $\text{Im}(z_2) = 0$

**Question 7**

Using a suitable substitution the definite integral  $\int_1^2 (x-3)(2x+1)^5 dx$  is equivalent to

- A.  $\int_1^2 (u^6 - 7u^5) du$
- B.  $\frac{1}{4} \int_3^5 (u^6 - 7u^5) du$
- C.  $\frac{1}{4} \int_1^2 (u^6 - 7u^5) du$
- D.  $\frac{1}{2} \int_3^5 (u^6 - 7u^5) du$
- E.  $\frac{1}{2} \int_1^2 (u^6 - 7u^5) du$

**Question 8**

The differential equation that best represents the direction field shown above is

- A.  $\frac{dy}{dx} = e^x$
- B.  $\frac{dy}{dx} = xe^x$
- C.  $\frac{dy}{dx} = xe^{-x}$
- D.  $\frac{dy}{dx} = xye^x$
- E.  $\frac{dy}{dx} = ye^{-x}$

**Question 9**

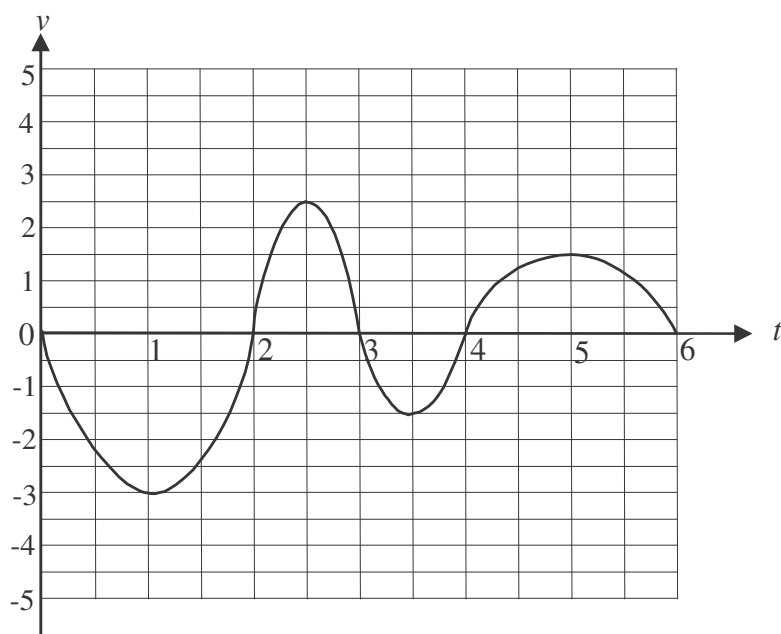
Consider the differential equation  $\frac{dy}{dx} = x^{\frac{3}{2}} + 2x$ , with  $x_0 = 1$  and  $y_0 = 2$ .

Using Euler's method with a step size of 0.1, the value of  $y_2$  correct to three decimal places is

- A. 1.508
- B. 1.635
- C. 2.300
- D. 2.635
- E. 3.109

**Question 10**

The velocity-time graph for a particle travelling in a straight line is shown below.



The particle is furthest from its initial position during the time interval

- A. (0.5, 1.5)
- B. (1.5, 2.5)
- C. (3, 4)
- D. (4.5, 5.5)
- E. (5, 6)

**Question 11**

The vector resolute of  $\underline{a} = 2\underline{i} + \underline{j} + \underline{k}$  in the direction of  $\underline{b} = \underline{i} - \underline{k}$  is

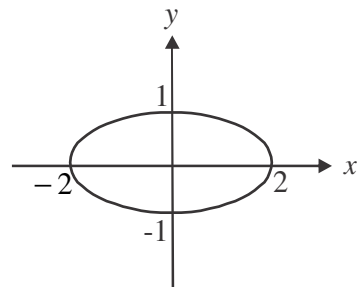
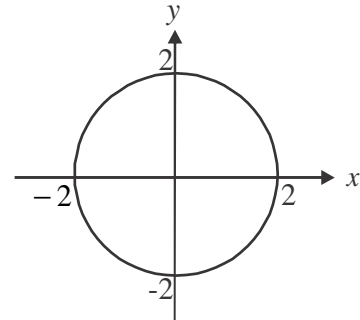
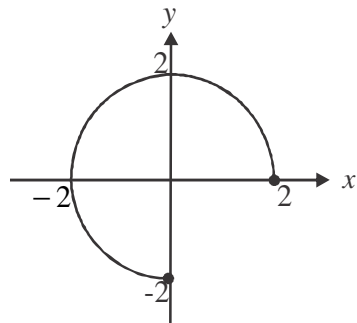
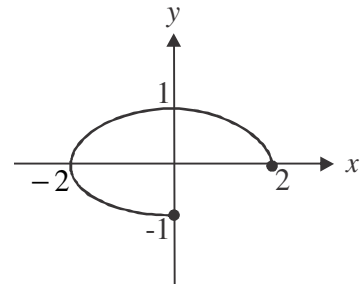
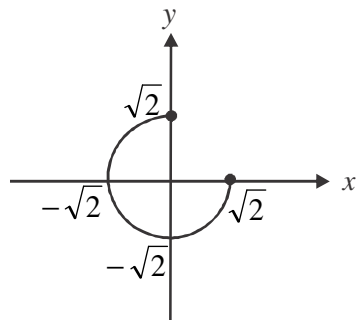
- A.  $\frac{1}{\sqrt{2}}$
- B.  $\frac{1}{2}$
- C.  $\frac{1}{2}(\underline{i} - \underline{k})$
- D.  $\underline{i} - \underline{k}$
- E.  $\frac{1}{2}(3\underline{i} + 2\underline{j} + 3\underline{k})$

**Question 12**

The position vector of a particle at time  $t$  seconds is given by

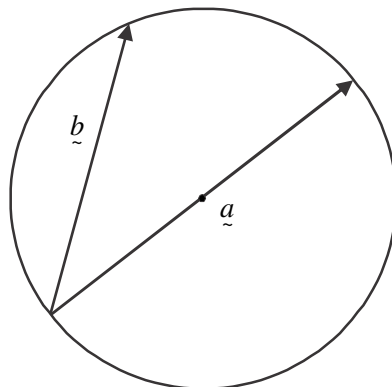
$$\underline{r}(t) = 2\cos(t)\underline{i} + \sin(t)\underline{j}, \quad 0 \leq t \leq \frac{3\pi}{2}.$$

The path of the particle is shown by

**A.****B.****C.****D.****E.**

**Question 13**

The diagram below shows a circle. The vector  $\underline{a}$  passes through the centre of the circle and vectors  $\underline{a}$  and  $\underline{b}$  meet at the circumference of the circle.



It can definitely be deduced from this diagram that

- A.  $\underline{a} \bullet \underline{b} = 0$
- B.  $\underline{a} \bullet \underline{b} = 1$
- C.  $\underline{a} \bullet \underline{a} = \underline{b} \bullet \underline{b}$
- D.  $|\underline{a}| |\underline{b}| = 1$
- E.  $(\underline{a} - \underline{b}) \bullet \underline{b} = 0$

**Question 14**

The position vectors of the points  $P$ ,  $Q$  and  $R$  are given respectively by  $\underline{p} = \underline{i} + \underline{k}$ ,  $\underline{q} = 2\underline{j} - \underline{k}$  and  $\underline{r} = \underline{i} - \underline{j} - 2\underline{k}$ .

The angle  $PQR$  is closest to

- A.  $6.2^\circ$
- B.  $59.8^\circ$
- C.  $72.5^\circ$
- D.  $123.6^\circ$
- E.  $134.7^\circ$



**Question 15**

A particle moving in a straight line has acceleration,  $a \text{ ms}^{-2}$ , given by  $a = \sqrt{v+4}$ , where  $v$  is the velocity of the particle in  $\text{ms}^{-1}$  at time  $t$  seconds. Initially the velocity of the particle is  $-3 \text{ ms}^{-1}$ .

The velocity of the particle at time  $t$  seconds is given by

- A.  $v = \left(\frac{3t}{2}\right)^{\frac{2}{3}} - 4$
- B.  $v = \left(\frac{3t+2}{2}\right)^{\frac{2}{3}} - 4$
- C.  $v = e^2 - 4$
- D.  $v = \frac{t^2 - 8}{2}$
- E.  $v = \frac{t^2 + 4t - 12}{4}$

**Question 16**

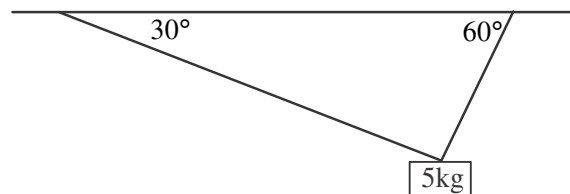
An 85 kg man stands in a lift which is accelerating downwards at  $1.8 \text{ ms}^{-2}$ . The reaction force, in newtons, of the lift floor on the man is

- A. 17
- B. 102
- C. 680
- D. 831.8
- E. 935

**Question 17**

A mass of 5 kg is held in equilibrium by two light strings which are each connected to a horizontal ceiling.

One of the strings makes an angle of  $30^\circ$  with the ceiling and has a tension  $T_1$  newtons. The other string makes an angle of  $60^\circ$  with the ceiling and has a tension of  $T_2$  newtons.



The ratio of  $T_2$  to  $T_1$  is

- A.  $1 : \frac{1}{\sqrt{3}}$
- B.  $1 : \sqrt{3}$
- C.  $1 : 2$
- D.  $1 : \sqrt{5}$
- E.  $1 : 2\sqrt{5}$

**Question 18**

A popular brand of pasta sells its penne in packets labelled as 750 grams.

A random sample of 40 packets of penne have a mean weight of 748.6 grams and a standard deviation of 2.7 grams.

An approximate 95% confidence interval for the population mean  $\mu$  is closest to

- A.  $747.8 < \mu < 749.4$
- B.  $747.9 < \mu < 748.1$
- C.  $748.2 < \mu < 749.0$
- D.  $748.5 < \mu < 751.5$
- E.  $749.05 < \mu < 750.95$

**Question 19**

A census found that the mean weekly earnings for adult full time workers in a particular region was \$1554 with a standard deviation of \$162.

A random sample of 50 adult full time workers in this region was taken. The probability that the mean weekly earnings for this group was less than \$1500 is closest to

- A. 0.0092
- B. 0.0522
- C. 0.2389
- D. 0.3694
- E. 0.4986

**Question 20**

A statistical test was conducted involving a null hypothesis  $H_0$  and an alternative hypothesis  $H_1$ . The results of the test indicated that  $H_0$  should be rejected.

Which one of the following statements **could not** be correct?

- A. A correct decision has been made.
- B. A type 1 error has been made.
- C. An incorrect decision has been made.
- D. A type 2 error has been made.
- E.  $H_1$  is true.

## SECTION B

### Question 1 (11 marks)

Consider  $f : [0, 2] \rightarrow \mathbb{R}$ ,  $f(x) = 1 + \frac{4}{\pi} \arctan(x-1)$ .

- a. Given that  $f''(x) = \frac{-8(x-1)}{\pi(x^2 - 2x + 2)^2}$ , show that the graph of  $f$  has a point of inflection at  $x = 1$ . 2 marks

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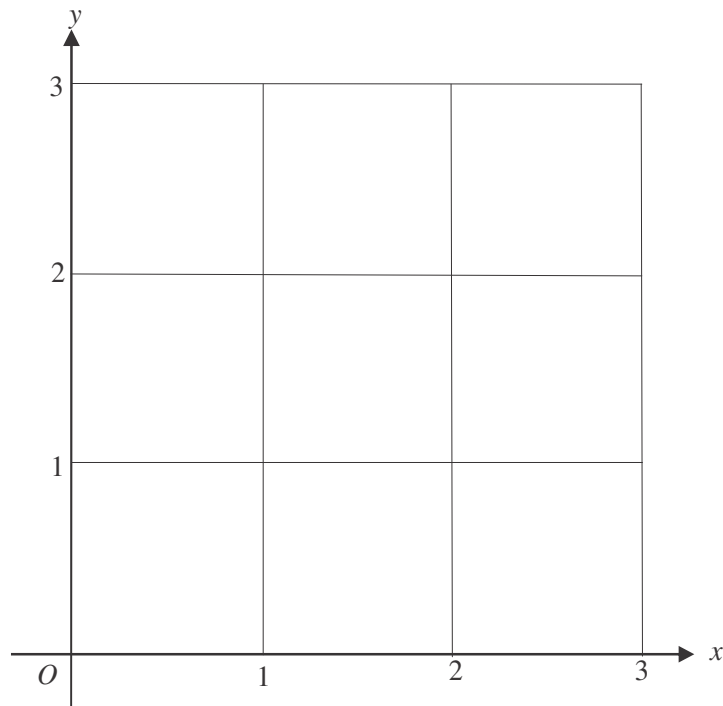


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- b. The inverse function of  $f$  is  $f^{-1}$ . Sketch the graphs of  $f$  and  $f^{-1}$  on the set of axes below. Indicate clearly endpoints and points of inflection. 2 marks



- c. Given that  $f^{-1}(x) = a + \tan\left(\frac{b}{c}(x-a)\right)$ , find the values of  $a$ ,  $b$ , and  $c$ . 2 marks

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Let  $A$  represent the area, in square units, enclosed by the graphs of  $f$  and  $f^{-1}$ .

- d. Given that  $\int_0^1 (f^{-1}(x) - g(x))dx = \frac{A}{4}$ , find the rule for the function  $g$ . 1 mark

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- e. Hence or otherwise find  $A$ . Give your answer correct to one decimal place. 2 marks

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- f. The region enclosed by the graph of  $f$ , the  $y$ -axis, and the line  $y = 1$  is rotated about the  $y$ -axis to form a solid of revolution. Find the volume of this solid. Give your answer correct to one decimal place. 2 marks

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**Question 2** (10 marks)

Let  $u = \frac{\sqrt{3}}{2} + \frac{1}{2}i$ .

- a. i.** Express  $u$  in polar form.

2 marks

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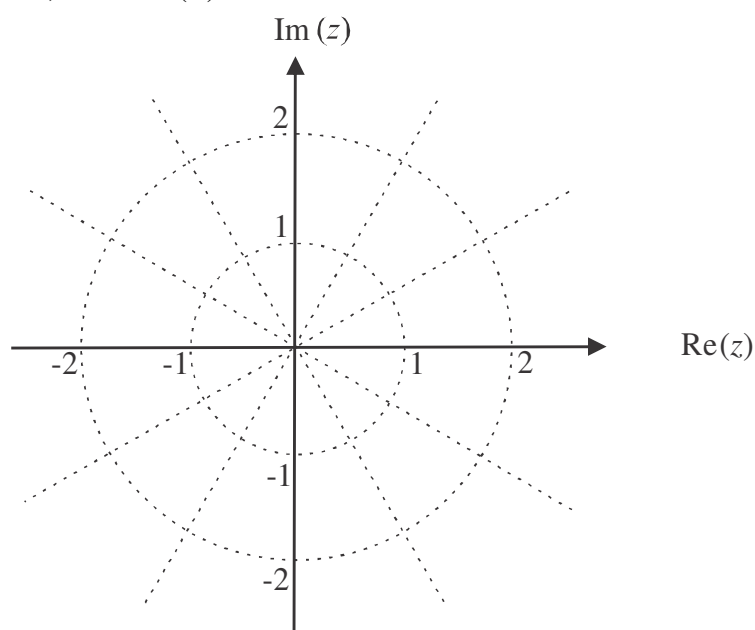
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- ii.** The complex number  $u$  is one root of the equation  $z^3 - i = 0$ .  
Plot all the roots of this equation on the Argand diagram below labelling them  $u$ ,  $v$  and  $w$ , where  $\text{Im}(w) < 0$ .

2 marks



- iii.** Let  $S = \{z : |z - u| = |z - w|\}$ .

Sketch  $S$  on the Argand diagram above.

1 mark

- iv.** Show algebraically that  $v$  lies on  $S$ .

2 marks

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- b.** Consider the equation  $z^2 + 6i \sin(\alpha)z - 9 = 0$ , where  $z \in \mathbb{C}$ ,  $\alpha$  is a real constant and  $0 < \alpha < \frac{\pi}{3}$ .

Find the roots  $z_1$  and  $z_2$  of this equation. Express them in polar form in terms of  $\alpha$ .

3 marks

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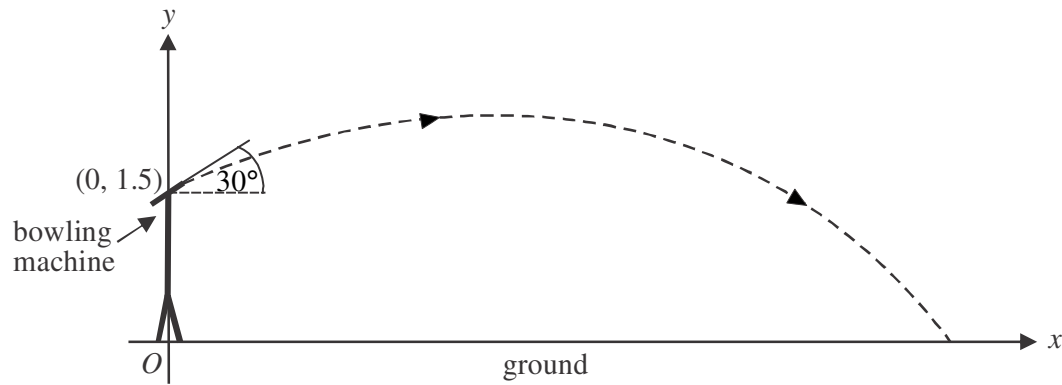
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**Question 3** (11 marks)

A bowling machine is programmed to project a ball from a height of 1.5 m above the ground at an angle of  $30^\circ$  to the horizontal as shown below. The ball's initial speed is  $8 \text{ ms}^{-1}$  and it travels in a vertical plane, landing on the ground  $T$  seconds later.

On a Cartesian graph, the origin  $O$  is located 1.5 m vertically below the point where the ball is projected. Let  $\underline{i}$  represent a unit vector in the positive  $x$  direction and let  $\underline{j}$  represent a unit vector in the positive  $y$  direction where displacement is measured in metres and time is in seconds.



- a.** Show that the ball has an initial velocity when projected of  $4\sqrt{3}\underline{i} + 4\underline{j}$  1 mark

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- b.** The acceleration of the ball  $t$  seconds after being projected is given by

$$\ddot{\mathbf{r}}(t) = -\frac{t}{50}\mathbf{i} + \left(\frac{t}{20} - g\right)\mathbf{j}, \quad 0 \leq t \leq T.$$

Show that the position vector of the ball  $t$  seconds after being projected by the machine is given by

$$\mathbf{r}(t) = \left(4\sqrt{3}t - \frac{t^3}{300}\right)\mathbf{i} + \left(\frac{t^3}{120} - \frac{gt^2}{2} + 4t + 1.5\right)\mathbf{j}, \quad 0 \leq t \leq T$$

3 marks

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- c.** How far does the ball travel horizontally before it hits the ground? Give your answer in metres correct to 1 decimal place.

2 marks

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- d.** What is the distance, in metres, that the ball travels from when it is projected until it hits the ground? Give your answer correct to one decimal place. 3 marks

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- e.** Find the speed of the ball, in metres per second, when it hits the ground. Give your answer correct to one decimal place. 2 marks

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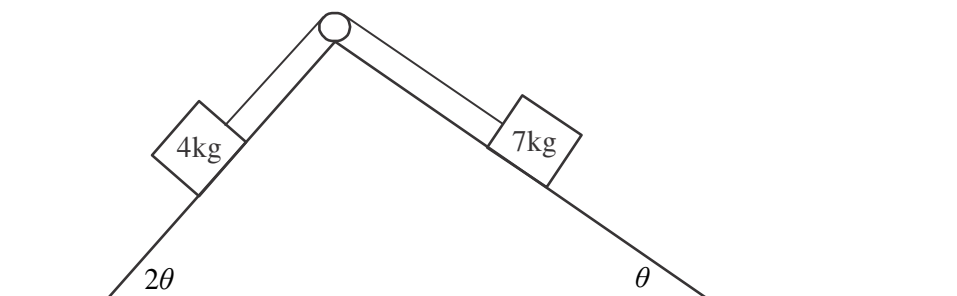
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**Question 4** (9 marks)

Particles of mass 4 kg and 7 kg are shown on adjoining smooth planes in the diagram below. The planes are inclined to the horizontal at angles of  $2\theta$  and  $\theta$  respectively where  $0^\circ < \theta < 45^\circ$ . A light inextensible string passes over a smooth pulley and connects the two particles. The tension in the string is  $T$  newtons. The 7 kg mass accelerates down the plane at  $a \text{ ms}^{-2}$ .



- a.** Find an equation for the motion of the 4 kg particle up the plane. 1 mark

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- b.** Hence show that the acceleration of the 7 kg mass down the other plane is given by  $a = \frac{g \sin(\theta)}{11}(7 - 8 \cos(\theta))$ . 3 marks

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- c.** Find the angle  $\theta$  when the system is in equilibrium. Give your answer correct to one decimal place. 2 marks

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Suppose  $\theta$  is fixed at  $30^\circ$  and a mass of  $m$  kg is added to the 4 kg mass. The 7 kg mass now accelerates up the plane with an acceleration of  $\frac{g(11\sqrt{3}-14)}{50}\text{ms}^{-2}$ .

**d.** Find the value of  $m$ .

3 marks

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**Question 5** (10 marks)

A tyre manufacturer produces thirteen inch car tyres using rubber and other components. The random variable  $R$  represents the weight, in kg, of rubber in one of these tyres.

The random variable  $C$  represents the weight, in kg, of the other components in one of these tyres.

Both variables are independent and are normally distributed with mean and variance as shown in the table below.

| random variable | mean ( $\mu$ ) | variance ( $\sigma^2$ ) |
|-----------------|----------------|-------------------------|
| $R$             | 2.2            | 0.18                    |
| $C$             | 4.9            | 0.31                    |

Let the random variable  $W$  represent four times the weight of one of these thirteen inch car tyres.

- a.** Find the mean and standard deviation of  $W$ .

2 marks

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The tyre manufacturer also produces fifteen inch car tyres. The distribution of the weights of these tyres is normal and it is claimed by the manufacturer that  $\mu = 8.6$  kg.

A motoring organization tests these claims by taking a random sample of 36 tyres. It finds the mean weight for the sample to be  $\bar{x} = 8.4$  kg and the standard deviation to be  $s = 0.54$  kg.

Using this sample data it checks the manufacturer's claims that  $\mu = 8.6$  kg by conducting a statistical test.

It is assumed that the population standard deviation ( $\sigma$ ) of these tyre weights can be estimated with sufficient accuracy by  $s$ .

- b.** Write down two hypotheses,  $H_0$  and  $H_1$ , that could be used to test whether the mean weight of the tyres is less than that being claimed by the manufacturer.

2 marks

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- c.** Find the  $p$  value for this statistical test. Give your answer correct to three decimal places. 2 marks

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- d.** State whether  $H_0$  should be rejected or not at the 5% level of significance. Give a reason to support your answer. 1 mark

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The motoring organization decides to repeat its test of the manufacturers claim that  $\mu = 8.6$  kg but this time it will be at the 1% level of significance.

Using the same hypotheses, it again takes a random sample of 36 tyres. The standard deviation of this second sample is found again to be 0.54 kg.

The motoring organization concludes that according to this second sample, the null hypothesis should not be rejected at the 1% level of significance.

- e.** Find the minimum value of the sample mean  $\bar{x}$  that the motoring organization could have found in its second sample. Give your answer correct to three decimal places. 3 marks

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**Question 6** (9 marks)

A tank initially contains 500 litres of brine which is a solution of water and salt. There is 100 grams of salt in the tank initially.

Water is pumped into the tank at the rate of 60 litres per minute and brine is pumped out of the tank at the rate of 20 litres per minute.

The amount of salt in the tank  $t$  minutes after the pumping begins is  $x$  grams. The solution in the tank is kept uniform by constant stirring.

- a. i.** Write down an expression for the concentration of salt in the tank, in grams per litre, in terms of  $x$  and  $t$ .

1 mark

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- ii.** Show that the differential equation which represents the rate of change of  $x$  with respect to  $t$  is given by  $\frac{dx}{dt} + \frac{x}{25+2t} = 0$ .

2 marks

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- b.** Show that  $x(t) = \frac{500}{\sqrt{25+2t}}$ .

2 marks

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- c. Verify by substitution that  $x(t)$  satisfies the differential equation  $\frac{dx}{dt} + \frac{x}{25+2t} = 0$  and the initial conditions.

2 marks

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- d. Find the amount of salt that flowed out of the tank in the first twenty minutes after the pumping began. Give your answer in grams correct to one decimal place.

2 marks

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## Specialist Mathematics Formulas

### Mensuration

|                                   |                                                             |
|-----------------------------------|-------------------------------------------------------------|
| area of a trapezium               | $\frac{1}{2}(a+b)h$                                         |
| curved surface area of a cylinder | $2\pi rh$                                                   |
| volume of a cylinder              | $\pi r^2 h$                                                 |
| volume of a cone                  | $\frac{1}{3}\pi r^2 h$                                      |
| volume of a pyramid               | $\frac{1}{3}Ah$                                             |
| volume of a sphere                | $\frac{4}{3}\pi r^3$                                        |
| area of a triangle                | $\frac{1}{2}bc \sin(A)$                                     |
| sine rule                         | $\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$ |
| cosine rule                       | $c^2 = a^2 + b^2 - 2ab \cos(C)$                             |

### Circular (trigonometric) functions

|                                                                      |                                                            |
|----------------------------------------------------------------------|------------------------------------------------------------|
| $\cos^2(x) + \sin^2(x) = 1$                                          |                                                            |
| $1 + \tan^2(x) = \sec^2(x)$                                          | $\cot^2(x) + 1 = \operatorname{cosec}^2(x)$                |
| $\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$                        | $\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$              |
| $\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$                        | $\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$              |
| $\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$           | $\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$ |
| $\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$ |                                                            |
| $\sin(2x) = 2\sin(x)\cos(x)$                                         | $\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$                |

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**Circular (trigonometric) functions – continued**

|                 |                                              |                      |                                              |
|-----------------|----------------------------------------------|----------------------|----------------------------------------------|
| <b>Function</b> | $\sin^{-1}(\arcsin)$                         | $\cos^{-1}(\arccos)$ | $\tan^{-1}(\arctan)$                         |
| <b>Domain</b>   | $[-1, 1]$                                    | $[-1, 1]$            | $R$                                          |
| <b>Range</b>    | $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ | $[0, \pi]$           | $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ |

**Algebra (complex numbers)**

|                                                                                |                                                                             |
|--------------------------------------------------------------------------------|-----------------------------------------------------------------------------|
| $z = x + iy = r(\cos(\theta) + i \sin(\theta)) = r \operatorname{cis}(\theta)$ |                                                                             |
| $ z  = \sqrt{x^2 + y^2} = r$                                                   | $-\pi < \operatorname{Arg}(z) \leq \pi$                                     |
| $z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$                    | $\frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$ |
| $z^n = r^n \operatorname{cis}(n\theta)$ (de Moivre's theorem)                  |                                                                             |

**Probability and statistics**

|                                              |                                                                                                                     |
|----------------------------------------------|---------------------------------------------------------------------------------------------------------------------|
| for random variables $X$ and $Y$             | $E(aX + b) = aE(x) + b$<br>$E(aX + bY) = aE(x) + bE(Y)$<br>$\operatorname{var}(aX + b) = a^2 \operatorname{var}(X)$ |
| for independent random variables $X$ and $Y$ | $\operatorname{var}(aX + bY) = a^2 \operatorname{var}(X) + b^2 \operatorname{var}(Y)$                               |
| approximate confidence interval for $\mu$    | $\left( \bar{x} - z \frac{s}{\sqrt{n}}, \bar{x} + z \frac{s}{\sqrt{n}} \right)$                                     |
| distribution of sample mean $\bar{X}$        | mean $E(\bar{X}) = \mu$<br>variance $\operatorname{var}(\bar{X}) = \frac{\sigma^2}{n}$                              |

**Calculus**

|                                                        |                                                                                                              |
|--------------------------------------------------------|--------------------------------------------------------------------------------------------------------------|
| $\frac{d}{dx}(x^n) = nx^{n-1}$                         | $\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$                                                          |
| $\frac{d}{dx}(e^{ax}) = ae^{ax}$                       | $\int e^{ax} dx = \frac{1}{a}e^{ax} + c$                                                                     |
| $\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$                | $\int \frac{1}{x} dx = \log_e x  + c$                                                                        |
| $\frac{d}{dx}(\sin(ax)) = a \cos(ax)$                  | $\int \sin(ax) dx = -\frac{1}{a}\cos(ax) + c$                                                                |
| $\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$                 | $\int \cos(ax) dx = \frac{1}{a}\sin(ax) + c$                                                                 |
| $\frac{d}{dx}(\tan(ax)) = a \sec^2(ax)$                | $\int \sec^2(ax) dx = \frac{1}{a}\tan(ax) + c$                                                               |
| $\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$  | $\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c, a > 0$                            |
| $\frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^2}}$ | $\int \frac{-1}{\sqrt{a^2-x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, a > 0$                           |
| $\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$         | $\int \frac{a}{a^2+x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$                                          |
|                                                        | $\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c, n \neq -1$                                             |
|                                                        | $\int (ax+b)^{-1} dx = \frac{1}{a}\log_e ax+b  + c$                                                          |
| product rule                                           | $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$                                                       |
| quotient rule                                          | $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$                       |
| chain rule                                             | $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$                                                                |
| Euler's method                                         | If $\frac{dy}{dx} = f(x)$ , $x_0 = a$ and $y_0 = b$ , then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$ |
| acceleration                                           | $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$          |
| arc length                                             | $\int_{x_1}^{x_2} \sqrt{1+(f'(x))^2} dx$ or $\int_{t_1}^{t_2} \sqrt{(x'(t))^2 + (y'(t))^2} dt$               |

**Vectors in two and three dimensions**

|                                                                                                                                             |
|---------------------------------------------------------------------------------------------------------------------------------------------|
| $\underline{r} = x \underline{i} + y \underline{j} + z \underline{k}$                                                                       |
| $ \underline{r}  = \sqrt{x^2 + y^2 + z^2} = r$                                                                                              |
| $\dot{\underline{r}} = \frac{d\underline{r}}{dt} = \frac{dx}{dt} \underline{i} + \frac{dy}{dt} \underline{j} + \frac{dz}{dt} \underline{k}$ |
| $\underline{r}_1 \cdot \underline{r}_2 = r_1 r_2 \cos(\theta) = x_1 x_2 + y_1 y_2 + z_1 z_2$                                                |

**Mechanics**

|                    |                                   |
|--------------------|-----------------------------------|
| momentum           | $\underline{p} = m \underline{v}$ |
| equation of motion | $\underline{R} = m \underline{a}$ |

SPECIALIST MATHS  
TRIAL EXAMINATION 2  
MULTIPLE - CHOICE ANSWER SHEET

STUDENT NAME:.....

INSTRUCTIONS

Fill in the letter that corresponds to your choice. Example: ☐ A ☒ B ☐ C ☐ D ☐ E

The answer selected is B. Only one answer should be selected.

1. ☐ A ☐ B ☐ C ☐ D ☐ E
2. ☐ A ☐ B ☐ C ☐ D ☐ E
3. ☐ A ☐ B ☐ C ☐ D ☐ E
4. ☐ A ☐ B ☐ C ☐ D ☐ E
5. ☐ A ☐ B ☐ C ☐ D ☐ E
6. ☐ A ☐ B ☐ C ☐ D ☐ E
7. ☐ A ☐ B ☐ C ☐ D ☐ E
8. ☐ A ☐ B ☐ C ☐ D ☐ E
9. ☐ A ☐ B ☐ C ☐ D ☐ E
10. ☐ A ☐ B ☐ C ☐ D ☐ E

11. ☐ A ☐ B ☐ C ☐ D ☐ E
12. ☐ A ☐ B ☐ C ☐ D ☐ E
13. ☐ A ☐ B ☐ C ☐ D ☐ E
14. ☐ A ☐ B ☐ C ☐ D ☐ E
15. ☐ A ☐ B ☐ C ☐ D ☐ E
16. ☐ A ☐ B ☐ C ☐ D ☐ E
17. ☐ A ☐ B ☐ C ☐ D ☐ E
18. ☐ A ☐ B ☐ C ☐ D ☐ E
19. ☐ A ☐ B ☐ C ☐ D ☐ E
20. ☐ A ☐ B ☐ C ☐ D ☐ E