VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2015

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER									Letter

SPECIALIST MATHEMATICS

Written examination 2

Monday 9 November 2015

Reading time: 3.00 pm to 3.15 pm (15 minutes)
Writing time: 3.15 pm to 5.15 pm (2 hours)

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	5	5	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 23 pages with a detachable sheet of miscellaneous formulas in the centrefold.
- Answer sheet for multiple-choice questions.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- All written responses must be in English.

At the end of the examination
- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

The ellipse $\frac{(x-2)^2}{9}+\frac{(y-3)^2}{4}=1$ can be expressed in parametric form as

A. $x = 2 + 3t$ and $y = 3 + 2\sqrt{1+t^2}$
B. $x = 2 + 3\sec(t)$ and $y = 3 + 2\tan(t)$
C. $x = 2 + 9\cos(t)$ and $y = 3 + 4\sin(t)$
D. $x = 3 + 2\cos(t)$ and $y = 2 + 3\sin(t)$
E. $x = 2 + 3\cos(t)$ and $y = 3 + 2\sin(t)$

Question 2

The range of the function with rule $f(x)=(2-x)\arcsin\left(\frac{x}{2}-1\right)$ is

A. $[-\pi, 0]$

B. $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

C. $\left[-\frac{(2-x)\pi}{2}, \frac{(2-x)\pi}{2}\right]$

D. $[0, 4]$

E. $[0, \pi]$

Question 3

If both a and c are non-zero real numbers, the relation $a^2x^2 + (1 - a^2)y^2 = c^2$ **cannot** represent

A. a circle.
B. an ellipse.
C. a hyperbola.
D. a single straight line.
E. a pair of straight lines.

Question 4

The two asymptotes of a particular hyperbola have gradients $\frac{2}{3}$ and $-\frac{2}{3}$ respectively and intersect at the point (2, 1). One branch of the hyperbola passes through the point (5, 5).

The equation of the hyperbola is

A. $\frac{(x-2)^2}{4}-\frac{(y-1)^2}{9}=1$

B. $\frac{(x-2)^2}{4}-\frac{(y-1)^2}{9}=\frac{17}{36}$

C. $\frac{(y-1)^2}{9}-\frac{(x-2)^2}{4}=\frac{17}{36}$

D. $\frac{(y-1)^2}{4}-\frac{(x-2)^2}{9}=3$

E. $\frac{(x-2)^2}{9}-\frac{(y-1)^2}{4}=3$

Question 5

Given $z=\frac{1+i\sqrt{3}}{1+i}$, the modulus and argument of the complex number z^5 are respectively

A. $2\sqrt{2}$ and $\frac{5\pi}{6}$

B. $4\sqrt{2}$ and $\frac{5\pi}{12}$

C. $4\sqrt{2}$ and $\frac{7\pi}{12}$

D. $2\sqrt{2}$ and $\frac{5\pi}{12}$

E. $4\sqrt{2}$ and $-\frac{\pi}{12}$

Question 6

Which one of the following relations has a graph that passes through the point $1 + 2i$ in the complex plane?

A. $z\bar{z}=\sqrt{5}$

B. $\text{Arg}(z)=\frac{\pi}{3}$

C. $|z-1|=|z-2i|$

D. $\text{Re}(z) = 2\text{Im}(z)$

E. $z+\bar{z}=2$

Question 7

If $z = \sqrt{3} + 3i$, then z^{63} is

A. real and negative

B. equal to a negative real multiple of i

C. real and positive

D. equal to a positive real multiple of i

E. a positive real multiple of $1 + i\sqrt{3}$

Question 8

A relation that does **not** represent a circle in the complex plane is

A. $z\overline{z} = 4$

B. $|z + 3i| = 2|z - i|$

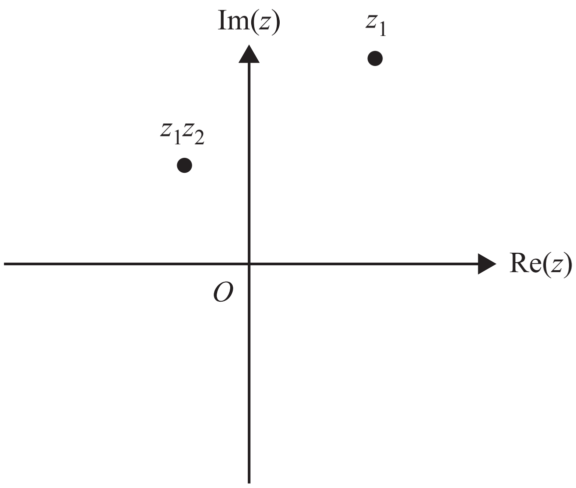
C. $|z - i| = |z + 2|$

D. $|z - 1 + i| = 4$

E. $|z| + 2|\overline{z}| = 4$

Question 9

Let $z_1 = r_1\text{cis}(\theta_1)$ and $z_2 = r_2\text{cis}(\theta_2)$, where z_1 and z_1z_2 are shown in the Argand diagram below; θ_1 and θ_2 are acute angles.



A statement that is **necessarily** true is

A. $r_2 > 1$

B. $\theta_1 < \theta_2$

C. $\left|\dfrac{z_1}{z_2}\right| > r_1$

D. $\theta_1 = \theta_2$

E. $r_1 > 1$

Question 10

Using a suitable substitution, the definite integral $\int_0^1 \left(x^2\sqrt{3x+1}\right)dx$ is equivalent to

A. $\frac{1}{9}\int_0^1 \left(u^{\frac{5}{2}} - 2u^{\frac{3}{2}} + u^{\frac{1}{2}}\right)du$

B. $\frac{1}{27}\int_1^4 \left(u^{\frac{5}{2}} - 2u^{\frac{3}{2}} + u^{\frac{1}{2}}\right)du$

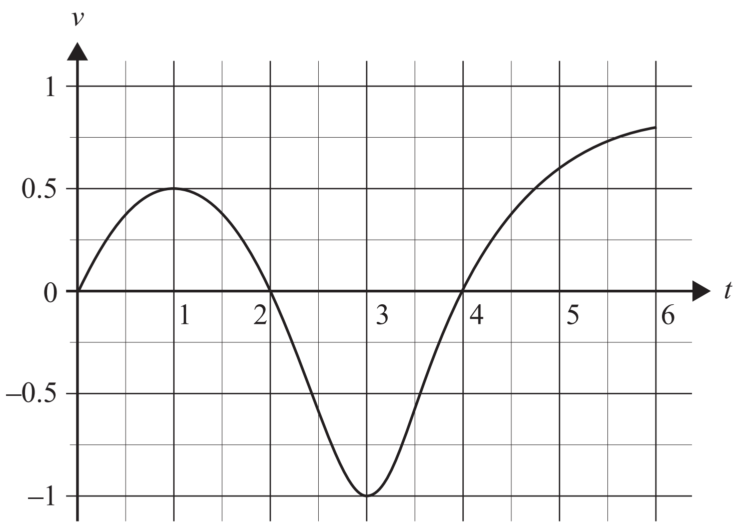
C. $\frac{1}{9}\int_1^4 \left(u^{\frac{5}{2}} - 2u^{\frac{3}{2}} + u^{\frac{1}{2}}\right)du$

D. $\frac{1}{27}\int_0^1 \left(u^{\frac{5}{2}} - 2u^{\frac{3}{2}} + u^{\frac{1}{2}}\right)du$

E. $\frac{1}{3}\int_1^4 \left(u^{\frac{5}{2}} - 2u^{\frac{3}{2}} + u^{\frac{1}{2}}\right)du$

Question 11

The velocity–time graph for a body moving along a straight line is shown below.



The body first returns to its initial position within the time interval

A. (0, 0.5)

B. (0.5, 1.5)

C. (1.5, 2.5)

D. (2.5, 3.5)

E. (3.5, 5)

Question 12

Given $\frac{dy}{dx} = 1 - \frac{y}{3}$ and $y = 4$ when $x = 2$, then

A. $y = e^{\frac{-(x-2)}{3}} - 3$

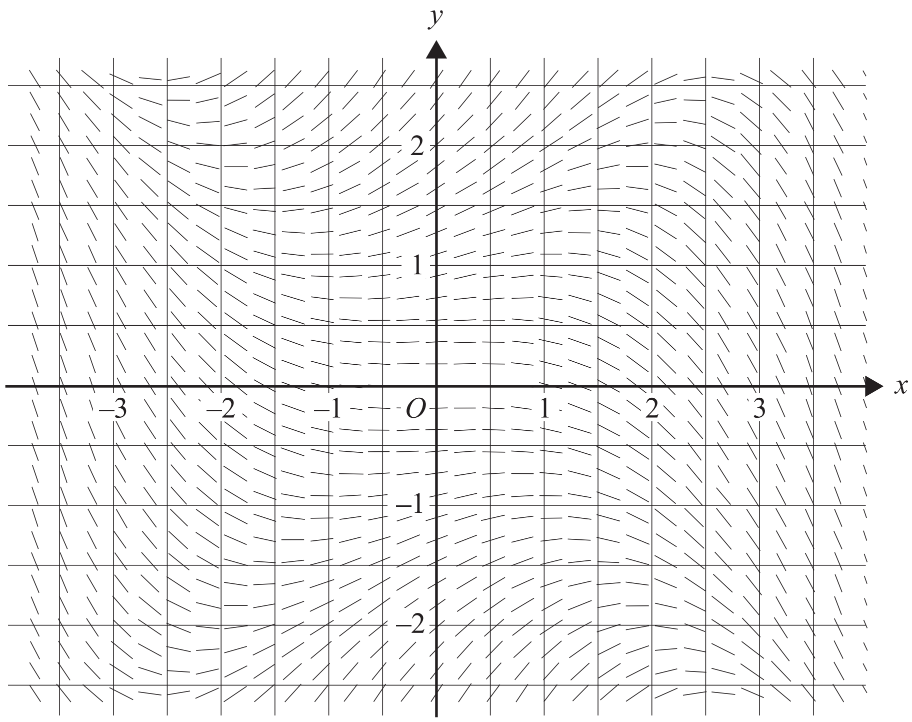
B. $y = e^{\frac{-(x-2)}{3}} + 3$

C. $y = 4e^{\frac{-(x-2)}{3}}$

D. $y = e^{\frac{4(y-x-2)}{3}}$

E. $y = e^{\frac{(x-2)}{3}} + 3$

Question 13



The direction field for a certain differential equation is shown above.

The solution curve to the differential equation that passes through the point (–2.5, 1.5) could also pass through

A. (0, 2)

B. (1, 2)

C. (3, 1)

D. (3, –0.5)

E. (–0.5, 2)

Question 14

A differential equation that has $y = x\sin(x)$ as a solution is

A. $\frac{d^2y}{dx^2} + y = 0$

B. $x\frac{d^2y}{dx^2} + y = 0$

C. $\frac{d^2y}{dx^2} + y = -\sin(x)$

D. $\frac{d^2y}{dx^2} + y = -2\cos(x)$

E. $\frac{d^2y}{dx^2} + y = 2\cos(x)$

Question 15

The component of the force $\underset{\sim}{F} = a\underset{\sim}{i} + b\underset{\sim}{j}$, where a and b are non-zero real constants, in the direction of the vector $\underset{\sim}{w} = \underset{\sim}{i} + \underset{\sim}{j}$, is

A. $\left(\frac{a+b}{2}\right)\underset{\sim}{w}$

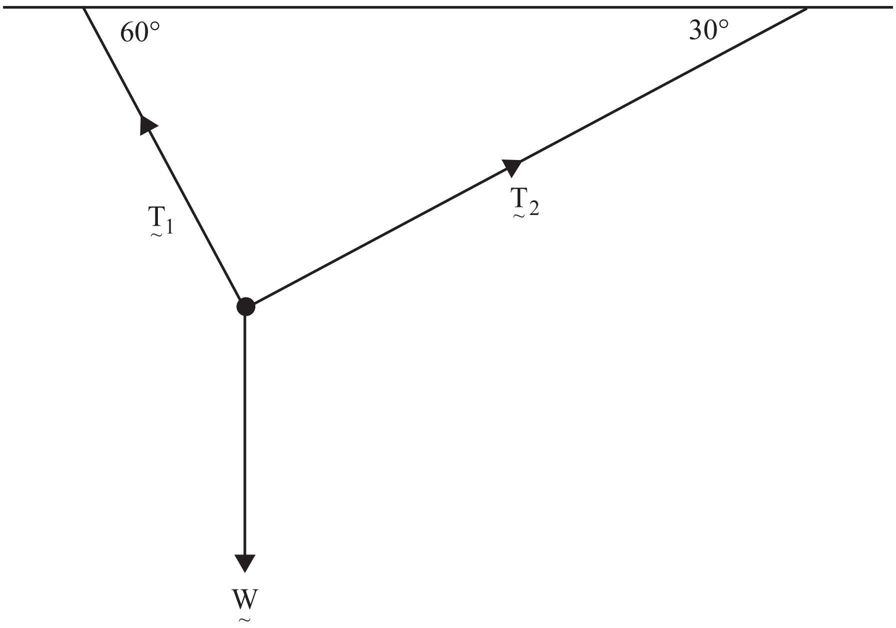
B. $\frac{\underset{\sim}{F}}{a+b}$

C. $\left(\frac{a+b}{a^2+b^2}\right)\underset{\sim}{F}$

D. $(a+b)\underset{\sim}{w}$

E. $\left(\frac{a+b}{\sqrt{2}}\right)\underset{\sim}{w}$

Question 16



The diagram above shows a mass suspended in equilibrium by two light strings that make angles of 60° and 30° with a ceiling. The tensions in the strings are $\underset{\sim}{T}_1$ and $\underset{\sim}{T}_2$, and the weight force acting on the mass is $\underset{\sim}{W}$. The correct statement relating the given forces is

A. $\underset{\sim}{T}_1 + \underset{\sim}{T}_2 + \underset{\sim}{W} = \underset{\sim}{0}$

B. $\underset{\sim}{T}_1 + \underset{\sim}{T}_2 - \underset{\sim}{W} = \underset{\sim}{0}$

C. $\underset{\sim}{T}_1 \times \frac{1}{2} + \underset{\sim}{T}_2 \times \frac{\sqrt{3}}{2} = \underset{\sim}{0}$

D. $\underset{\sim}{T}_1 \times \frac{\sqrt{3}}{2} + \underset{\sim}{T}_2 \times \frac{1}{2} = \underset{\sim}{W}$

E. $\underset{\sim}{T}_1 \times \frac{1}{2} + \underset{\sim}{T}_2 \times \frac{\sqrt{3}}{2} = \underset{\sim}{W}$

Question 17

Points *A*, *B* and *C* have position vectors $\underset{\sim}{a} = 2\underset{\sim}{i} + \underset{\sim}{j}$, $\underset{\sim}{b} = 3\underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{c} = -3\underset{\sim}{j} + \underset{\sim}{k}$ respectively.
The cosine of angle *ABC* is equal to

A. $\frac{5}{\sqrt{6}\sqrt{10}}$

B. $\frac{7}{\sqrt{6}\sqrt{13}}$

C. $-\frac{1}{\sqrt{6}\sqrt{13}}$

D. $-\frac{7}{\sqrt{21}\sqrt{6}}$

E. $-\frac{2}{\sqrt{6}\sqrt{13}}$

Question 18

The position vectors of two moving particles are given by $\underset{\sim}{r}_1(t) = (2+4t^2)\underset{\sim}{i} + (3t+2)\underset{\sim}{j}$ and $\underset{\sim}{r}_2(t) = (6t)\underset{\sim}{i} + (4+t)\underset{\sim}{j}$, where $t \geq 0$.

The particles will collide at

A. $3\underset{\sim}{i} + 3.5\underset{\sim}{j}$

B. $6\underset{\sim}{i} + 5\underset{\sim}{j}$

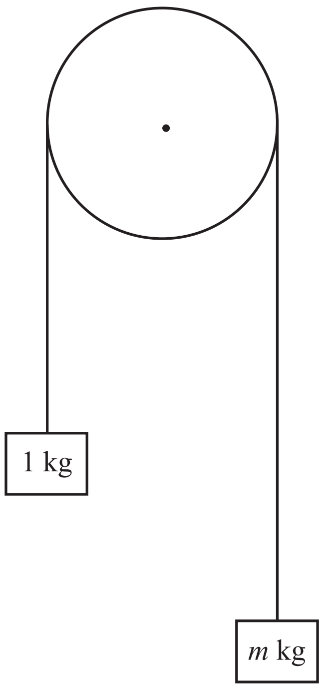
C. $3\underset{\sim}{i} + 4.5\underset{\sim}{j}$

D. $0.5\underset{\sim}{i} + \underset{\sim}{j}$

E. $5\underset{\sim}{i} + 6\underset{\sim}{j}$

Question 19

A light inextensible string passes over a smooth pulley, as shown below, with particles of mass 1 kg and *m* kg attached to the ends of the string.



If the acceleration of the 1 kg particle is 4.9 ms^{-2} **upwards**, then *m* is equal to

A. 1

B. 2

C. 3

D. 4

E. 5

Question 20

An object is moving in a straight line, initially at 5 ms^{-1}. Sixteen seconds later, it is moving at 11 ms^{-1} in the **opposite** direction to its initial velocity.

Assuming that the acceleration of the object is constant, after 16 seconds the distance, in metres, of the object from its starting point is

A. 24

B. 48

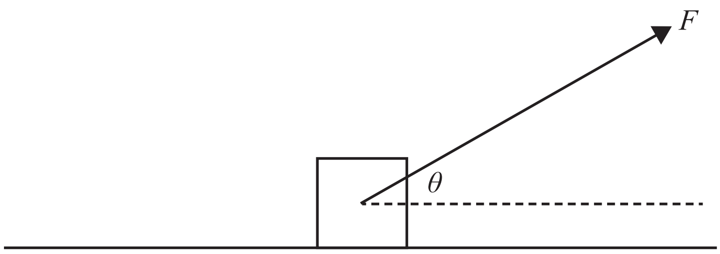
C. 73

D. 96

E. 128

Question 21

A block of mass M kg is on a rough horizontal plane. A constant force of F newtons is applied to the block at an angle of θ to the horizontal, as shown below. The block has acceleration a ms^{-2} and the coefficient of friction between the block and the plane is μ.



The equation of motion of the block in the horizontal direction is

A. $F - \mu Mg = Ma$

B. $F\cos(\theta) - \mu Mg = Ma$

C. $F\sin(\theta) - \mu(Mg - F\cos(\theta)) = Ma$

D. $F\cos(\theta) - \mu(F\sin(\theta) - Mg) = Ma$

E. $F\cos(\theta) - \mu(Mg - F\sin(\theta)) = Ma$

Question 22

A ball is thrown vertically up with an initial velocity of $7\sqrt{6}$ ms^{-1}, and is subject to gravity and air resistance.

The acceleration of the ball is given by $\ddot{x} = -\left(9.8 + 0.1v^2\right)$, where x metres is its vertical displacement, and v ms^{-1} is its velocity at time t seconds.

The time taken for the ball to reach its maximum height is

A. $\frac{\pi}{3}$

B. $\frac{5\pi}{21\sqrt{2}}$

C. $\log_e(4)$

D. $\frac{10\pi}{21\sqrt{2}}$

E. $10\log_e(4)$

END OF SECTION 1

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.
Unless otherwise specified, an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1 (12 marks)

Consider $y = \sqrt{2 - \sin^2(x)}$.

a. Use the relation $y^2 = 2 - \sin^2(x)$ to find $\frac{dy}{dx}$ in terms of x and y. 1 mark

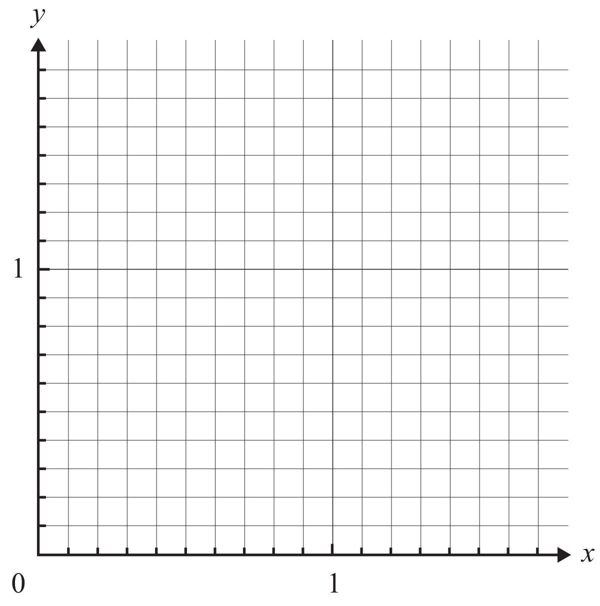
b. **i.** Write down the values of y where $x = 0$ and where $x = \frac{\pi}{2}$. 1 mark

ii. Write down the values of $\frac{dy}{dx}$ where $x = 0$ and where $x = \frac{\pi}{2}$. 1 mark

Now consider the function f with rule $f(x) = \sqrt{2 - \sin^2(x)}$ for $0 \le x \le \frac{\pi}{2}$.

c. Find the rule for the inverse function f^{-1}, and state the domain and range of f^{-1}. 3 marks

d. Sketch and label the graphs of f and f^{-1} on the axes below. 2 marks



e. The graphs of f and f^{-1} intersect at the point $P(a, a)$.

Find a, correct to three decimal places. 1 mark

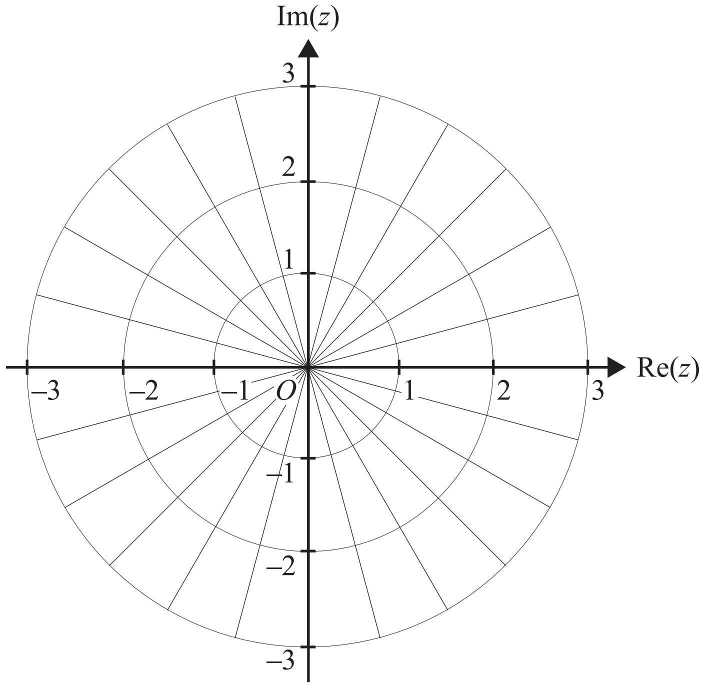
The region bounded by the graph of f, the coordinate axes and the line $x = 1$ is rotated about the x-axis to form a solid of revolution.

f. **i.** Write down a definite integral in terms of x that gives the volume of this solid of revolution. 2 marks

ii. Find the volume of this solid, correct to one decimal place. 1 mark

Question 2 (12 marks)

a. **i.** On the Argand diagram below, plot and label the points $0 + 0i$ and $1+i\sqrt{3}$. 2 marks



ii. On the same Argand diagram above, sketch the line $\left|z-\left(1+i\sqrt{3}\right)\right| = |z|$ and the circle $|z-2| = 1$. 2 marks

iii. Use the fact that the line $\left|z-\left(1+i\sqrt{3}\right)\right| = |z|$ passes through the point $z = 2$, or otherwise, to find the equation of this line in cartesian form. 1 mark

iv. Find the points of intersection of the line and the circle, expressing your answers in the form $a + ib$. 3 marks

b. i. Consider the equation $z^2 - 4\cos(\alpha)z + 4 = 0$, where α is a real constant and $0 < \alpha < \frac{\pi}{2}$.

Find the roots z_1 and z_2 of this equation, in terms of α, expressing your answers in polar form. 3 marks

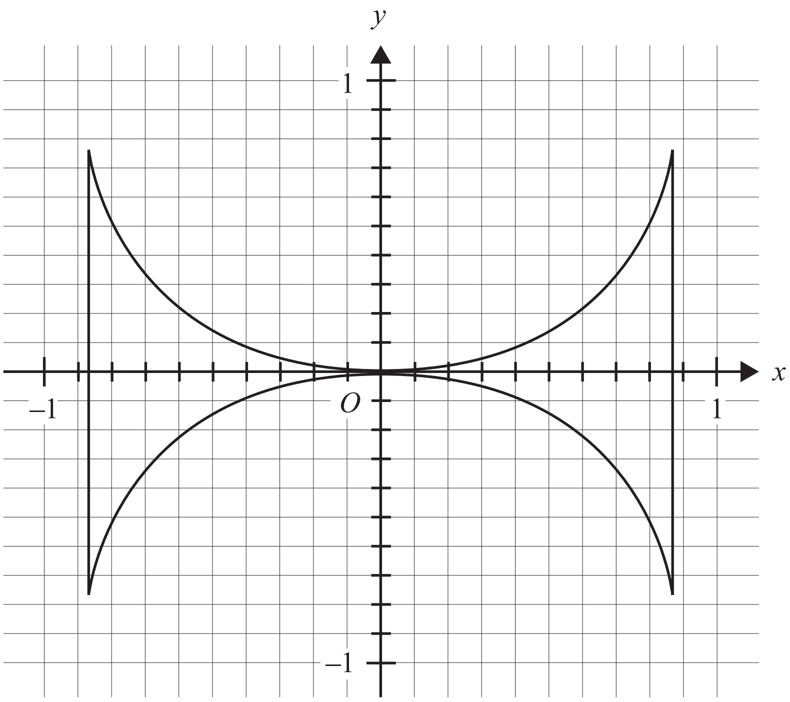
ii. Find the value of α for which $\left|\text{Arg}\left(\frac{z_1}{z_2}\right)\right| = \frac{5\pi}{6}$. 1 mark

Question 3 (10 marks)

A manufacturer of bow ties wishes to design an advertising logo, represented below, where the upper boundary curve in the first and second quadrants is given by the parametric relations

$x = \sin(t),\ y = \frac{1}{2}\sin(t)\tan(t)$ for $t \in \left[-\frac{\pi}{3}, \frac{\pi}{3}\right]$.

The logo is symmetrical about the x-axis.



a. Find an expression for $\frac{dy}{dx}$ in terms of t. 2 marks

b. Find the slope of the upper boundary curve where $t = \frac{\pi}{6}$. Give your answer in the form $\frac{a\sqrt{b}}{c}$, where a, b and c are positive integers. 1 mark

c. **i.** Verify that the cartesian equation of the upper boundary curve is $y = \frac{x^2}{2\sqrt{1-x^2}}$. 1 mark

ii. State the domain for x of the upper boundary curve. 1 mark

d. Show that $\frac{d}{dx}\left(\arcsin(x)\right) = \frac{2x^2}{\sqrt{1-x^2}} + \frac{d}{dx}\left(x\sqrt{1-x^2}\right)$ by simplifying the right-hand side of this equation. 2 marks

e. **Hence** write down an antiderivative in terms of x, to be evaluated between two appropriate terminals, and find the area of the advertising logo. 3 marks

Question 4 (12 marks)

The position vector $\underset{\sim}{r}(t)$, from origin O, of a model helicopter t seconds after leaving the ground is given by

$$\underset{\sim}{r}(t) = \left(50 + 25\cos\left(\frac{\pi t}{30}\right)\right)\underset{\sim}{i} + \left(50 + 25\sin\left(\frac{\pi t}{30}\right)\right)\underset{\sim}{j} + \frac{2t}{5}\underset{\sim}{k}$$

where $\underset{\sim}{i}$ is a unit vector to the east, $\underset{\sim}{j}$ is a unit vector to the north and $\underset{\sim}{k}$ is a unit vector vertically up. Displacement components are measured in metres.

a. **i.** Find the time, in seconds, required for the helicopter to gain an altitude of 60 m. 1 mark

ii. Find the angle of elevation from O of the helicopter when it is at an altitude of 60 m. Give your answer in degrees, correct to the nearest degree. 2 marks

b. After how many seconds will the helicopter first be directly above the point of take-off? 1 mark

c. Show that the velocity of the helicopter is perpendicular to its acceleration. 3 marks

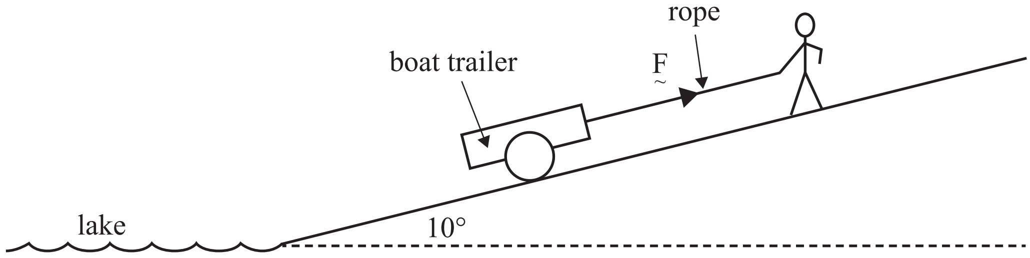
d. Find the speed of the helicopter in ms^{-1}, giving your answer correct to two decimal places. 2 marks

e. A treetop has position vector $\underset{\sim}{r} = 60\underset{\sim}{i} + 40\underset{\sim}{j} + 8\underset{\sim}{k}$.

Find the distance of the helicopter from the treetop after it has been travelling for 45 seconds. Give your answer in metres, correct to one decimal place. 3 marks

Question 5 (12 marks)

A boat ramp at the edge of a deep lake is inclined at an angle of 10° to the horizontal. A 250 kg boat trailer on the ramp is unhitched from a car and a man attempts to lower the trailer down the ramp using a rope parallel to the ramp, as shown in the diagram below.



Assume negligible friction forces in this situation.

a. Calculate the constant force, F newtons, that would be required to prevent the trailer from moving down the ramp. Give your answer correct to the nearest newton. 1 mark

b. If the man exerts a force of 200 N via the rope, find the acceleration of the trailer down the ramp, assuming negligible friction forces and air resistance. Give your answer in ms^{-2}, correct to three decimal places. 2 marks

c. Using your result for acceleration from **part b.**, find the speed of the trailer in ms^{-1}, correct to two decimal places, after it has moved 30 m down the ramp, having started from rest. 2 marks

When the trailer rolls into the water, it stops, then sinks vertically from rest so that its depth x metres after t seconds is given by the differential equation

$$\frac{d^2x}{dt^2} = 1.4\left(7 - \frac{dx}{dt}\right)$$

d. **i.** Show that the above differential equation can be written as

$1.4\frac{dx}{dv} = -1 + \frac{7}{7-v}$, where $v = \frac{dx}{dt}$. 2 marks

ii. **Hence**, show by integration that $1.4x = -v - 7\log_e(7 - v) + 7\log_e(7)$. 1 mark

When the trailer has sunk to a depth of D metres, it is descending at a rate of 5 ms^{-1}.

iii. Find D, correct to one decimal place. 1 mark

iv. Write down a definite integral for the time, in seconds, taken for the trailer to sink to the depth of *D* metres and evaluate this integral correct to one decimal place. 3 marks

END OF QUESTION AND ANSWER BOOK

SPECIALIST MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Instructions

Detach this formula sheet during reading time.
This formula sheet is provided for your reference.

Specialist Mathematics formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of a triangle:	$\frac{1}{2}bc\sin A$
sine rule:	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
cosine rule:	$c^2 = a^2 + b^2 - 2ab\cos C$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ hyperbola: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Circular (trigonometric) functions

$\cos^2(x) + \sin^2(x) = 1$

$1 + \tan^2(x) = \sec^2(x)$ $\cot^2(x) + 1 = \operatorname{cosec}^2(x)$

$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$ $\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$

$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$ $\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$

$\tan(x+y) = \frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$ $\tan(x-y) = \frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$

$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$

$\sin(2x) = 2\sin(x)\cos(x)$ $\tan(2x) = \frac{2\tan(x)}{1-\tan^2(x)}$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (complex numbers)

$z = x + yi = r(\cos\theta + i\sin\theta) = r\,\text{cis}\,\theta$

$|z| = \sqrt{x^2 + y^2} = r$

$-\pi < \text{Arg}\, z \le \pi$

$z_1 z_2 = r_1 r_2\,\text{cis}(\theta_1 + \theta_2)$

$\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}\text{cis}(\theta_1 - \theta_2)$

$z^n = r^n\,\text{cis}(n\theta)$ (de Moivre's theorem)

Calculus

$\dfrac{d}{dx}\left(x^n\right) = nx^{n-1}$

$\int x^n dx = \dfrac{1}{n+1}x^{n+1} + c, n \ne -1$

$\dfrac{d}{dx}\left(e^{ax}\right) = ae^{ax}$

$\int e^{ax} dx = \dfrac{1}{a}e^{ax} + c$

$\dfrac{d}{dx}\left(\log_e(x)\right) = \dfrac{1}{x}$

$\int \dfrac{1}{x} dx = \log_e|x| + c$

$\dfrac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$

$\int \sin(ax)\,dx = -\dfrac{1}{a}\cos(ax) + c$

$\dfrac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$

$\int \cos(ax)\,dx = \dfrac{1}{a}\sin(ax) + c$

$\dfrac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$

$\int \sec^2(ax)\,dx = \dfrac{1}{a}\tan(ax) + c$

$\dfrac{d}{dx}\left(\sin^{-1}(x)\right) = \dfrac{1}{\sqrt{1-x^2}}$

$\int \dfrac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\left(\dfrac{x}{a}\right) + c, a > 0$

$\dfrac{d}{dx}\left(\cos^{-1}(x)\right) = \dfrac{-1}{\sqrt{1-x^2}}$

$\int \dfrac{-1}{\sqrt{a^2-x^2}}dx = \cos^{-1}\left(\dfrac{x}{a}\right) + c, a > 0$

$\dfrac{d}{dx}\left(\tan^{-1}(x)\right) = \dfrac{1}{1+x^2}$

$\int \dfrac{a}{a^2+x^2}dx = \tan^{-1}\left(\dfrac{x}{a}\right) + c$

product rule: $\dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

quotient rule: $\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

chain rule: $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx}$

Euler's method: If $\dfrac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$

acceleration: $a = \dfrac{d^2x}{dt^2} = \dfrac{dv}{dt} = v\dfrac{dv}{dx} = \dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right)$

constant (uniform) acceleration: $v = u + at$ $\quad s = ut + \dfrac{1}{2}at^2$ $\quad v^2 = u^2 + 2as$ $\quad s = \dfrac{1}{2}(u+v)t$

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r$

$\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos\theta = x_1 x_2 + y_1 y_2 + z_1 z_2$

$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$

Mechanics

momentum: $\underset{\sim}{p} = m\underset{\sim}{v}$

equation of motion: $\underset{\sim}{R} = m\underset{\sim}{a}$

friction: $F \le \mu N$

END OF FORMULA SHEET