

**‘2016 Examination Package’ -
Trial Examination 5 of 5**

STUDENT
NUMBER

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Letter

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SPECIALIST MATHEMATICS

Written examination 1

(TSSM’s 2015 trial exam updated for the current study design)

Reading time: 15 minutes

Writing time: 1 hour

QUESTION & ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
8	8	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper and/or white out liquid/tape.
- No calculator is permitted in this examination.

Materials supplied

- Question and answer book of 12 pages.

Instructions

- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.

Unless otherwise specified, an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagram in this book are not drawn to scale.

Take the acceleration due to gravity to have magnitude $g \text{ m/s}^2$, where $g = 9.8$.

Question 1 (6 marks)

Consider vectors $\vec{u} = 2\vec{i} - \vec{j} + 2\vec{k}$, $\vec{v} = (8 + 3\sqrt{11})\vec{i} + 7\vec{j}$.

a. Show that the magnitude of $\vec{v}$ is $|\vec{v}| = 6 + 4\sqrt{11}$

2 marks

b. Find the angle between $\vec{u}$ and $\vec{v}$.

2 marks

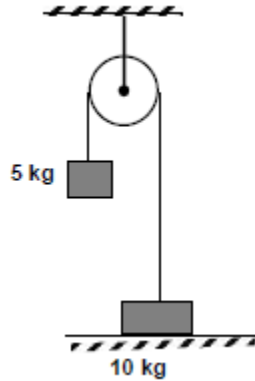
c. Find the resolute vector of $\vec{v}$ in the direction of $\vec{u}$.

2 marks

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Question 2 (5 marks)

Two masses of 5 kg and 10 kg connected by a massless string passing over a frictionless pulley are in equilibrium as shown in the figure below. One of the mass is resting on the surface.



- a.** Label all the forces in the diagram above and hence find tension in the string.

4 marks

- b.** Find the normal reaction on the 10kg block.

1 mark

Question 3 (7 marks)

The position vector of a particle at time $t \geq 0$ is given by

$$\tilde{r}(t) = 4(e^t + e^{-t})\tilde{i} + 3(e^t - e^{-t})\tilde{j}$$

- a.** Show that the Cartesian equation of the path followed by the particle is

$$\frac{x^2}{64} - \frac{y^2}{36} = 1$$

2 marks

- b.** Write down the domain and range of the path.

1 mark

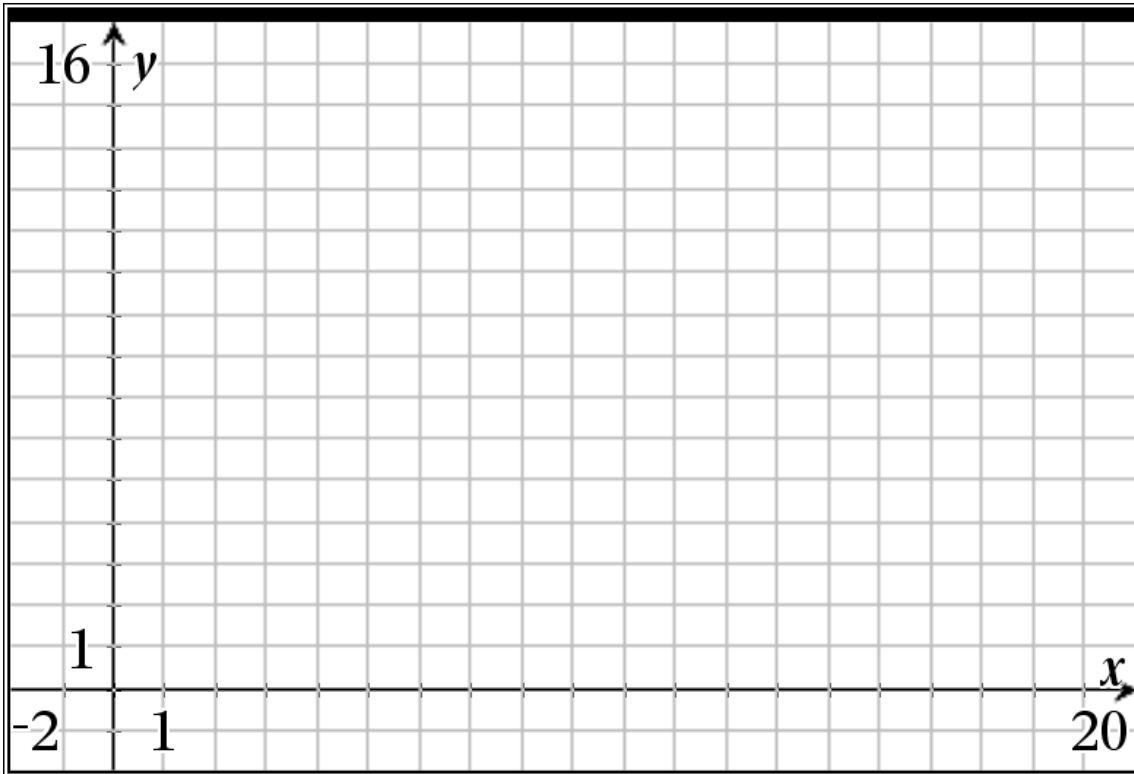
- c.** Find the equation of the asymptote of the path.

1 mark

TURN OVER

- d. Sketch the graphs of the path and its asymptote in the coordinate plane below, labelling the x-intercept.

3 marks



Question 4 (3 marks)

$P(z)$ is a polynomial given by $P(z) = 2z^3 + 5z^2 + 6z + 2$ over the complex field.

- a. Show that $z = -\frac{1}{2}$ is a root of $P(z)$.

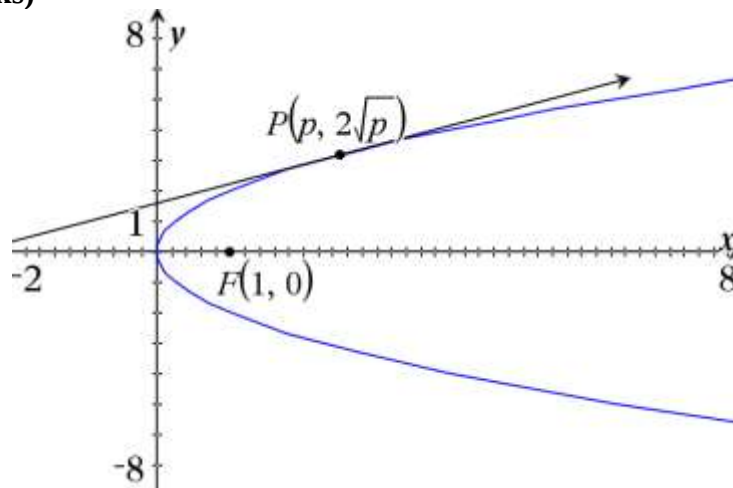
1 mark

- b. Find the other roots of $P(z)$ in the form $rcis(\theta)$.

2 marks

TURN OVER

Question 5 (4 marks)



The equation of the curve shown above is given by $y^2 = 4x$. $P(p, 2\sqrt{p})$ is a point on the graph and $F(1, 0)$ is a point on the x-axis.

- a. Find the gradient of the tangent at the point P .

1 mark

- b. α is the angle between the tangent and the positive direction of the x-axis and β is the angle between the line PF and the positive direction of the x-axis. **Show** that $\beta = 2\alpha$.

3 marks

Question 6 (5 marks)

- a. Express $1 + \cos(4x)$ in terms of $\cos(2x)$.

1 mark

- b. Use an appropriate substitution u to write the following integral in terms of u only.

$$\int \frac{\sin(2x)}{1 + \cos(4x)} dx$$

2 marks

- c. Evaluate the definite integral

$$\int_{\frac{\pi}{8}}^{\frac{\pi}{6}} \frac{\sin(2x)}{1 + \cos(4x)} dx$$

2 marks

TURN OVER

Question 7 (4 marks)

- a.** Find an antiderivative of the function $f(x) = \frac{16 \arctan(x)}{1+x^2}$

2 marks

- b.** Find the exact area of the region enclosed by $y = f(x)$, $x = -1$, $x = 1$ and the x-axis.

2 marks

Question 8 (6 marks)

- a.** Write $2x - x^2$ in the form $a - (x - b)^2$.

1 mark

- b.** Find an antiderivative of

$$\int \frac{1}{x(x-2)} dx$$

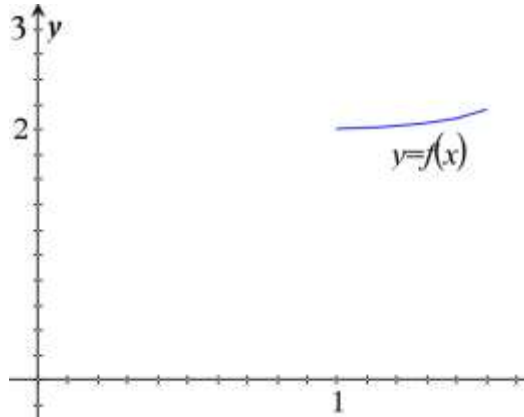
2 marks

Question 8 - continued
TURN OVER

- c. A solid revolution is formed by rotating the region enclosed by

$$0 \leq y \leq 1 + \frac{1}{\sqrt{2x-x^2}} \text{ and } 1 \leq x \leq \frac{3}{2}$$

about the x-axis. The graph of $f(x) = 1 + \frac{1}{\sqrt{2x-x^2}}$, $1 \leq x \leq \frac{3}{2}$ is shown below.



Find the volume of the solid revolution.

3 marks

END OF QUESTION AND ANSWER BOOK