

Trial Examination 2015

VCE Specialist Mathematics Units 3&4

Written Examination 1

Question and Answer Booklet

Reading time: 15 minutes

Writing time: 1 hour

Student's Name: _____

Teacher's Name: _____

Structure of Booklet

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
9	9	40

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.

Students are not permitted to bring into the examination room: notes of any kind, a calculator of any type, blank sheets of paper and/or white out liquid/tape.

Materials supplied

Question and answer booklet of 9 pages and a sheet of miscellaneous formulas.

Working space is provided throughout the booklet.

Instructions

Write your **name** and your **teacher's name** in the space provided above on this page.

All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2015 VCE Specialist Mathematics Units 3&4 Written Examination 1.

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Instructions

Answer **all** questions in the spaces provided.

Unless otherwise specified, an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ m/s}^2$, where $g = 9.8$.

Question 1 (4 marks)

Find the equation of the tangent to the curve $y^2 = (2x + 3)^4$ at the point $\left(-\frac{1}{2}, 4\right)$. Give your answer in the form $y = mx + c$.

Question 2 (3 marks)

Solve the equation $\cos^2(x) = \sin^2(x) - 15 \sin(x) + 8$, $0 < x < 2\pi$.

Question 3 (5 marks)

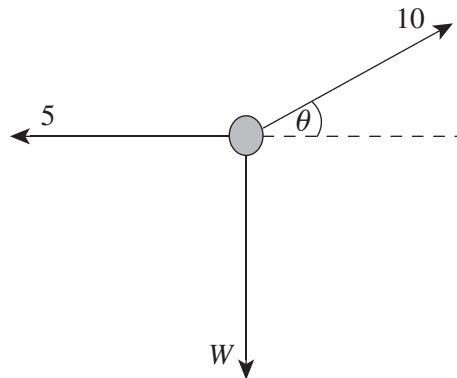
Consider $z = x + yi$, $x, y \in \mathbb{R}$, $z \in \mathbb{C}$.

- a.** Show that $\bar{z}^2 = x^2 - y^2 - 2xyi$. 2 marks

- b.** If $z^2 = \bar{z}^2$, show that z must be either real or purely imaginary. 3 marks

Question 4 (3 marks)

The diagram below shows a particle of weight W newtons that is held in equilibrium by two forces. The force of magnitude 5 newtons acts in a horizontal direction and the force of magnitude 10 newtons acts at an angle θ above the horizontal. All three forces act in the same vertical plane.



Find the mass of the particle.

Question 5 (5 marks)

A particle moves in a straight line so that its displacement, x , from a fixed origin, O , is related to its velocity, v , by the equation $v = 3\sqrt{4 - x^2}$.

- a.** Show that the acceleration, $\ddot{x}$, of the particle is given by the differential equation $\ddot{x} = -9x$. 3 marks

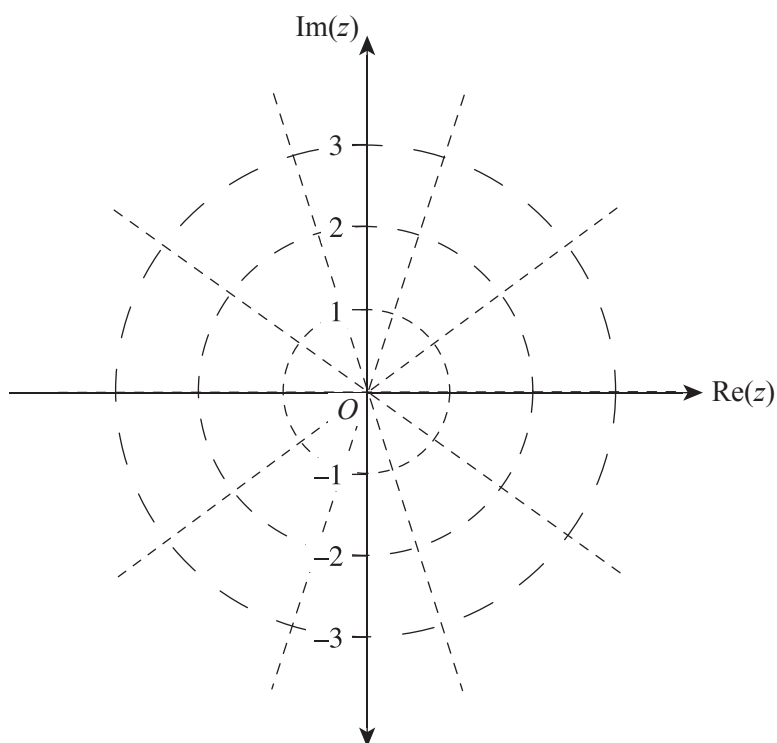
- b.** Hence verify that the particle's displacement at time t is given by $x = 2\sin(3t)$. 2 marks

Question 6 (3 marks)

Let $z = \cos(\theta) + i\sin(\theta)$, $z \in \mathbb{C}$.

- a. On the Argand diagram below, plot the roots of the equation $z^5 = 1$.

1 mark



When plotted on an Argand diagram, the roots of the equation $z^5 = 1$ form a regular pentagon.

- b. Find the area of the regular pentagon, giving your answer in the form $k\sin(\alpha)$.

2 marks

Question 7 (6 marks)

Find $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \tan^3(x) dx$. Give your answer in the form $a - b \log_e(c)$, where a , b and c are rational numbers.

Question 8 (5 marks)

The position vector of a particle at time t , $t \geq 0$ is defined by $\underline{r}(t) = a \cos(2t)\underline{i} + a \sin(2t)\underline{j} + bt\underline{k}$, $a, b > 0$.

- a.** Find the particle's velocity. 1 mark

- b.** Show that the particle moves in such a way that it always makes a fixed angle, θ , with the $\underline{k}$ direction. 4 marks
