

**The Mathematical Association of Victoria**

**Trial Exam 2015**

**SPECIALIST MATHEMATICS**

**Written Examination 1**

**STUDENT NAME** \_\_\_\_\_

**Reading time: 15 minutes**  
**Writing time: 1 hour**

**QUESTION AND ANSWER BOOK**

**Structure of Book**

| <i>Number of questions</i> | <i>Number of questions to be answered</i> | <i>Number of marks</i> |
|----------------------------|-------------------------------------------|------------------------|
| 9                          | 9                                         | 40                     |

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers,
- Students are NOT permitted to bring into the examination room: notes of any kind, a calculator of any type, blank sheets of paper and/or white out liquid/tape.

**Materials supplied**

- Question and answer book of 13 pages with a detachable sheet of miscellaneous formulas in the centrefold.

**Instructions**

- Detach the formula sheet from the centre of this book during reading time.
- Write your **name** in the space provided above on this page.
- All written responses must be in English.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**

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**Question 1** (5 marks)

Consider the polynomial  $2z^3 + 9iz^2 + 10z + 7i$  where  $z \in \mathbb{C}$ .

- a.** Show that  $2z + i$  is a factor of the polynomial.

2 marks

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- b.** Hence find the remaining linear factors of the polynomial.

3 marks

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A hyperbola is defined by the relation  $2x^2 - y^2 + 4y - 8 = 0$ .

- 2 marks

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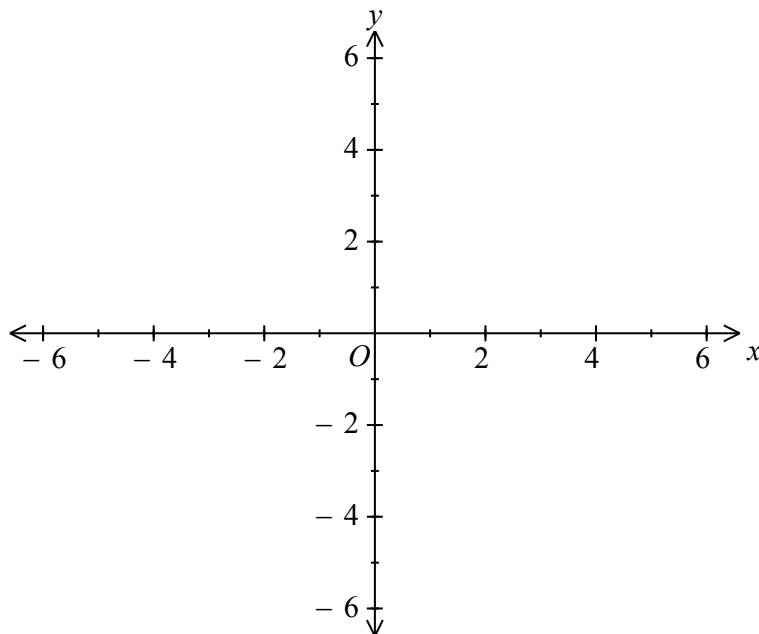
- 1 mark

[illegible]

**Question 3** (4 marks)

Consider the function  $h : (-\infty, 0) \rightarrow \mathbb{R}$ ,  $h(x) = \frac{x^3 + 1}{2x^2}$  and let  $h^{-1}$  be the **inverse** function of  $h$ .

Sketch the graph of  $y = h^{-1}(x)$ . Label all axes intercepts with their coordinates and all asymptotes with their equation.



**Working space**

Let  $g(x) = 2 \arctan(x)$ .

- 4 marks

[illegible]

- 4 marks

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**Question 5** (7 marks)

Consider the function  $f$  with rule  $f(x) = \frac{1}{\frac{\pi}{4} - \arccos(2x)}$ .

- a. i.** Find the domain of  $f$ .

2 marks

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- ii.** Hence or otherwise find the range of  $f$ .

2 marks

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3 marks

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The vectors  $(b - a^2) \underset{\sim}{i} + \underset{\sim}{j} - b \underset{\sim}{k}$  and  $2(a - b) \underset{\sim}{i} + 4 \underset{\sim}{j} + 3(a - b) \underset{\sim}{k}$ , where  $a, b \in \mathbb{R} \setminus \{0\}$ , are parallel.

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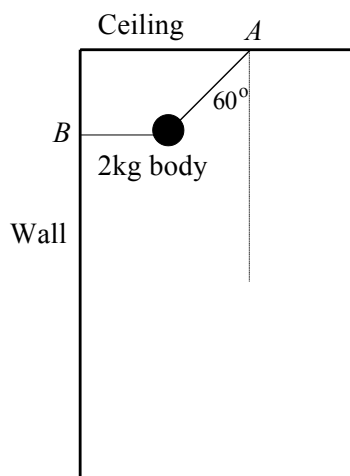


The acceleration, in  $\text{ms}^{-2}$ , of a 5 kg mass moving in a straight line is given by  $\sqrt{4-v^2}$  where  $v \text{ ms}^{-1}$  is its velocity. The displacement of the mass from a fixed origin is  $x$  metres. If the mass is at rest where  $x = 2$ , calculate the magnitude of the momentum of the mass where  $x = \frac{5}{2}$ .

[illegible]

**Question 9 (3 marks)**

A body of mass 2 kg is held by two light inextensible wires in a lift. One wire is attached to the roof of the lift at  $A$  and the other to a wall at  $B$ . The wire attached to the roof is at an angle  $60^\circ$  to the vertical and the other wire is horizontal. Both wires are made of the same material.



The type of wire used will break if it is subjected to a tension of more than  $9g$  N. Find in terms of  $g$  the maximum allowable upwards acceleration of the lift so that neither wire will break.

This image shows a full page of white paper with horizontal blue or grey ruling lines. The lines are evenly spaced and run across the width of the page, providing a template for writing. There are no margins, text, or other markings on the page.