

**SPECIALIST MATHEMATICS UNIT 4**  
**SAC 3: INTEGRAL CALCULUS AND APPLICATIONS TEST**

NAME: \_\_\_\_\_

**PAPER TWO: Technology Active**

**Time: 25 Minutes    Total = 18 marks**

**SECTION A: MULTIPLE CHOICE**

*Please circle the correct answer.*

**Question 1**

An antiderivative of  $\frac{1}{x^2 - 2x + 2}$  is:

- A.  $-(x^2 - 2x + 2)^{-2}$
- B.  $\log_e(x^2 - 2x + 2)$
- C.  $\log_e \left| \frac{x-2}{x+1} \right|$
- D.  $\operatorname{arcsec}(x-1)$
- E.  $\arctan(x-1)$

**Question 2**

A solid is constructed by rotating the function  $y = 1 - \cos(2x)$ , where  $0 \leq x \leq \frac{\pi}{2}$ , about the y-axis. The volume of this solid is:

- A.  $\frac{\pi(\pi^2 - 4)}{4}$
- B.  $\frac{-\pi(\pi^2 - 20)}{4}$
- C.  $\frac{\pi^3}{2}$
- D.  $\frac{-\pi(\pi^2 - 4\pi - 4)}{4}$
- E.  $\frac{\pi(\pi^2 + 4)}{4}$

### Question 3

The region enclosed by the graph of  $y = x^2 + 1$  and the lines  $y = 1$  and  $y = 4$  is rotated about the  $y$ -axis to form a solid of revolution. The volume of the solid is given by

- A.  $\pi \int_0^{\sqrt{3}} (x^2 + 1) dx$
- B.  $\pi \int_1^4 (y - 1) dx$
- C.  $\pi \int_1^4 (x^2 + 1) dx$
- D.  $\pi \int_1^4 (y - 1) dy$
- E.  $\pi \int_0^{\sqrt{3}} (y - 1) dy$

### Question 4

Using a suitable substitution,  $\int_{-\frac{\pi}{6}}^{\frac{\pi}{3}} (\tan(x) \log_e(\sec(x))) dx$  can be expressed completely in terms of  $u$  as:

- A.  $\int_{\frac{2}{\sqrt{3}}}^2 (\log_e(u)) du$
- B.  $\int_{-\log\left(\frac{\sqrt{3}}{2}\right)}^{\log_e(2)} (u) du$
- C.  $-\int_{-\frac{\sqrt{3}}{2}}^{\frac{1}{2}} (u) du$
- D.  $\int_{-\frac{\pi}{6}}^{\frac{\pi}{3}} (\log_e(u)) du$
- E.  $\int_{-\frac{\pi}{6}}^{\frac{\pi}{3}} (u) du$

### Question 5

If the substitutions  $u = \frac{x}{2}$  is made, the integral  $\int_2^4 \frac{1 - \left(\frac{x}{2}\right)^2}{x} dx$  becomes:

A.  $\int_1^2 \frac{1 - u^2}{u} du$

B.  $\int_2^4 \frac{1 - u^2}{u} du$

C.  $\int_1^2 \frac{1 - u^2}{2u} du$

D.  $\int_1^2 \frac{1 - u^2}{4u} du$

E.  $\int_2^4 \frac{1 - u^2}{2u} du$

## SECTION B: SHORT ANSWER/ANALYSIS

### Question 6 (7 marks)

A wine glass is formed by rotating, around the  $y$ -axis, the graph defined by function

$$f : [0, 2] \rightarrow \mathbb{R}, f(x) = \frac{1}{10}(2 + 5x^3). \text{ All measurements are in cm.}$$

- a) Sketch the graph of  $f(x)$  clearly labelling coordinates of endpoints.

2 marks

- b) State a definite integral that would find the volume of the glass formed, when full, after it is rotated around the  $y$ -axis.

2 marks

- c) Evaluate this volume, in cubic centimetres.

1 mark

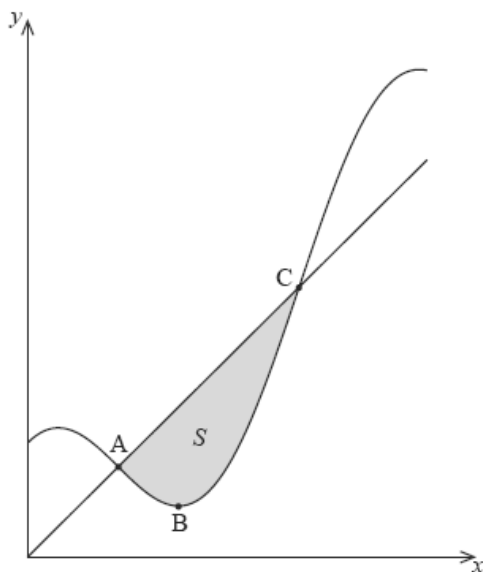
The curve  $f : [0, a] \rightarrow \mathbb{R}, f(x) = \frac{1}{10}(2 + 5x^3)$  is rotated about the  $x$ -axis now and the volume of the solid obtained in this way is equal by  $\frac{22\pi}{175}$  cubic centimetres.

- d) Find the value of  $a$ .

2 marks

**Question 7** (6 marks)

Let  $f$  be a function defined by  $f(x) = x + 2 \cos x$ ,  $x \in [0, 2\pi]$ . The diagram below shows a region  $S$  bound by the graph of  $f$  and the line  $y = x$ .



$A$  and  $C$  are the points of intersection of the line  $y = x$  and the graph of  $f$ , and  $B$  is the minimum point of  $f$ .

- a) If  $A$ ,  $B$  and  $C$  have  $x$ -coordinates  $\frac{a\pi}{2}$ ,  $\frac{b\pi}{2}$  and  $\frac{c\pi}{2}$ , where  $a, b, c \in \mathbb{Z}^+$ , find the values of  $a$ ,  $b$  and  $c$ .

3 marks

- b) Write down a definite integral which would find the area of region  $S$ .

2 marks

- c) Hence find the area of the region  $S$ .

1 mark

**END OF PAPER TWO**