

Year 2015

VCE

Specialist Mathematics

Trial Examination 1



KILBAHA MULTIMEDIA PUBLISHING
PO BOX 2227
KEW VIC 3101
AUSTRALIA

TEL: (03) 9018 5376
FAX: (03) 9817 4334
kilbaha@gmail.com
<http://kilbaha.com.au>

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Victorian Certificate of Education 2015

STUDENT NUMBER

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SPECIALIST MATHEMATICS

Trial Written Examination 1

Reading time: 15 minutes

Total writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are not permitted to bring into the examination room: notes of any kind, a calculator, blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 18 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.
- Working space is provided throughout the booklet.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Take the **acceleration due to gravity** to have magnitude $g \text{ m/s}^2$, where $g = 9.8$.

Find an antiderivative of $\frac{2x-5}{x^2-25}$, $x \in \mathbb{R} \setminus \{-5, 5\}$.

[illegible]

Question 2 (4 marks)

Let $f(z) = z^3 + (2 - 2\sqrt{3})z^2 + cz + 8$, where c is a real constant and $z \in \mathbb{C}$.

- a.** Given that $z = 2\operatorname{cis}\left(-\frac{\pi}{6}\right)$ is a solution of $f(z) = 0$, find a quadratic factor of $f(z)$. 2 marks

- b.** Hence find all the roots of $f(z) = 0$ and the value of c . 2 marks

Question 3 (6 marks)

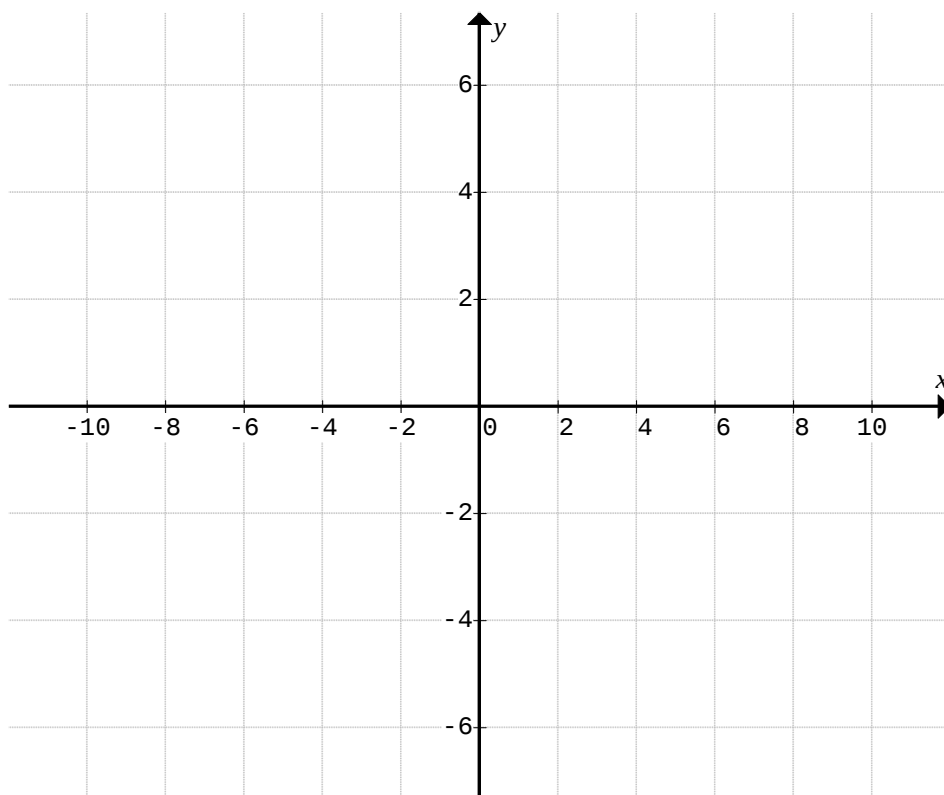
The position vector of a particle at a time t is given by $\underline{r}(t) = 2\left(t + \frac{1}{t}\right)\underline{i} + \left(t - \frac{1}{t}\right)\underline{j}$, $t > 0$

- a. Show that the particle moves on the hyperbola $\frac{x^2}{16} - \frac{y^2}{b^2} = 1$ and find the value of the positive real constant b .

2 marks

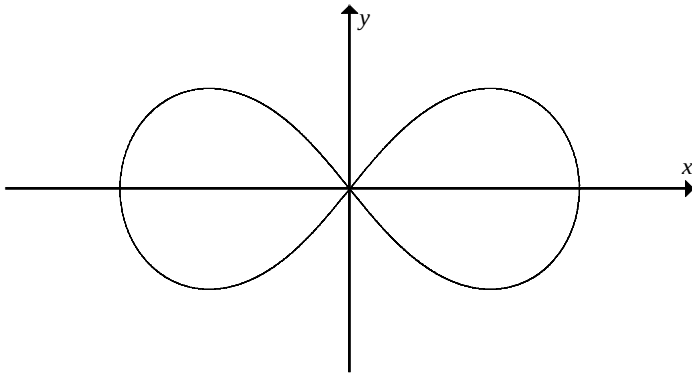
- b. Sketch the path of the particle on the axes below, showing all important features.

2 marks



Question 5 (3 marks)

The relation shown has the equation $(x^2 + y^2)^2 = 4(x^2 - y^2)$.



- a. Find an expression for $\frac{dy}{dx}$ in terms of both x and y . 2 marks

- b. The tangent line is horizontal when $x = \pm \frac{\sqrt{c}}{2}$. Determine the value of c . 1 mark

Question 6 (3 marks)

A tank contains V_0 litres of pure water. A salt solution of concentration b grams per litre is poured into the tank at a rate of g litres per minute, and the well-stirred mixture flows out at a rate of f litres per minute. The amount of salt in the tank at a time t minutes, where $t \geq 0$ is Q grams and satisfies the differential equation $\frac{dQ}{dt} = 12 - \frac{4Q}{5+2t}$.

- a. State the values of V_0, b, g and f .

1 mark

- b. Use Euler's method with a step size of 0.5, to find the value of Q when $t = 1$.

2 marks

Question 7 (5 marks)

Consider the vectors $\vec{a} = 3\vec{i} - 2\vec{j} - 4\vec{k}$ and $\vec{b} = -2\vec{i} + \vec{j} + t\vec{k}$.

- a. Find the value of the scalar t if the vectors $\vec{a}$ and $\vec{b}$ are perpendicular.

1 mark

- b. Find the value of the scalar t if the vectors $\vec{a}$ and $\vec{b}$ are equal in length.

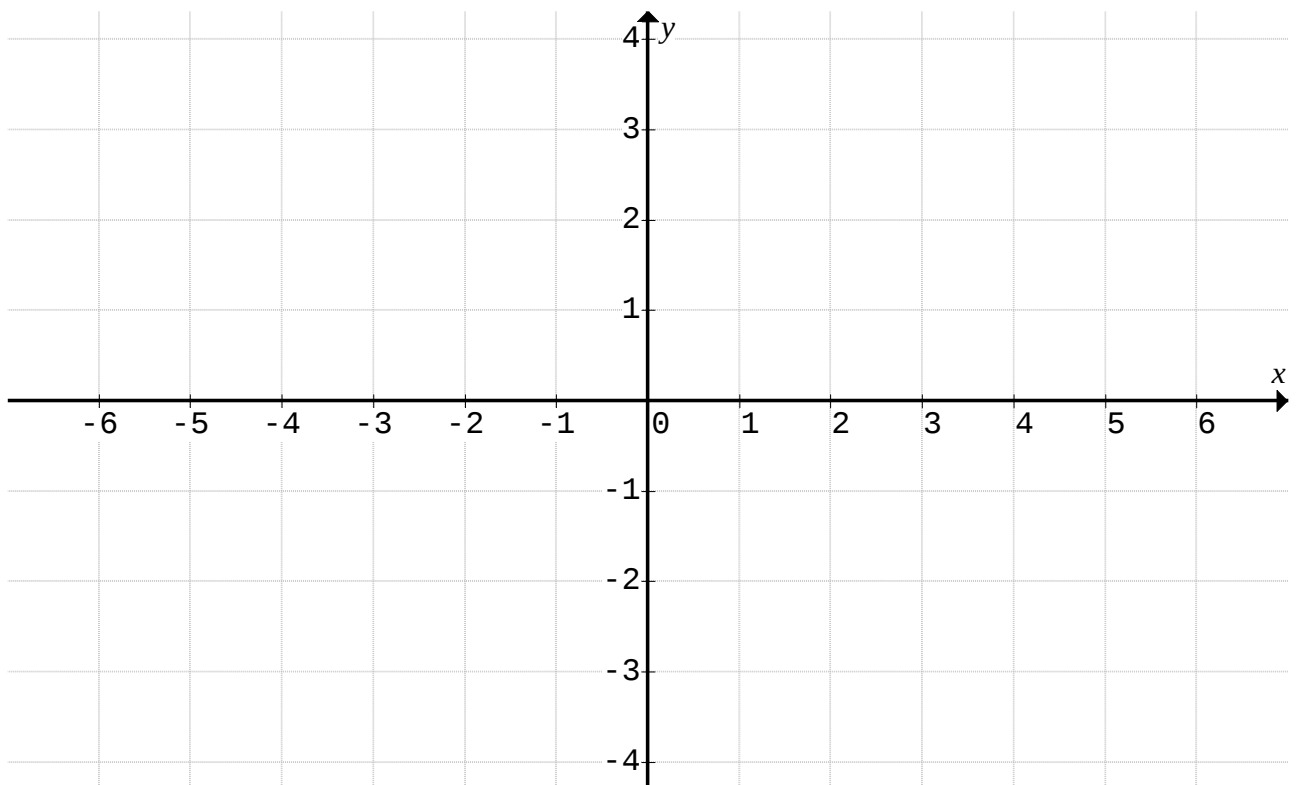
2 marks

- c. Find the value of the scalar t if the vector $\vec{b}$ makes an angle of 150° with the z -axis.

2 marks

Question 8 (4 marks)

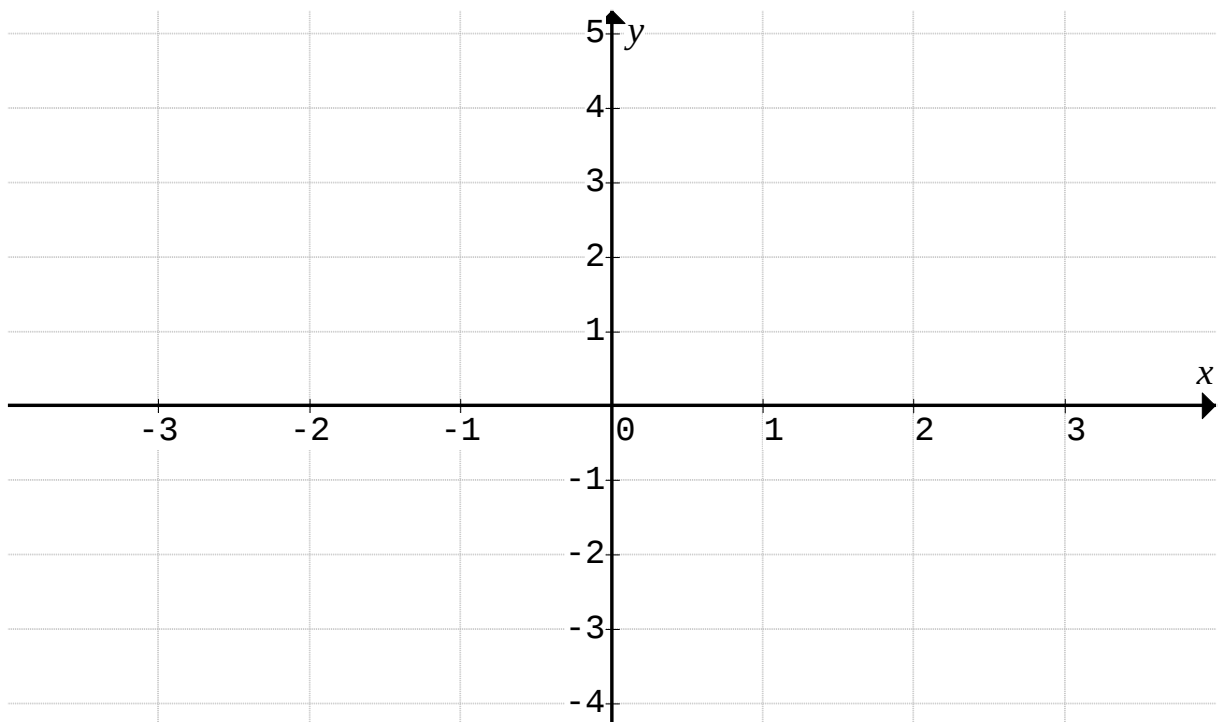
Sketch the graph of $y = \frac{x^2 + 9}{3x}$ on the axes below. Give the coordinates of any turning points and axial intercepts and state the equations of all straight line asymptotes.



Question 10 (6 marks)

- a. Sketch the graph of the function $f(x) = \frac{4}{\pi} \arccos\left(\frac{x}{3}\right)$ on the axes below, stating the coordinates of all axial intercepts and the endpoints.

2 marks



SPECIALIST MATHEMATICS

Written examination 1

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

curved surface area of a cylinder: $2\pi rh$

volume of a cylinder: $\pi r^2 h$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a pyramid: $\frac{1}{3}Ah$

volume of a sphere: $\frac{4}{3}\pi r^3$

area of triangle: $\frac{1}{2}bc \sin(A)$

sine rule: $\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$

cosine rule: $c^2 = a^2 + b^2 - 2ab \cos(C)$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ hyperbola: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Circular (trigonometric) functions

$$\cos^2(x) + \sin^2(x) = 1$$

$$1 + \tan^2(x) = \sec^2(x)$$

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

$$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$$

$$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$$

$$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$$

$$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$$

$$\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$$

$$\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (Complex Numbers)

$$z = x + yi = r(\cos(\theta) + i \sin(\theta)) = r \operatorname{cis}(\theta)$$

$$|z| = \sqrt{x^2 + y^2} = r$$

$$-\pi < \operatorname{Arg}(z) \leq \pi$$

$$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$$

$$z^n = r^n \operatorname{cis}(n\theta) \text{ (de Moivre's theorem)}$$

Vectors in two and three dimensions

$$\underline{r} = x\underline{i} + y\underline{j} + z\underline{k}$$

$$|\underline{r}| = \sqrt{x^2 + y^2 + z^2} = r$$

$$\underline{r}_1 \cdot \underline{r}_2 = r_1 r_2 \cos(\theta) = x_1 x_2 + y_1 y_2 + z_1 z_2$$

$$\dot{\underline{r}} = \frac{d\underline{r}}{dt} = \frac{dx}{dt} \underline{i} + \frac{dy}{dt} \underline{j} + \frac{dz}{dt} \underline{k}$$

Mechanics

momentum: $\underline{p} = m\underline{v}$

equation of motion: $\underline{R} = m\underline{a}$

sliding friction: $F \leq \mu N$

constant (uniform) acceleration:

$$v = u + at \quad s = ut + \frac{1}{2}at^2 \quad v^2 = u^2 + 2as \quad s = \frac{1}{2}(u + v)t$$

acceleration: $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx} \left(\frac{1}{2}v^2 \right)$

Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$$

$$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$$

$$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$$

$$\frac{d}{dx}(\tan(ax)) = a \sec^2(ax)$$

$$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, \quad n \neq -1$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$$

$$\int \frac{1}{x} dx = \log_e(x) + c, \quad \text{for } x > 0$$

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$$

$$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$$

$$\int \sec^2(ax) dx = \frac{1}{a} \tan(ax) + c$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c, \quad a > 0$$

$$\int \frac{-1}{\sqrt{a^2 - x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, \quad a > 0$$

$$\int \frac{a}{a^2 + x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$$

product rule: $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$

Euler's method If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$

END OF FORMULA SHEET