

2015 VCAA Specialist Mathematics Exam 1 Solutions

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Q1a $|\vec{OA}| = |\vec{OC}|$, $a = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$

Q1b $\vec{AC} = \vec{OC} - \vec{OA} = (1 - \sqrt{3})\hat{i} + \hat{j} + \hat{k}$

$\vec{OB} = \vec{OC} + \vec{OA} = (1 + \sqrt{3})\hat{i} + \hat{j} + \hat{k}$

$\vec{AC} \cdot \vec{OB} = (1 - \sqrt{3})(1 + \sqrt{3}) + 1 + 1 = 0$, $\therefore \vec{AC} \perp \vec{OB}$

Hence the diagonals are perpendicular.

Q2a Let R newtons be the reaction force.

$R - 20 \times 9.8 = 20 \times 1.2$, $R = 220$

Q2b Let a m s⁻² be the acceleration.

$166 - 20 \times 9.8 = 20a$, $a = -1.5$

The downward acceleration is 1.5 m s⁻².

Q3a $\tilde{r}(t) = (4t - 3)\hat{i} + 2t\hat{j} - 5\hat{k}$

$\tilde{r}(t) = (2t^2 - 3t)\hat{i} + t^2\hat{j} - 5t\hat{k} + \tilde{c}$

$\tilde{r}(0) = \tilde{c} = \tilde{i} - 2\hat{k}$, $\therefore \tilde{r}(t) = (2t^2 - 3t + 1)\hat{i} + t^2\hat{j} - (5t + 2)\hat{k}$

When $t = 2$, $\tilde{r}(2) = 3\hat{i} + 4\hat{j} - 12\hat{k}$

$|\tilde{r}(2)| = \sqrt{3^2 + 4^2 + (-12)^2} = 13$

The distance from the origin = 13 metres

Q4a $z^3 = 8i$, $z^3 - 8i = 0$, $-2i$ is a solution by inspection

$\therefore z^3 - 8i = (z + 2i)(z^2 - 2iz - 4) = 0$

$\therefore z^2 - 2iz - 4 = 0$, $z = \frac{2i \pm \sqrt{-4 + 16}}{2} = \pm\sqrt{3} + i$

The solutions are: $-2i$, $\pm\sqrt{3} + i$

Q4b Let $z - 2i = -2i$, $z - 2i = \pm\sqrt{3} + i$

$\therefore z = 0$ or $z = \pm\sqrt{3} + 3i$

Q5 $y = 2x^2 - 3$ cuts the y -axis at -3 , $x^2 = \frac{y+3}{2}$

$V = \int_{-3}^5 \pi x^2 dy = \int_{-3}^5 \frac{\pi}{2} (y+3) dy = \frac{\pi}{2} \left[\frac{(y+3)^2}{2} \right]_{-3}^5 = 16\pi$

Q6a $a = 4v^2$, $\frac{1}{2} \frac{dv^2}{dx} = 4v^2$, $\frac{dx}{dv^2} = \frac{1}{8} \times \frac{1}{v^2}$, $x = \frac{1}{8} \int \frac{1}{v^2} dv^2$

$\therefore x = \frac{1}{8} \log_e v^2 + c$, when $x = 1$, $v = e$

$\therefore c = \frac{3}{4}$ and $x = \frac{1}{8} \log_e v^2 + \frac{3}{4}$

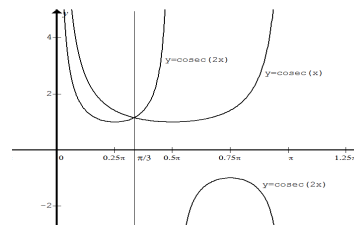
When $x = 2$, $2 = \frac{1}{8} \log_e v^2 + \frac{3}{4}$, $\log_e v^2 = 10$, $v^2 = e^{10}$, $v = e^5$

Q7a $\sin(2x) = \sin x$, $2 \sin x \cos x - \sin x = 0$

$\sin x(2 \cos x - 1) = 0$, $\therefore \sin x = 0$ or $\cos x = \frac{1}{2}$

$\therefore x = 0, \pi, 2\pi$ or $x = \frac{\pi}{3}, \frac{5\pi}{3}$

Q7b Sketch the graphs of $y = \operatorname{cosec}(2x)$ and $y = \operatorname{cosec}(x)$.



$\operatorname{cosec}(2x) < \operatorname{cosec}(x)$ for
 $x \in \left(0, \frac{\pi}{3}\right) \cup \left(\frac{\pi}{2}, \pi\right)$

Q8a $\int \tan(2x) dx = \int \frac{\sin(2x)}{\cos(2x)} dx$

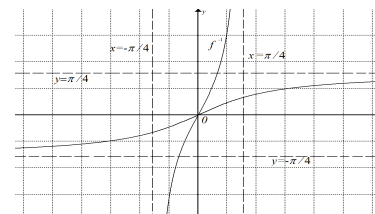
$= -\frac{1}{2} \int \frac{1}{u} du = -\frac{1}{2} \log_e |u| + c$

$= -\frac{1}{2} \log_e |\cos(2x)| + c = \frac{1}{2} \log_e |\sec(2x)| + c$

$u = \cos(2x)$
 $\frac{du}{dx} = -2 \sin(2x)$
 $-\frac{1}{2} \times \frac{du}{dx} = \sin(2x)$

Q8bi $y = -\frac{\pi}{4}$, $y = \frac{\pi}{4}$

Q8bii



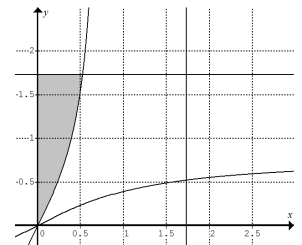
Q8c $f(\sqrt{3}) = \frac{1}{2} \arctan(\sqrt{3}) = \frac{1}{2} \times \frac{\pi}{3} = \frac{\pi}{6}$

Q8d Area of the required region = area of the shaded region

$= \sqrt{3} \times \frac{\pi}{6} - \int_0^{\frac{\pi}{6}} \tan(2x) dx$

$= \frac{\sqrt{3}\pi}{6} - \left[\frac{1}{2} \log_e |\sec(2x)| \right]_0^{\frac{\pi}{6}}$

$= \frac{\sqrt{3}\pi}{6} - \log_e \sqrt{2}$



Q9a $x^2 - xy + \frac{3}{2}y^2 = 9$ $\therefore 2x - y - x \frac{dy}{dx} + 3y \frac{dy}{dx} = 0$

$\therefore 2x - y = (x - 3y) \frac{dy}{dx}$, $\frac{dy}{dx} = \frac{2x - y}{x - 3y}$

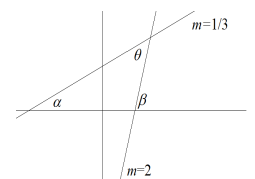
Q9b At $(3, 0)$, $m = \frac{dy}{dx} = 2$. Tangent: $y = 2(x - 3)$, $y = 2x - 6$

At $(0, \sqrt{6})$, $m = \frac{1}{3}$. Tangent: $y = \frac{1}{3}x + \sqrt{6}$

Q9c $\beta = \tan^{-1} 2$, $\alpha = \tan^{-1}(\frac{1}{3})$, $\theta = \beta - \alpha = \tan^{-1} 2 - \tan^{-1}(\frac{1}{3})$

$\therefore \tan \theta = \tan(\tan^{-1} 2 - \tan^{-1}(\frac{1}{3}))$
 $= \frac{\tan(\tan^{-1} 2) - \tan(\tan^{-1}(\frac{1}{3}))}{1 + \tan(\tan^{-1} 2) \tan(\tan^{-1}(\frac{1}{3}))} = \frac{2 - \frac{1}{3}}{1 + 2 \times \frac{1}{3}} = 1$

$\therefore \theta = \frac{\pi}{4}$



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