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Specialist Mathematics

2015

Trial Examination 2 (2 hours)

SECTION 1 Multiple-choice questions

Instructions for Section 1

Answer **all** questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ ms}^{-2}$, where $g = 9.8$.

Question 1 $(1, 1)$ is a point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

The line which is **NOT** an asymptote of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is

- A. $bx + ay = 0$
- B. $x - \sqrt{1 - a^2} y = 0$
- C. $\sqrt{1 + b^2} x - y = 0$
- D. $x - \sqrt{a^2 - 1} y = 0$
- E. $\sqrt{b^2 + 1} x + y = 0$

Question 2 The range of function f with the rule $f(x) = \cos^{-1}(ax) - \frac{\pi}{2}$ is $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$.

For $a > 0$, the domain of f is

- A. $\left[-\frac{a}{\sqrt{2}}, \frac{a}{\sqrt{2}}\right]$
- B. $\left[-\frac{1}{\sqrt{2}a}, \frac{1}{\sqrt{2}a}\right]$
- C. $\left[-\frac{\sqrt{3}a}{2}, \frac{\sqrt{3}a}{2}\right]$
- D. $\left[-\frac{\sqrt{3}}{2a}, \frac{\sqrt{3}}{2a}\right]$
- E. $\left[\frac{\sqrt{3}a}{2}, -\frac{\sqrt{3}a}{2}\right]$

Question 3 For $b \in R^+$, $\left\{z : \text{Arg}(z) = \frac{\pi}{3}\right\} \cap \{z : |z - ib\sqrt{3}| = |z - b|\}$ is

A. $\left\{\frac{b}{2} + \frac{ib\sqrt{3}}{2}\right\}$

B. $\left(\frac{b\sqrt{3}}{2}, \frac{b}{2}\right)$

C. $\left\{\frac{b\sqrt{3}}{2} + \frac{ib}{2}\right\}$

D. $\left(\frac{\sqrt{3}}{2}, \frac{i}{2}\right)$

E. $\left\{\frac{1}{2} - \frac{i\sqrt{3}}{2}\right\}$

Question 4 A possible factor of $z^5 + i$ is

A. $z + 1$

B. $z - i$

C. $z - 1$

D. $z^4 - iz^3 - z^2 + iz + 1$

E. $z^4 - iz^3 + z^2 - iz + 1$

Question 5 Given $f : \left[\frac{\pi}{2}, \frac{3\pi}{2}\right] \rightarrow R$, $f(x) = \sin x$,

A. $f^{-1}(x) = \sin^{-1} x + \frac{\pi}{2}$

B. $f^{-1}(x) = \sin^{-1} x$

C. $f^{-1}(x) = \cos^{-1} x + \frac{\pi}{2}$

D. $f^{-1}(x) = \cos^{-1} x$

E. $f^{-1}(x) = \sin^{-1} x - \frac{\pi}{2}$

Question 6 For non-zero $a, b \in R$, $\frac{d}{dx} \left[\cot^2 \left(\frac{a+2x}{b\sqrt{x}+1} \right) - \operatorname{cosec}^2 \left(\frac{a+2x}{b\sqrt{x}+1} \right) \right] =$

- A. 1
- B. $\frac{2}{b} \sin \left(\frac{a+2x}{b\sqrt{x}+1} \right)$
- C. 0
- D. $\frac{2}{b} \cos \left(\frac{a+2x}{b\sqrt{x}+1} \right)$
- E. -1

Question 7 Given $\frac{z+1}{z+i} = i$, $\operatorname{Arg}(z) =$

- A. 0.79
- B. 2.4
- C. -0.79
- D. -0.75π
- E. -0.25π

Question 8 For $x \in \left[-\frac{\pi}{2}, \frac{3\pi}{2} \right]$, the exact solution(s) to $(\sin x + 1) \left(\tan x + \frac{3}{2} \right) = 0$ is/are

- A. $-\frac{\pi}{2}, \frac{3\pi}{2}, \tan^{-1} \left(-\frac{3}{2} \right)$
- B. $\frac{3\pi}{2}, \tan^{-1} \left(-\frac{3}{2} \right)$
- C. $-\frac{\pi}{2}, \tan^{-1} \left(-\frac{3}{2} \right)$
- D. $-\frac{\pi}{2}, \frac{3\pi}{2}$
- E. $\tan^{-1} \left(-\frac{3}{2} \right)$

Question 9 For $a, b, \alpha, \beta \in R^+$, the area of the smallest rectangle which can cover the graph of

$$\frac{(x+\alpha)^2}{a^2} + \frac{(y-\beta)^2}{b^2} = 1 \text{ is}$$

- A. $(a-\alpha)(b+\beta)$
- B. $4(a+\alpha)(b-\beta)$
- C. $4ab$
- D. $4\alpha\beta$
- E. $\alpha\beta ab$

Question 10 The line $y = mx$ is a tangent to the curve $y = \tan^{-1} x - \frac{\pi}{4}$.

The value of m is closest to

- A. 0.08
- B. 0.12
- C. 0.16
- D. 0.20
- E. 0.24

Question 11 A vector makes the same angle θ with three mutually perpendicular vectors $\tilde{a}, \tilde{b}$ and $\tilde{c}$.
A possible value of θ is

- A. $\frac{\pi}{4}$
- B. $\frac{3\pi}{4}$
- C. $\cos^{-1}\left(-\frac{1}{\sqrt{3}}\right)$
- D. $\sin^{-1}\left(-\frac{1}{\sqrt{3}}\right)$
- E. $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Question 12 Vectors $\tilde{a} = \tilde{i} + \sqrt{2}\tilde{j} - \tilde{k}$ and $\tilde{b} = -\sqrt{2}\tilde{i} + 2\tilde{j} + \sqrt{2}\tilde{k}$ are on the same plane. Another possible vector on the plane is

- A. $\frac{1}{2}\tilde{j}$
- B. $\frac{1}{2}\tilde{i} + \frac{1}{2}\tilde{k}$
- C. $\frac{1}{2}\tilde{i} + \frac{1}{\sqrt{2}}\tilde{j} + \frac{1}{2}\tilde{k}$
- D. $-\frac{1}{2}\tilde{i} + \frac{1}{\sqrt{2}}\tilde{j}$
- E. $-\frac{1}{\sqrt{2}}\tilde{i}$

Question 13 Given $\overrightarrow{AB} = 4\tilde{i} - 4\tilde{j}$ and $\overrightarrow{AC} = -\tilde{j} + 12\tilde{k}$, the shortest distance from point C to vector $\overrightarrow{AB}$ is closest to

- A. 11.5
- B. 12.0
- C. 12.5
- D. 13.0
- E. 13.5

Question 14 Let A be the area of the region enclosed by the x -axis, the line $x = a$ and the curve $y = \tan^{-1}\left(\frac{x}{a}\right)$, where $a \in R^+$. Which one of the following statements is correct?

- A. $A = \int_0^{\frac{\pi}{4}} \tan^{-1}\left(\frac{x}{a}\right) dx$
- B. $A = a\left(\frac{\pi}{4} - \int_0^{\frac{\pi}{4}} \tan x dx\right)$
- C. $A = \frac{\pi}{4} - a \int_0^{\frac{\pi}{4}} \tan x dx$
- D. $A = \frac{a\pi}{4} - \int_0^{\frac{\pi}{4}} \tan x dx$
- E. $A = \frac{\pi}{4} - \int_0^{\frac{a\pi}{4}} \tan x dx$

Question 15 The region enclosed by the curve $y = \sin^{-1}\left(\frac{x}{x^2+1}\right)$ and the x -axis from $x = 0$ to $x = 4$ is rotated about the x -axis. The volume of the solid of revolution is closest to

- A. 0.9
- B. 1.2
- C. 1.5
- D. 1.8
- E. 2.1

Question 16 Given $f'(x) = \log_e \sqrt{x^2+1}$ and $f(2) = 2$, the value of $f(0)$ is closest to

- A. 0.7
- B. 0.9
- C. 1.1
- D. 1.3
- E. 1.5

Question 17 Given $f(x) = \log_e \left(\frac{a \cos^{-1} x}{b} \right)$, $a, b \in R^+$, the value of $f'\left(\frac{1}{2}\right)$ is

- A. $\frac{6b}{\sqrt{3}a\pi}$
- B. $-\frac{6}{a\pi}$
- C. $\frac{a\pi}{\sqrt{3}b}$
- D. $-\frac{2\sqrt{3}}{\pi}$
- E. $\frac{6}{\sqrt{3}\pi}$

Question 18 A particle moves in a plane with position $\tilde{r}(t)$ and velocity $\tilde{v}(t) = 2t\tilde{i} - \tilde{j}$, $t \geq 0$. The straight-line distance from $\tilde{r}(1)$ to $\tilde{r}(2)$ is closest to

- A. $2\sqrt{2}$
- B. 3
- C. $\sqrt{10}$
- D. $\sqrt{11}$
- E. $2\sqrt{3}$

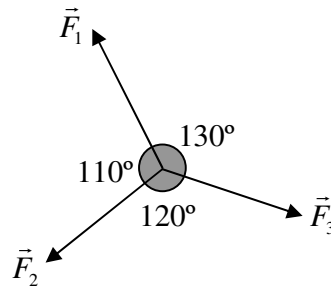
Question 19 The position of a particle is given by $\tilde{r} = 4.9t\tilde{i} + 4.9(\sqrt{3}t - t^2)\tilde{j}$ where $t \geq 0$. The minimum speed of the particle is

- A. 4.9
- B. 9.8
- C. $4.9(1 + \sqrt{3})$
- D. $9.8\sqrt{3}$
- E. 19.6

Question 20 A particle moves with constant acceleration. It has an initial velocity of $10\tilde{i} \text{ m s}^{-1}$. It has a velocity of $(-5)\tilde{i} \text{ m s}^{-1}$ after a displacement of $\tilde{s} = 7.5\tilde{i} \text{ m}$. When the velocity is $(-5)\tilde{i} \text{ m s}^{-1}$, the total distance (m) travelled by the particle is closest to

- A. 7.5
- B. 10.0
- C. 12.5
- D. 15.0
- E. 17.5

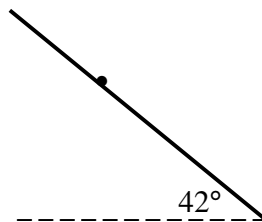
Question 21 A particle is in **equilibrium** under the action of three forces $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$ (see diagram below).



Which one of the following statements is correct?

- A. $|\vec{F}_3|\cos 110^\circ = |\vec{F}_2|\cos 130^\circ$
- B. $|\vec{F}_3|\sin 30^\circ = |\vec{F}_2|\sin 20^\circ$
- C. $|\vec{F}_3|\cos 30^\circ = |\vec{F}_2|\cos 20^\circ$
- D. $|\vec{F}_3|\sin 50^\circ = |\vec{F}_2|\sin 70^\circ$
- E. $|\vec{F}_3|\cos 50^\circ = |\vec{F}_2|\cos 70^\circ$

Question 22 A 0.5-kg particle slides *at constant speed* down a rough plane inclined at 42° to the horizontal. The particle exerts a force of F newtons on the plane.



The value of F is closest to

- A. 4.9
- B. 3.6
- C. 3.3
- D. 0.5
- E. 0.4

SECTION 2 Extended-answer questions

Instructions for Section 2

Answer **all** questions.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ m s}^{-2}$, where $g = 9.8$.

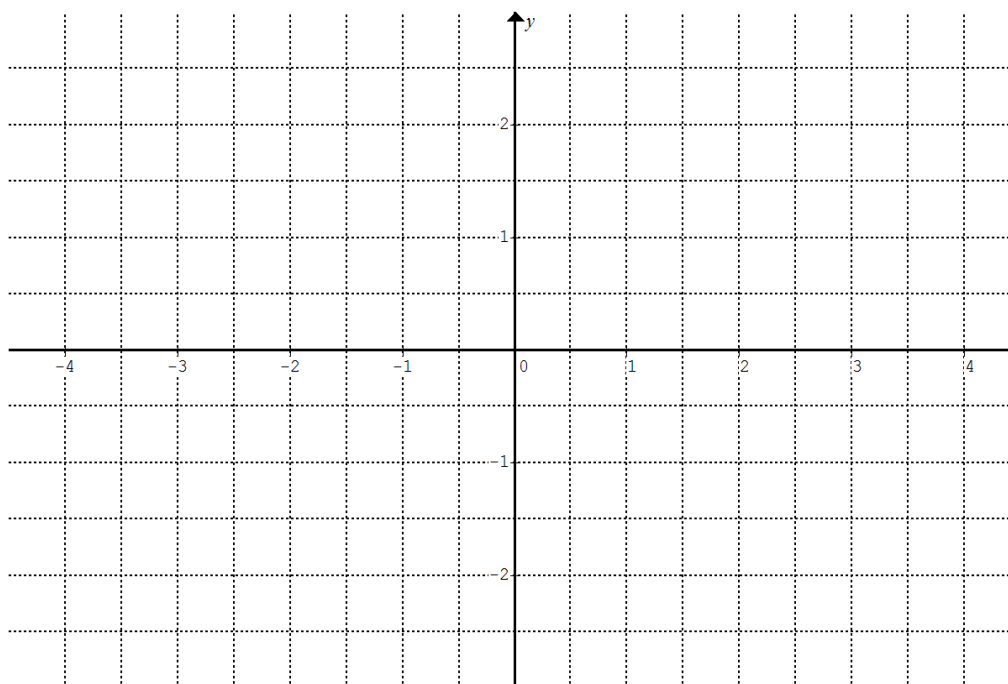
Question 1 Consider relations $x^2 + 4y^2 = 4$ and $4\left(x - \frac{3}{2}\right)^2 - 8y^2 = 1$.

- a. The graphs of the two relations intersect at some points. Find the exact coordinates of these points.

3 marks

- b. Sketch the graphs of the relations on the same axes. Show and label the coordinates of the axis intercepts and intersections, and the asymptotes with equations.

3 marks



- c. Find the equations of the tangents to the graph of $4\left(x - \frac{3}{2}\right)^2 - 8y^2 = 1$ at $x = 0$. 2 marks

The region enclosed by the tangents in part c and the curve $4\left(x - \frac{3}{2}\right)^2 - 8y^2 = 1$ is rotated about the x -axis by 180° .

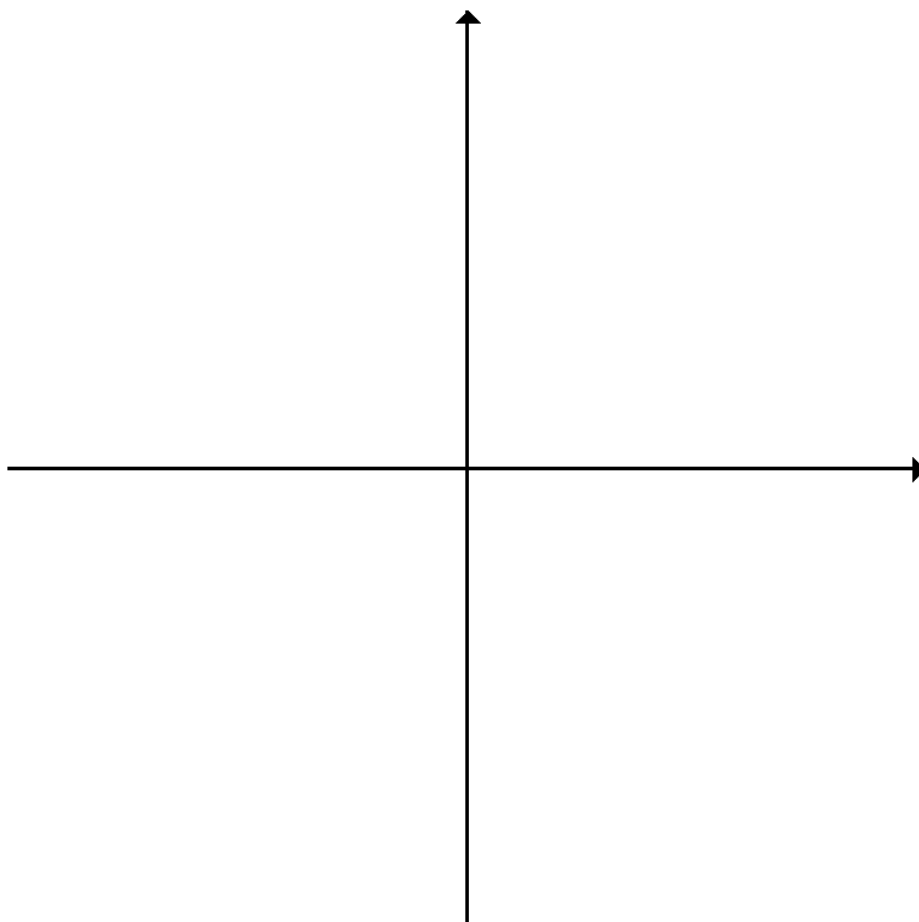
- d. Find the exact volume of the solid of revolution. 3 marks

Question 2 The volume of water in a tank is given by $V = \frac{5\pi}{6} - \cos^{-1}\left(h - \frac{\sqrt{3}}{2}\right)$ where V is measured in m^3 and h in metres is the depth of water in the tank.

- a. Find the exact values of the maximum depth and maximum volume of the tank. 2 marks

- b. Sketch the graph of $V = \frac{5\pi}{6} - \cos^{-1}\left(h - \frac{\sqrt{3}}{2}\right)$.

Label the axes. Show and label the endpoints and point of inflection with their exact coordinates. 3 marks



At time $t = 0$ water is pumped into the empty tank at a rate of 50 litres per minute. t is measured in minutes.

c. Express h in terms of t . 3 marks

d. Find the exact time in minutes required to fill the tank. 1 mark

e. Find the rate of increase of the depth when the water in the tank is $\sqrt{3}$ metres deep. 3 marks

Question 3 Given unit vector $\tilde{i}$ is in the positive x direction, and unit vector $\tilde{j}$ is in the positive y direction. The position of a motorcyclist is given by $\tilde{r}_c(t) = \left(10 - 100\cos\frac{\pi t}{15}\right)\tilde{i} + \left(160 + 150\sin\frac{\pi t}{15}\right)\tilde{j}$ where time $t \geq 0$ and it is measured in seconds.

a. Find the Cartesian equation of the path of the motorcyclist.

2 marks

b. Find the exact time when the motorcyclist first returns to the position $\tilde{r}_c(0)$.

1 mark

c. Find the exact maximum speed of the motorcyclist.

2 marks

A spectator is at position $\tilde{r}_s = 10\tilde{i} + \frac{40}{3}\tilde{k}$.

d i. Find the exact time when the motorcyclist is first closest to the spectator.

3 marks

d ii. Find the exact value of the closest distance.

1 mark

e. Show that the acceleration of the motorcyclist is $\tilde{a} = k(\tilde{r}_c - \tilde{r}_0)$ where $k \in R$ and $\tilde{r}_0 = 10\tilde{i} + 160\tilde{j}$.

2 marks

Question 4 Consider $z - \frac{1}{z}$ where $z = x + yi$.

a. Find the real part and the imaginary part of $z - \frac{1}{z}$ in terms of x and y .

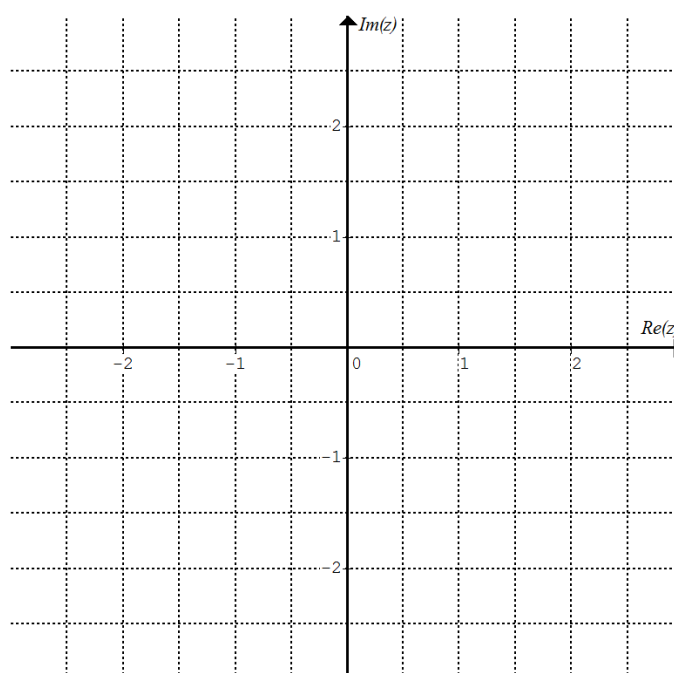
2 marks

b i. Find the Cartesian equation of $\left\{ z : i 2 \operatorname{Im}(z) = z - \frac{1}{z} \right\}$.

2 marks

b ii. Sketch $\left\{ z : \left| i 2 \operatorname{Im}(z) - \left(z - \frac{1}{z} \right) \right| > 0 \right\}$.

3 marks



- c. Find the Cartesian equation of $\left\{z: 2\operatorname{Re}(z) = z + \frac{1}{z}\right\}$. 1 mark

- d. Simplify $\left\{z: \left|i2\operatorname{Im}(z) - \left(z - \frac{1}{z}\right)\right| > 0\right\} \cap \left\{z: 2\operatorname{Re}(z) = z + \frac{1}{z}\right\}$. 1 mark

- e. Find the Cartesian equation of $\left\{z: 2\operatorname{Re}(z - 1 - i) = z + \frac{1}{z - 1 - i} - 1 - i\right\}$. 1 mark

Question 5 A 1-kg particle is projected vertically upwards with a speed of 20 m s^{-1} at $t = 0$. Air resistance against its motion is $0.01v^2$ newtons where v is the speed of the particle in m s^{-1} .

a. Draw a diagram showing the forces on the particle while it moves upwards. Label the forces. 1 mark

b. Write an appropriate equation of motion by which the maximum height reached by the particle can be calculated. Let the upward displacement from the point of projection be x metres. 1 mark

c i. Use calculus to solve the equation in part b to show $x = 50 \log_e \left(\frac{13.8}{9.8 + 0.01v^2} \right)$. 3 marks

c ii. Find the maximum height reached by the particle, in metres, correct to 2 decimal places. 1 mark

In the downward motion back to the point of projection, the particle experiences the same air resistance of $0.01v^2$ newtons.

- d. Write an appropriate equation by which the speed of the particle after falling x metres can be calculated. 1 mark

- e i. Use calculus to solve the equation in part d to show $v^2 = 980\left(1 - e^{-\frac{x}{50}}\right)$. 3 marks

- e ii. Find the speed of the particle when it returns to its point of projection, in metres per second, correct to 2 decimal places. 1 mark

- f. Find the time taken for the downward motion of the particle, correct to 2 decimal places. 3 marks

End of Exam 2