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Specialist Mathematics

2015

Trial Examination 1 (1 hour)

Instructions

Answer **all** questions. Do **not** use calculators.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working or explanation **must** be shown.

Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.

Question 1

Consider the set of complex numbers $S = \left\{ z : \frac{1}{\bar{z}} - \frac{1}{z} = i \right\}$.

- a. Find the Cartesian equation representing S in the x - y plane.

2 marks

- b. For $z \in S$, find the possible values of $\operatorname{Re}(z)$.

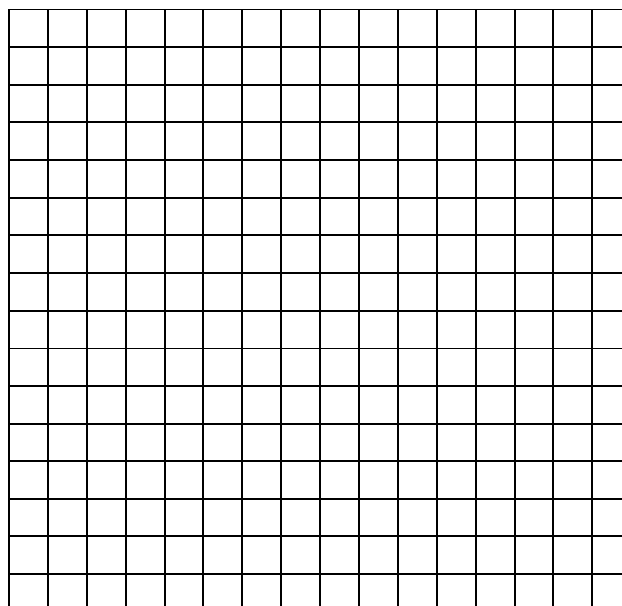
1 mark

- c. For $z \in S$, find the possible values of $\operatorname{Im}(z)$.

1 mark

- d. Sketch the graph of S on the Argand plane.

2 marks



Question 2

Consider polynomial $P(z) = z^3 - 2iz^2 + 2z - 2i$. Let α , β and γ be the roots of $P(z) = 0$.

a. Show that $\alpha + \beta + \gamma = 2i$.

1 mark

b. Show that $(i - \alpha)(i - \beta)(i - \gamma) = i$.

1 mark

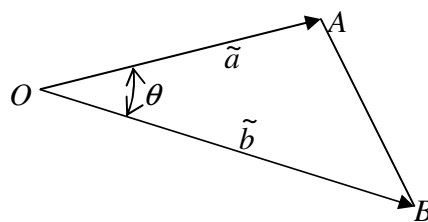
c. Use part a and part b to find the value of $(\alpha + \beta - \gamma)(\beta + \gamma - \alpha)(\gamma + \alpha - \beta)$.

2 marks

Question 3

Triangle OAB is defined by two non-parallel and non-zero vectors $\overrightarrow{OA} = \tilde{a}$ and $\overrightarrow{OB} = \tilde{b}$.

The area of the triangle is given by $\frac{1}{2} |\tilde{a}| |\tilde{b}| \sin \theta$ where θ is the angle between vectors $\tilde{a}$ and $\tilde{b}$.



Express the area of $\triangle OAB$ in terms of dot products of $\tilde{a}$ and $\tilde{b}$.

3 marks

Question 4

Let $\tilde{p} = \frac{2}{3}\tilde{i} + \frac{1}{3}\tilde{j} + \frac{2}{3}\tilde{k}$, $\tilde{q} = -\frac{1}{3}\tilde{i} - \frac{2}{3}\tilde{j} + \frac{2}{3}\tilde{k}$ and $\tilde{r} = \frac{2}{3}\tilde{i} - \frac{2}{3}\tilde{j} - \frac{1}{3}\tilde{k}$ be vectors in 3-dimensional space defined by perpendicular unit vectors $\tilde{i}$, $\tilde{j}$ and $\tilde{k}$.

a. Show that $\tilde{p}$, $\tilde{q}$ and $\tilde{r}$ are unit vectors.

1 mark

b. Show that $\tilde{p}$, $\tilde{q}$ and $\tilde{r}$ are perpendicular to each other.

2 marks

c. Show that $\tilde{s} = \tilde{i} - \tilde{j} + \tilde{k}$ can be expressed as $\tilde{s} = \tilde{p} + \tilde{q} + \tilde{r}$.

1 mark

d. Given $\tilde{t} = \frac{2}{\sqrt{3}}\tilde{p} - \sqrt{3}\tilde{q} + \frac{1}{\sqrt{3}}\tilde{r}$, find $\tilde{s} \cdot \tilde{t}$.

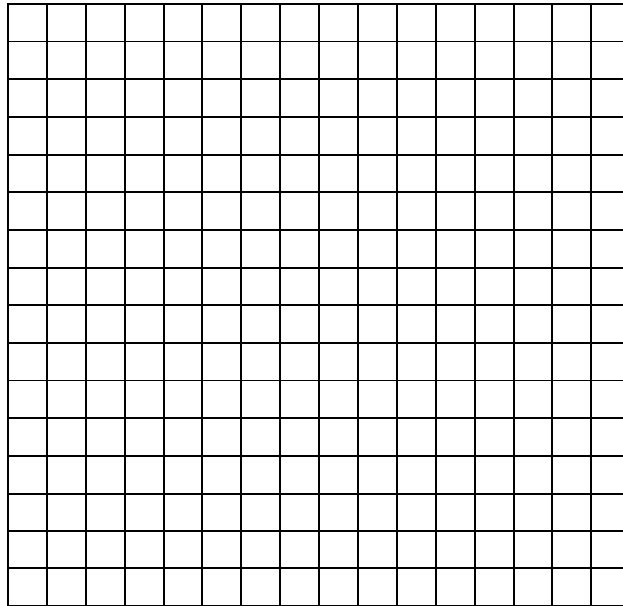
2 marks

Question 5

Consider $f : \left[\pi, \frac{3\pi}{2} \right] \rightarrow \mathbb{R}, f(x) = \frac{1}{2} \cos(2x)$.

- a. Sketch the graph of f^{-1} . Show and label endpoint(s) and intercept(s).

2 marks



- b. Find the rule of f^{-1} in terms of the arccos (i.e. $\cos^{-1}$) function.

2 marks

- c. Find the equation of the normal to the graph of f^{-1} at $x = 0$.

2 marks

Question 6

The solution curve to the differential equation $\frac{dy}{dx} - \frac{x^2}{2y} = 0$ passes through $(2, 1)$.

- a. Use Euler's method (first order approximation) to estimate the value of y at $x = 3$.
Choose 0.5 as the step size.

2 marks

- b. Show that $y = \sqrt{\frac{x^3 - 5}{3}}$ is the equation of the solution curve.

1 mark

Question 7

A solid of revolution is formed by revolving $y = f(x)$ about the x -axis, where $f(x) \geq 0$ for $0 \leq x \leq 2$.

The volume of the solid is given by $V = \int_0^2 \pi(2x - x^2) dx$.

- a. Find $f(x)$.

1 mark

- b. Find the area enclosed by $y = f(x)$ and $y = -f(x)$.

2 marks

Question 8

A 0.4 kg particle moves with acceleration $\tilde{a} = \frac{g}{4}(\sqrt{3}\tilde{i} - \tilde{j})$ in a vertical x - y plane, where $g = 9.8 \text{ m s}^{-2}$.

$\tilde{i}$ is a horizontal unit vector in the positive x -direction, and $\tilde{j}$ is a unit vector pointing vertically upwards in the positive y -direction.

The particle starts from rest at position $\tilde{r} = 10\tilde{j}$.

- a. Find the exact time t in seconds when the particle is at $\tilde{r} = 10\sqrt{3}\tilde{i}$. Express your answer in terms of g .

3 marks

- b. Find the exact speed in m s^{-1} of the particle at $\tilde{r} = 10\sqrt{3}\tilde{i}$. Express your answer in terms of g .

2 marks

- c. Find the magnitude of the resultant force (net force) in newtons at $t = 1$ second.

1 mark

Question 9

A particle is in equilibrium under the action of three forces $\tilde{F}_1$, $\tilde{F}_2$ and $\tilde{F}_3$.

$\tilde{F}_1 = \sqrt{7}$ newtons, $|\tilde{F}_3| = 3|\tilde{F}_2|$, and the angle between $\tilde{F}_2$ and $\tilde{F}_3$ is 120° .

- a. Find the exact magnitude of $\tilde{F}_3$ in newtons.

2 marks

- b. $\tilde{F}_2$ and $\tilde{F}_1$ make an acute angle θ where $\sin \theta = \alpha\sqrt{\beta}$. Find the exact value of α and β .

2 marks

End of Exam 1