

2015 Specialist Mathematics Trial Exam 1 Solutions

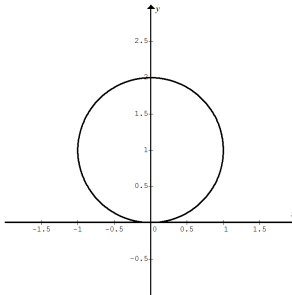
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Q1a $\frac{1}{\bar{z}} - \frac{1}{z} = i, \frac{z - \bar{z}}{z\bar{z}} = i, \frac{2yi}{x^2 + y^2} = i, x^2 + y^2 - 2y = 0$
 $x^2 + (y-1)^2 = 1$

Q1b $\operatorname{Re}(z) \in [-1, 1]$

Q1c $\operatorname{Im}(z) \in [0, 2]$

Q1d



Q2a

$P(z) = (z - \alpha)(z - \beta)(z - \gamma) = z^3 - (\alpha + \beta + \gamma)z^2 + \dots$
 $= z^3 - 2iz^2 + 2z - 2i$
 $\therefore \alpha + \beta + \gamma = 2i$

Q2b $P(z) = (z - \alpha)(z - \beta)(z - \gamma) = z^3 - 2iz^2 + 2z - 2i$
 $\therefore P(i) = (i - \alpha)(i - \beta)(i - \gamma) = i^3 - 2i^3 + 2i - 2i$
 $\therefore (i - \alpha)(i - \beta)(i - \gamma) = i$

Q2c $(\alpha + \beta - \gamma)(\beta + \gamma - \alpha)(\gamma + \alpha - \beta)$
 $= (2i - \gamma - \gamma)(2i - \alpha - \alpha)(2i - \beta - \beta)$
 $= (2i - 2\gamma)(2i - 2\alpha)(2i - 2\beta)$
 $= 8(i - \gamma)(i - \alpha)(i - \beta) = 8i$

Q3 Let A be the area of the triangle. Given $A = |\tilde{a}||\tilde{b}|\sin\theta$

$A^2 = |\tilde{a}|^2|\tilde{b}|^2\sin^2\theta,$
 $A^2 = |\tilde{a}|^2|\tilde{b}|^2(1 - \cos^2\theta) = |\tilde{a}|^2|\tilde{b}|^2 - |\tilde{a}|^2|\tilde{b}|^2\cos^2\theta$
 $= |\tilde{a}|^2|\tilde{b}|^2 - (|\tilde{a}||\tilde{b}|\cos\theta)^2 = (\tilde{a} \cdot \tilde{a})(\tilde{b} \cdot \tilde{b}) - (\tilde{a} \cdot \tilde{b})^2$
 $\therefore A = \sqrt{(\tilde{a} \cdot \tilde{a})(\tilde{b} \cdot \tilde{b}) - (\tilde{a} \cdot \tilde{b})^2}$

Q4a $|\tilde{p}| = |\tilde{q}| = |\tilde{r}| = \sqrt{\left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2} = 1$

Q4b $\tilde{p} \cdot \tilde{q} = 0, \tilde{q} \cdot \tilde{r} = 0, \tilde{r} \cdot \tilde{p} = 0, \therefore \tilde{p}, \tilde{q}, \tilde{r}$ are $\perp$ to each other.

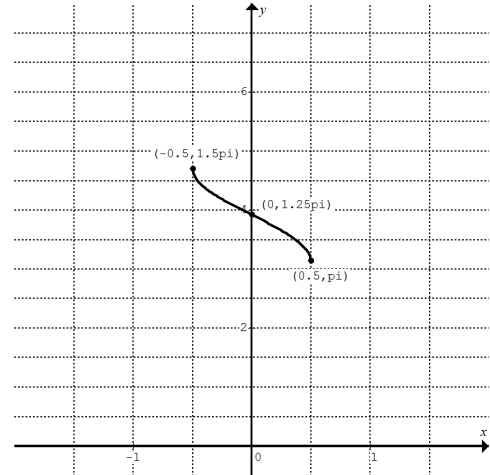
Q4c

$\tilde{p} + \tilde{q} + \tilde{r} = \left(\frac{2}{3}\tilde{i} + \frac{1}{3}\tilde{j} + \frac{2}{3}\tilde{k}\right) + \left(-\frac{1}{3}\tilde{i} - \frac{2}{3}\tilde{j} + \frac{2}{3}\tilde{k}\right) + \left(\frac{2}{3}\tilde{i} - \frac{2}{3}\tilde{j} - \frac{1}{3}\tilde{k}\right)$
 $= \tilde{i} - \tilde{j} + \tilde{k} = \tilde{s}$

Q4d $\tilde{s} \cdot \tilde{t} = (\tilde{i} - \tilde{j} + \tilde{k}) \cdot \left(\frac{2}{\sqrt{3}}\tilde{p} - \sqrt{3}\tilde{q} + \frac{1}{\sqrt{3}}\tilde{r}\right)$

$= (\tilde{p} + \tilde{q} + \tilde{r}) \cdot \left(\frac{2}{\sqrt{3}}\tilde{p} - \sqrt{3}\tilde{q} + \frac{1}{\sqrt{3}}\tilde{r}\right) = \frac{2}{\sqrt{3}} - \sqrt{3} + \frac{1}{\sqrt{3}} = 0$

Q5a



Q5b Given $y = f(x) = \frac{1}{2}\cos(2x)$ for $\pi \leq x \leq \frac{3\pi}{2}$

$\therefore$ the inverse is $x = \frac{1}{2}\cos(2y - 2\pi), 2y - 2\pi = \cos^{-1}(2x)$

$\therefore y = \frac{1}{2}\cos^{-1}(2x) + \pi, f^{-1}(x) = \frac{1}{2}\cos^{-1}(2x) + \pi$

Q5c $(f^{-1})' = \frac{1}{2} \times \frac{-2}{\sqrt{1 - (2x)^2}} = \frac{-1}{\sqrt{1 - 4x^2}}$

At $x = 0$, gradient of the tangent $= \frac{-1}{\sqrt{1 - 4x^2}} = -1$

$\therefore$ gradient of the normal $= -\frac{1}{m_t} = 1$

$\therefore$ equation of the normal: $y = x + \frac{5\pi}{4}$

Q6a $\frac{dy}{dx} = \frac{x^2}{2y}$

$x = 2, \quad y = 1, \quad \frac{dy}{dx} = \frac{2^2}{2(1)} = 2$

$x = 2.5, \quad y \approx 1 + 0.5 \times 2 = 2, \quad \frac{dy}{dx} = \frac{2.5^2}{2(2)} = \frac{6.25}{4}$

$x = 3, \quad y \approx 2 + 0.5 \times \frac{6.25}{4} = \frac{89}{32}$

Q6b $\frac{dy}{dx} = \frac{1}{2\sqrt{\frac{x^3-5}{3}}} \times x^2 = \frac{x^2}{2y}, \therefore y = \sqrt{\frac{x^3-5}{3}}$ satisfies the

differential equation $\frac{dy}{dx} = \frac{x^2}{2y}$

Q7a $f(x) = \sqrt{2x-x^2}$

Q7b $y = \pm\sqrt{2x-x^2}, \quad y^2 = 2x-x^2, \quad x^2-2x+1+y^2=1$
 $(x-1)^2 + y^2 = 1$, a circle of radius 1, $\therefore$ area is π .

Q8a The particle starts from rest, $\therefore \tilde{v} = \frac{g}{4}(\sqrt{3}\tilde{i} - \tilde{j})t$.

Displacement at time t is given by

$$\int_0^t \frac{g}{4}(\sqrt{3}\tilde{i} - \tilde{j})t \, dt = \frac{g}{8}(\sqrt{3}\tilde{i} - \tilde{j})t^2$$

$$\therefore 10\sqrt{3}\tilde{i} - 10\tilde{j} = \frac{g}{8}(\sqrt{3}\tilde{i} - \tilde{j})t^2$$

$$\therefore t^2 = \frac{80}{g}, \quad t = 4\sqrt{\frac{5}{g}}$$

Q8b Note: This is a 1 mark question, not 2 as indicated in the trial exam.

At $\tilde{r} = 10\sqrt{3}\tilde{i}$, $t = 4\sqrt{\frac{5}{g}}$

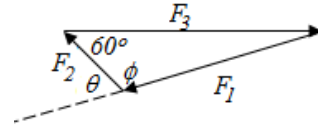
$$\tilde{v} = \frac{g}{4}(\sqrt{3}\tilde{i} - \tilde{j})4\sqrt{\frac{5}{g}} = \sqrt{5g}(\sqrt{3}\tilde{i} - \tilde{j})$$

Speed $= |\tilde{v}| = \sqrt{5g}\sqrt{3+1} = 2\sqrt{5g}$

Q8c $\tilde{F} = m\tilde{a} = 0.4 \times \frac{g}{4}(\sqrt{3}\tilde{i} - \tilde{j}) = 0.1g(\sqrt{3}\tilde{i} - \tilde{j})$

$$|\tilde{F}| = 0.1g\sqrt{3+1} = \frac{g}{5} \text{ newtons}$$

Q9a



$$F_2^2 + F_3^2 - 2F_2F_3 \cos 60^\circ = F_1^2$$

$$\left(\frac{F_3}{3}\right)^2 + F_3^2 - 2\left(\frac{F_3}{3}\right)F_3 \cos 60^\circ = F_1^2$$

$$\frac{F_3^2}{9} + F_3^2 - \frac{2F_3^2}{3} \times \frac{1}{2} = 7, \quad F_3 = 3$$

Q9b

$$\frac{\sin \phi}{3} = \frac{\sin 60^\circ}{\sqrt{7}}, \quad \sin \phi = \frac{3}{\sqrt{7}} \times \frac{\sqrt{3}}{2} = \frac{3}{2} \sqrt{\frac{3}{7}}$$

$$\therefore \sin \theta = \sin \phi = \frac{3}{2} \sqrt{\frac{3}{7}}$$

$$\therefore \alpha = \frac{3}{2} \text{ and } \beta = \frac{3}{7}, \text{ or other equivalent forms.}$$

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