

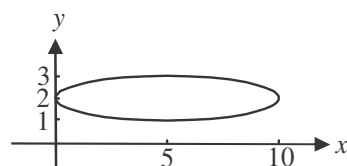
SECTION 1 – Multiple-choice answers

- | | | | |
|------|-------|-------|-------|
| 1. A | 7. E | 13. A | 19. E |
| 2. C | 8. B | 14. D | 20. A |
| 3. E | 9. E | 15. D | 21. D |
| 4. C | 10. B | 16. C | 22. E |
| 5. B | 11. B | 17. D | |
| 6. C | 12. C | 18. D | |

SECTION 1 - Multiple-choice solutions

Question 1

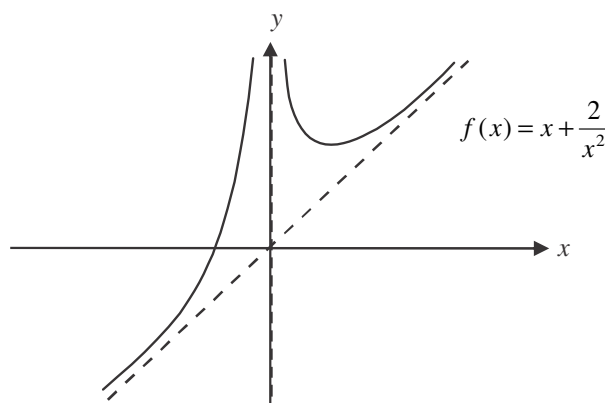
$$\begin{aligned}
 x^2 - 10x + 25y^2 - 100y + 100 &= 0 \\
 x^2 - 10x + 25 - 25 + 25(y^2 - 4y + 4) &= 0 \\
 (x - 5)^2 + 25(y - 2)^2 &= 25 \\
 \frac{(x - 5)^2}{25} + \frac{(y - 2)^2}{1} &= 1
 \end{aligned}$$



The only axis intercept occurs at (0,2).
The answer is A.

Question 2

The graph of $f(x) = x + \frac{2}{x^2}$ has two asymptotes, one with equation $y = x$ and the other with equation $x = 0$. Immediately we see that option C cannot be true as both the asymptotes are straight lines.



The answer is C.

Question 3

$$x = 3\sec(\theta) - 1 \quad y = \frac{4}{\cot(\theta)}$$

$$\frac{x+1}{3} = \sec(\theta) \quad \frac{y}{4} = \tan(\theta)$$

$$\frac{(x+1)^2}{9} = \sec^2(\theta) \quad \frac{y^2}{16} = \tan^2(\theta)$$

$$1 + \tan^2(\theta) = \sec^2(\theta) \quad (\text{formula sheet})$$

$$1 + \frac{y^2}{16} = \frac{(x+1)^2}{9}$$

$$\frac{(x+1)^2}{9} - \frac{y^2}{16} = 1$$

The answer is E.

Question 4

For $f(x) = a \sin^{-1}(bx+1)$

$$-1 \leq bx+1 \leq 1$$

$$-2 \leq bx \leq 0$$

$$\frac{-2}{b} \leq x \leq 0$$

Since $d_f = \left[-\frac{2}{b}, 0\right]$ and $d_f = [-6, 0]$,

$$\text{then } -\frac{2}{b} = -6$$

$$b = \frac{1}{3}$$

$$\text{Also } -\frac{\pi}{2} \leq \frac{y}{a} \leq \frac{\pi}{2}$$

$$-\frac{\pi a}{2} \leq y \leq \frac{\pi a}{2}$$

$$\text{Since } r_f = [-\pi, \pi]$$

$$\frac{\pi a}{2} = \pi$$

$$a = 2$$

The answer is C.

Question 5

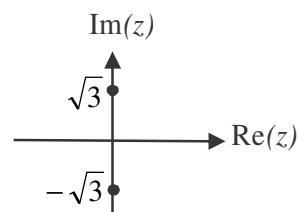
The modulus of z is the distance from the origin to the point on the Argand diagram represented by z .

Since z lies on the imaginary axis, there are two possible values of z , as shown on the diagram.

One is $\sqrt{3}i$ and the other is $-\sqrt{3}i$.

Only the latter is offered in the answers.

The answer is B.



Question 6

$$\begin{aligned}
 z &= \operatorname{cis}\left(-\frac{\pi}{3}\right) \\
 &= \cos\left(-\frac{\pi}{3}\right) + i \sin\left(-\frac{\pi}{3}\right) \\
 &= \frac{1}{2} - \frac{\sqrt{3}}{2}i \\
 z + \frac{i}{\sqrt{3}} &= \frac{1}{2} - \frac{\sqrt{3}}{2}i + \frac{i}{\sqrt{3}} \\
 &= \frac{1}{2} + \frac{2i - 3i}{2\sqrt{3}} \\
 &= \frac{1}{2} - \frac{1}{2\sqrt{3}}i
 \end{aligned}$$



The imaginary part is $-\frac{1}{2\sqrt{3}}$.

The answer is C.

Question 7

$$\begin{aligned}
 z_1 &= a \operatorname{cis}(\theta) & \text{and } z_2 &= b \operatorname{cis}(2\theta) \\
 z_1^3 &= a^3 \operatorname{cis}(3\theta) & \text{so } \bar{z}_2 &= b \operatorname{cis}(-2\theta)
 \end{aligned}$$

$$\begin{aligned}
 \arg\left(\frac{z_1^3}{\bar{z}_2}\right) &= 3\theta - -2\theta \\
 &= 5\theta
 \end{aligned}$$

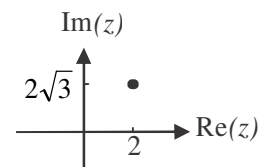
The answer is E.

Question 8

$$\begin{aligned}
 z^2 &= 2 + 2\sqrt{3}i \\
 &= 4 \operatorname{cis}\left(\frac{\pi}{3} + 2k\pi\right) \quad k \in \mathbb{Z}
 \end{aligned}$$

$$\begin{aligned}
 \text{So } z &= \left(4 \operatorname{cis}\left(\frac{\pi}{3} + 2k\pi\right)\right)^{\frac{1}{2}} \\
 &= 2 \operatorname{cis}\left(\frac{1}{2}\left(\frac{\pi}{3} + 2k\pi\right)\right) \\
 &= 2 \operatorname{cis}\left(\frac{\pi}{6} + k\pi\right)
 \end{aligned}$$

$$\begin{aligned}
 r &= \sqrt{4 + 4 \times 3} = 4 \\
 \theta &= \tan^{-1}\left(\frac{2\sqrt{3}}{2}\right) \\
 &= \frac{\pi}{3}
 \end{aligned}$$



For the principle valued argument, $-\pi < \theta \leq \pi$. (formula sheet)

$$\text{So } z = 2 \operatorname{cis}\left(\frac{\pi}{6}\right) \text{ or } z = 2 \operatorname{cis}\left(-\frac{5\pi}{6}\right).$$

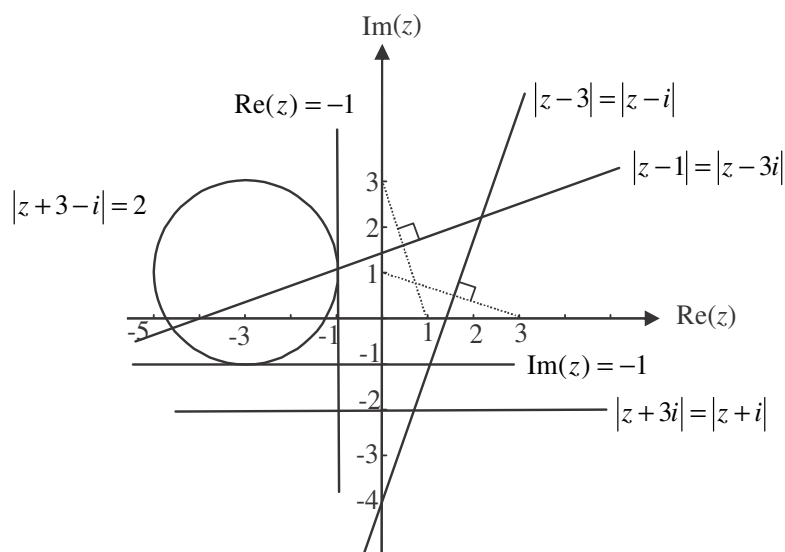
The answer is B.

Question 9

Do a quick sketch.

$$|z + 3 - i| = 2$$

$$|z - (-3 + i)| = 2$$



The lines $\text{Re}(z) = -1$ and $\text{Im}(z) = -1$ are both tangents to the circle.

The lines given in options B and D don't intersect with the circle at all.

The line $|z - 1| = |z - 3i|$ intersects twice.

The answer is E.

Question 10

$$V = 5\pi h$$

$$\frac{dV}{dh} = 5\pi$$

$$\frac{dh}{dt} = \frac{dh}{dV} \cdot \frac{dV}{dt}$$

$$= \frac{1}{5\pi} (0.1 - 0.02\sqrt{h})$$

$$= \frac{10 - 2\sqrt{h}}{500\pi}$$

$$= \frac{5 - \sqrt{h}}{250\pi}$$

The answer is B.

Question 11Method 1

At $x = 0$, $\frac{dy}{dx}$ is undefined, so eliminate options C and E.

For $x \in \mathbb{R} \setminus \{0\}$, $\frac{dy}{dx} > 0$, so eliminate options A and D.

The answer is B.

Method 2

The slopes $\left(\frac{dy}{dx}\right)$ are influenced by x -values only, so A and B are the only possibilities.

The slopes are non-negative so it must be B.

The answer is B.

Question 12

$$\frac{dy}{dx} = x^2y + 3x, \quad y=1 \text{ when } x=2$$

$$\text{So, } x_0 = 2 \quad y_0 = 1$$

$$x_1 = 2.1 \quad y_1 = 1 + 0.1(2^2 \times 1 + 3 \times 2) \\ = 2$$

$$x_2 = 2.2 \quad y_2 = 2 + 0.1(2.1^2 \times 2 + 3 \times 2.1) \\ = 3.512$$

The answer is C.

Question 13

$$\int_1^2 \frac{x-1}{\sqrt{1+2x}} dx$$

$$\text{Let } u = 1 + 2x$$

$$\frac{du}{dx} = 2$$

$$\text{Since } u = 1 + 2x$$

$$2x = u - 1$$

$$x = \frac{u-1}{2}$$

$$x-1 = \frac{u-3}{2}$$

$$\text{We have } \int_3^5 \frac{u-3}{2} \times u^{-\frac{1}{2}} \times \frac{1}{2} \frac{du}{dx} dx$$

$$= \frac{1}{4} \int_3^5 \frac{u-3}{\sqrt{u}} du$$

The answer is A.

Question 14

Draw a diagram.

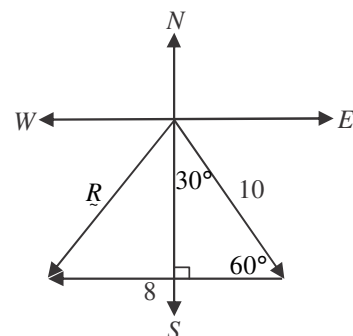
Add tip to tail to get the resultant vector $\underline{R}$.

$$|\underline{R}|^2 = 8^2 + 10^2 - 2 \times 10 \times 8 \cos(60^\circ) \quad (\text{cosine rule})$$

$$= 84$$

$$|\underline{R}| = 2\sqrt{21}$$

The answer is D.



Question 15

$$\underline{r}(t) = (2t^3 + 3)\underline{i} + t^2\underline{j} - (1 - t^3)\underline{k}$$

$$\underline{\dot{r}}(t) = 6t^2\underline{i} + 2t\underline{j} + 3t^2\underline{k}$$

$$\underline{\ddot{r}}(t) = 12t\underline{i} + 2\underline{j} + 6t\underline{k}$$

$$\underline{\ddot{r}}(2) = 24\underline{i} + 2\underline{j} + 12\underline{k}$$

$$|\underline{\ddot{r}}(2)| = \sqrt{576 + 4 + 144}$$

$$= \sqrt{724}$$

$$= 2\sqrt{181} \text{ m/s}^2$$

The answer is D.

Question 16

$$\underline{a} \bullet \underline{b} = -4 + 2\sqrt{2} - 2\sqrt{2} = -4$$

$$\text{Also, } \underline{a} \bullet \underline{b} = |\underline{a}||\underline{b}|\cos(\theta)$$

$$= \sqrt{4 + 2 + 4}\sqrt{4 + 4 + 2}\cos(\theta)$$

$$= 10\cos(\theta)$$

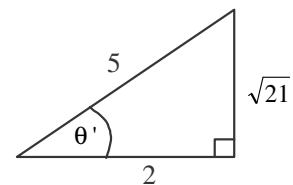
$$\text{So } 10\cos(\theta) = -4$$

$$\cos(\theta) = \frac{-2}{5}$$

θ must be a second quadrant angle.

$$\text{So } \sin(\theta) = \frac{\sqrt{21}}{5} \quad (\sin \text{ is positive in the second quadrant})$$

The answer is C.

**Question 17**

$$\underline{v}(t) = e^{3t}\underline{i} - 4\underline{j} + \sin(t)\underline{k}, \quad t \geq 0$$

$$\underline{r}(t) = \frac{e^{3t}}{3}\underline{i} - 4t\underline{j} - \cos(t)\underline{k} + \underline{c}$$

$$\underline{r}(0) = \frac{1}{3}\underline{i} - \underline{k} + \underline{c} = 0\underline{i} + 0\underline{j} + 0\underline{k}$$

$$\text{So } \underline{c} = -\frac{1}{3}\underline{i} + \underline{k}$$

$$\text{So } \underline{r}(t) = \frac{e^{3t} - 1}{3}\underline{i} - 4t\underline{j} + (1 - \cos(t))\underline{k}$$

The answer is D.

Question 18

Draw in the forces.

Find T_1 .

Resolving horizontally:

$$\begin{aligned} T_1 &= T_2 \sin(30^\circ) \\ &= \frac{T_2}{2} \quad \text{--- (1)} \end{aligned}$$

Resolving vertically:

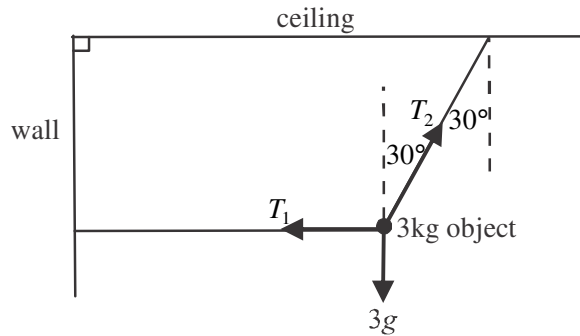
$$T_2 \cos(30^\circ) = 3g$$

$$\frac{\sqrt{3}}{2} T_2 = 3g$$

$$T_2 = \frac{6g}{\sqrt{3}}$$

$$\begin{aligned} \text{In (1)} \quad T_1 &= \frac{6g}{2\sqrt{3}} \\ &= \frac{3\sqrt{3}g}{3} \\ &= \sqrt{3}g \end{aligned}$$

The answer is D.

**Question 19**

Since $3m > m$, the $3m$ kg particle will accelerate downwards and so the m kg particle will accelerate upwards.

For the $3m$ kg particle,

$$3mg - T = 3ma \quad \text{--- (1)}$$

For the m kg particle,

$$T - mg = ma \quad \text{--- (2)}$$

$$(1)+(2) \quad 2mg = 4ma$$

$$a = \frac{2mg}{4m}$$

$$= \frac{g}{2}$$

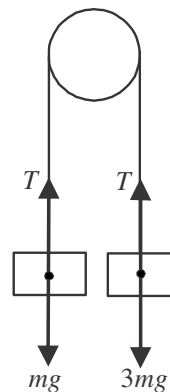
$$a = 4.9$$

$$v = 24.5 \quad v = u + at$$

$$u = 0 \quad 24.5 = 0 + 4.9 \times t$$

$$t = ? \quad t = 5$$

The answer is E.



Question 20

$$\underline{R} = m \underline{a}$$

$$-9 = 4a \quad (\text{the direction of the } 9N \text{ force is opposite to the direction of the particle's motion})$$

$$a = -\frac{9}{4}$$

$$u = 6, \quad v^2 = u^2 + 2as$$

$$v = 0 \quad 0 = 36 + 2 \times -\frac{9}{4}s$$

$$s = ? \quad s = 8\text{m}$$

The answer is A.

Question 21

$$\begin{aligned} \text{distance travelled} &= 2 \times 7 + \int_2^3 v(t)dt - \int_3^9 v(t)dt + \int_9^{10} v(t)dt \\ &= \frac{170}{3} \text{ metres} \end{aligned}$$

The answer is D.

Question 22

$$a = 3 - x^2$$

$$\frac{d}{dx} \left(\frac{1}{2} v^2 \right) = 3 - x^2 \quad (\text{formula sheet})$$

$$\frac{1}{2} v^2 = \int (3 - x^2) dx$$

$$\frac{1}{2} v^2 = 3x - \frac{x^3}{3} + c$$

$$\text{When } x = 0, v = 0.$$

$$0 = 0 - 0 + c$$

$$c = 0$$

$$\text{So } \frac{1}{2} v^2 = 3x - \frac{x^3}{3}$$

$$\text{When } v = 0$$

$$0 = \frac{9x - x^3}{3}$$

$$0 = \frac{x(9 - x^2)}{3}$$

The particle will also be at rest where $x = \pm 3$.

The answer is E.

SECTION 2

Question 1 (11 marks)

a. $x = t + \frac{2}{t} + 1$ $y = t - \frac{2}{t}$

$$(x-1)^2 = \left(\frac{t^2+2}{t}\right)^2 \quad y^2 = \left(\frac{t^2-2}{t}\right)^2$$

$$(x-1)^2 - y^2 = \frac{t^4 + 4t^2 + 4 - (t^4 - 4t^2 + 4)}{t^2} \quad (1 \text{ mark})$$

$$(x-1)^2 - y^2 = \frac{8t^2}{t^2}$$

$$(x-1)^2 - y^2 = 8$$

$$\frac{(x-1)^2}{8} - \frac{y^2}{8} = 1 \text{ as required.}$$

(1 mark)

b. $x = t + 2t^{-1} + 1$ $y = t - 2t^{-1}$

$$\frac{dx}{dt} = 1 - 2t^{-2} \quad \frac{dy}{dt} = 1 + 2t^{-2} \quad (1 \text{ mark})$$

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$$

$$= \left(1 + \frac{2}{t^2}\right) \times \frac{t^2}{t^2 - 2}$$

$$= \frac{t^2 + 2}{t^2} \times \frac{t^2}{t^2 - 2}$$

$$= \frac{t^2 + 2}{t^2 - 2} \quad (1 \text{ mark})$$

When $\frac{dy}{dx} = 2$,

$$2 = \frac{t^2 + 2}{t^2 - 2}$$

Method 1 - solve for t using CAS

$$t = \pm\sqrt{6} \quad (1 \text{ mark})$$

Method 2 - solve for t by hand

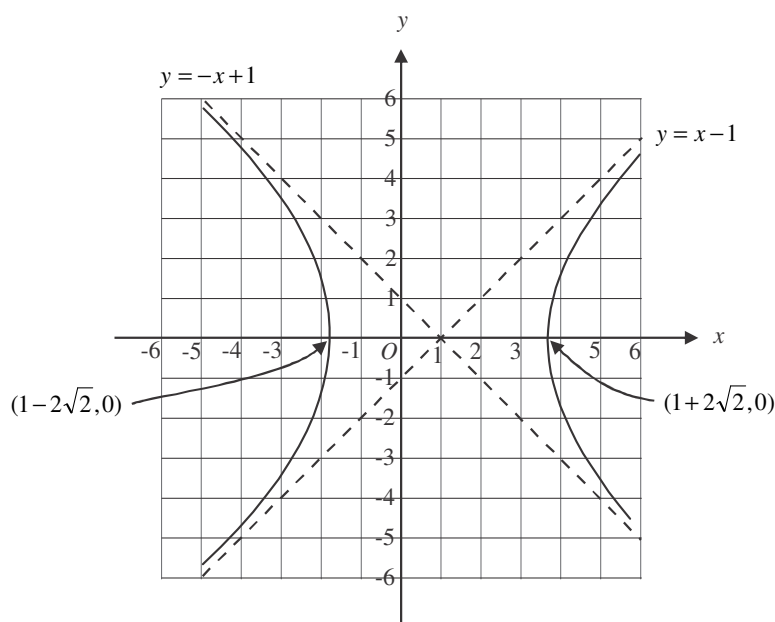
$$2(t^2 - 2) = t^2 + 2$$

$$t^2 = 6$$

$$t = \pm\sqrt{6}$$

(1 mark)

c.



Asymptotes: $y = \pm \frac{\sqrt{8}}{\sqrt{8}}(x-1)$

$$y = \pm (x-1)$$

x-intercepts occur when $y = 0$

$$\frac{(x-1)^2}{8} = 1$$

$$x = 1 - 2\sqrt{2} \text{ or } x = 1 + 2\sqrt{2}$$

(1 mark) – correct asymptotes

(1 mark) – correct x-intercepts

(1 mark) – correct shape

d. $\text{volume} = \pi \int_{1+2\sqrt{2}}^5 y^2 dx$

$$= \pi \int_{1+2\sqrt{2}}^5 ((x-1)^2 - 8) dx$$

$$= \frac{32\pi}{3} (\sqrt{2} - 1) \text{ cubic units}$$

$$\frac{(x-1)^2}{8} - \frac{y^2}{8} = 1$$

$$\frac{y^2}{8} = \frac{(x-1)^2}{8} - 1$$

$$y^2 = (x-1)^2 - 8$$

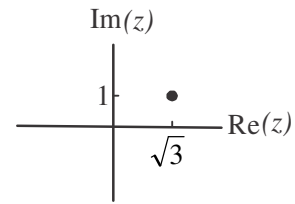
(1 mark) – correct integrand

(1 mark) – correct terminals

(1 mark) – correct answer

Question 2 (13 marks)

a. $z_1 = \sqrt{3} + i$
 $r = \sqrt{3+1} = 2$
 $\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$
 $= \frac{\pi}{6}$ (since z_1 is in the first quadrant)
 $z_1 = 2\text{cis}\left(\frac{\pi}{6}\right)$

**(1 mark)** – correct modulus**(1 mark)** – correct argument

b. $z_1^4 = -8 + 8\sqrt{3}i$
 $LS = z_1^4$
 $= \left(2\text{cis}\left(\frac{\pi}{6}\right)\right)^4$ (from part a.)
 $= 2^4 \text{cis}\left(\frac{4\pi}{6}\right)$
 $= 16\left(\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)\right)$
 $= 16\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$
 $= -8 + 8\sqrt{3}i$
 $= RS$

So z_1 satisfies the equation.**(1 mark)**

c. $\text{Arg}(z_1^p) = \frac{\pi}{2}, \quad p \in \mathbb{R}$

So, $\arg(z_1^p) = \dots, -\frac{3\pi}{2}, \frac{\pi}{2}, \frac{5\pi}{2}, \dots$

or $\arg(z_1^p) = \frac{\pi}{2} + 2k\pi$ where $k \in \mathbb{Z}$

Also $\arg(z_1^p) = p \times \arg(z_1)$

(1 mark)

$$= p \times \frac{\pi}{6} \quad (\text{from part a.})$$

So $p \times \frac{\pi}{6} = \dots, -\frac{3\pi}{2}, \frac{\pi}{2}, \frac{5\pi}{2}, \dots$

or $\frac{p\pi}{6} = \frac{\pi}{2} + 2k\pi$

$$p = \dots, -9, 3, 15, \dots$$

$$\frac{p}{6} = \frac{1}{2} + 2k$$

$$p = 12k + 3$$

(1 mark)

d. i.

$$\begin{aligned}
 |z - 2z_1| &= |z| \\
 |x + iy - 2(\sqrt{3} + i)| &= |x + iy| & (1 \text{ mark}) \\
 |(x - 2\sqrt{3}) + (y - 2)i| &= |x + iy| \\
 \sqrt{(x - 2\sqrt{3})^2 + (y - 2)^2} &= \sqrt{x^2 + y^2} \\
 x^2 - 4\sqrt{3}x + 12 + y^2 - 4y + 4 &= x^2 + y^2 \\
 -4\sqrt{3}x - 4y + 16 &= 0 \\
 \sqrt{3}x + y &= 4 \quad \text{as required}
 \end{aligned}$$

(1 mark)

ii. Method 1

$$\begin{aligned}
 \bar{z}_1 + \frac{2}{\sqrt{3}} &= \sqrt{3} - i + \frac{2}{\sqrt{3}} \\
 &= \frac{5}{\sqrt{3}} - i
 \end{aligned}$$

So $\bar{z}_1 + \frac{2}{\sqrt{3}}$ corresponds to the point $\left(\frac{5}{\sqrt{3}}, -1\right)$ on the Cartesian plane.

(1 mark)

From part i., the Cartesian equation of the relation $|z - 2z_1| = |z|$ is $\sqrt{3}x + y = 4$.

Substituting the point into this relation gives

$$\begin{aligned}
 LS &= \sqrt{3} \times \frac{5}{\sqrt{3}} - 1 \\
 &= 4 \\
 &= RS
 \end{aligned}$$

(1 mark)

Method 2

$$\text{To Show } \left| \bar{z}_1 + \frac{2}{\sqrt{3}} - 2z_1 \right| = \left| \bar{z}_1 + \frac{2}{\sqrt{3}} \right|$$

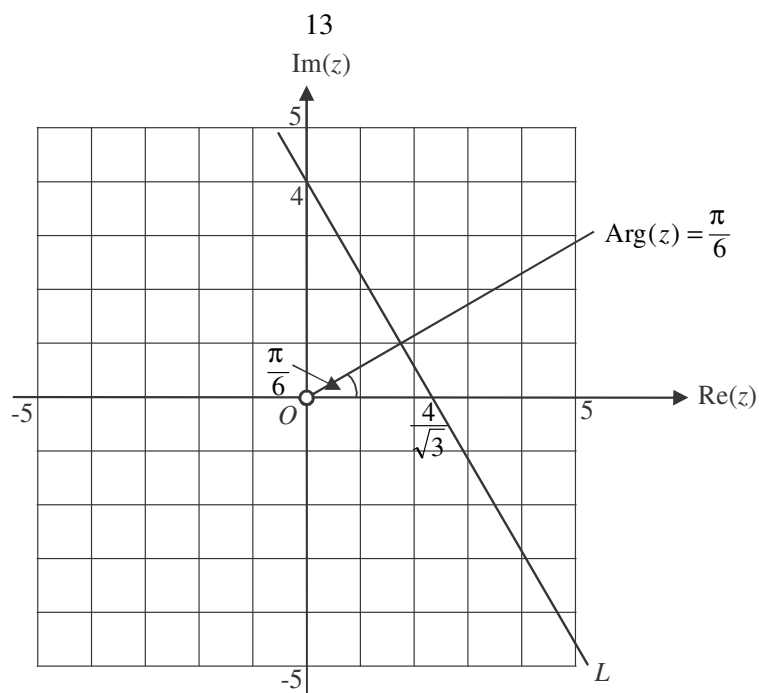
$$\begin{aligned}
 LS &= \left| \bar{z}_1 + \frac{2}{\sqrt{3}} - 2z_1 \right| \\
 &= \left| \sqrt{3} - i + \frac{2}{\sqrt{3}} - 2(\sqrt{3} + i) \right| \\
 &= \left| \sqrt{3} + \frac{2}{\sqrt{3}} - 2\sqrt{3} - 3i \right| \\
 &= \left| \frac{3+2-6}{\sqrt{3}} - 3i \right| \\
 &= \left| \frac{-1}{\sqrt{3}} - 3i \right| \\
 &= \sqrt{\frac{1}{3} + 9} \\
 &= \sqrt{\frac{28}{3}}
 \end{aligned}$$

(1 mark)

$$\begin{aligned}
 RS &= \left| \bar{z}_1 + \frac{2}{\sqrt{3}} \right| \\
 &= \left| \sqrt{3} - i + \frac{2}{\sqrt{3}} \right| \\
 &= \left| \frac{3+2}{\sqrt{3}} - i \right| \\
 &= \sqrt{\frac{25}{3} + 1} \\
 &= \sqrt{\frac{28}{3}} \\
 &= LS
 \end{aligned}$$

(1 mark)

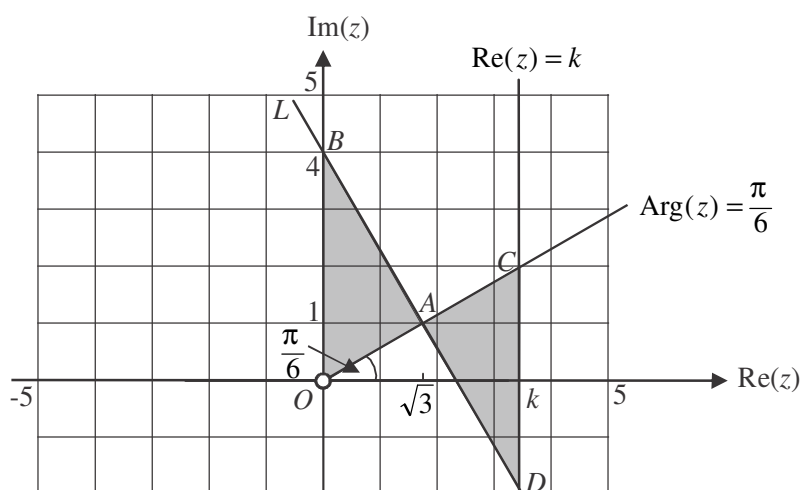
e. i.



(1 mark) for L including axes intercepts

(1 mark) for $\text{Arg}(z) = \frac{\pi}{6}$ including excluded origin

ii.



Start by finding the point of intersection of L and $\text{Arg}(z) = \frac{\pi}{6}$.

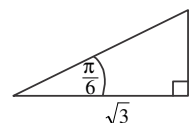
$\text{Arg}(z) = \frac{\pi}{6}$ can be expressed in Cartesian form as $y = \frac{1}{\sqrt{3}}x$.

L is $\sqrt{3}x + y = 4$ or $y = -\sqrt{3}x + 4$

So $\frac{1}{\sqrt{3}}x = -\sqrt{3}x + 4$

Solve using CAS, $x = \sqrt{3}$ so $y = 1$.

A is the point $(\sqrt{3}, 1)$.



(1 mark)

$\triangle ABO$ and $\triangle ACD$ are similar because $\angle BAO = \angle CAD$ (vertically opposite angles), $\angle ABO = \angle ADC$ (alternate angles) and $\angle AOB = \angle ACD$ (alternate angles). They will be congruent, and therefore have the same area, when $k = 2\sqrt{3}$.

(1 mark)

Question 3 (12 marks)

a. $\underline{s}(t) = \cos(\pi t) \underline{i} + \sqrt{3} \sin(\pi t) \underline{j}$

$$x = \cos(\pi t) \quad y = \sqrt{3} \sin(\pi t)$$

$$x^2 = \cos^2(\pi t) \quad y^2 = 3 \sin^2(\pi t)$$

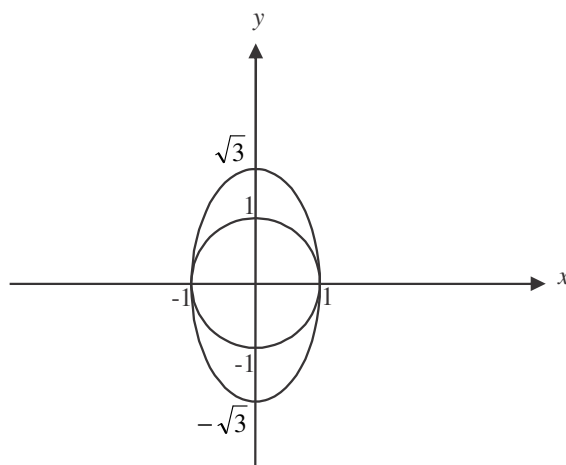
$$\frac{y^2}{3} = \sin^2(\pi t)$$

$$x^2 + \frac{y^2}{3} = \cos^2(\pi t) + \sin^2(\pi t)$$

$$x^2 + \frac{y^2}{3} = 1$$

(1 mark)

b.

**(1 mark)****(1 mark)** – correct axes intercepts**(1 mark)** – correct shape

c. $\underline{r}(0) = \sin(0) \underline{i} + \cos(0) \underline{j}$

$$= 0 \underline{i} + \underline{j}$$

For drone R, the starting position is (0,1).

(1 mark)

$$\underline{s}(0) = \cos(0) \underline{i} + \sqrt{3} \sin(0) \underline{j}$$

$$= \underline{i} + 0 \underline{j}$$

For drone S, the starting position is (1,0).

(1 mark)

d. For the drones to meet,

$$\sin(\pi t) = \cos(\pi t) \quad \underline{\text{AND}} \quad \cos(\pi t) = \sqrt{3} \sin(\pi t)$$

$$\tan(\pi t) = 1 \quad \tan(\pi t) = \frac{1}{\sqrt{3}}$$

$$\pi t = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \dots \quad \pi t = \frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}, \dots$$

$$t = \frac{1}{4}, \frac{5}{4}, \frac{9}{4}, \dots \quad t = \frac{1}{6}, \frac{7}{6}, \frac{13}{6}, \dots$$

**(1 mark)**

Since the displacement components are not equal at the same time, the drones never meet.

(1 mark)

e. $\underline{r}(t) = \sin(\pi t) \underline{i} + \cos(\pi t) \underline{j}$
 $\underline{\dot{r}}(t) = \pi \cos(\pi t) \underline{i} - \pi \sin(\pi t) \underline{j}$
 $\underline{\dot{r}}(3) = \pi \cos(3\pi) \underline{i} - \pi \sin(3\pi) \underline{j}$
 $= -\pi \underline{i} + 0 \underline{j}$
 $|\underline{\dot{r}}(3)| = \sqrt{(-\pi)^2} = \pi \text{ km/hr}$

(1 mark)

f. $\underline{r}(t) = \sin(\pi t) \underline{i} + \cos(\pi t) \underline{j}$
 $\underline{\dot{r}}(t) = \pi \cos(\pi t) \underline{i} - \pi \sin(\pi t) \underline{j}$
 $\underline{\ddot{r}}(t) = -\pi^2 \sin(\pi t) \underline{i} - \pi^2 \cos(\pi t) \underline{j}$
 $\underline{\ddot{r}}(2) = -\pi^2 \sin(2\pi) \underline{i} - \pi^2 \cos(2\pi) \underline{j}$
 $= 0 \underline{i} - \pi^2 \underline{j}$
 $|\underline{\ddot{r}}(2)| = \sqrt{(-\pi^2)^2} = \pi^2 \text{ km/hr}^2$

(1 mark)

g. $\underline{\dot{r}}(t) = \pi \cos(\pi t) \underline{i} - \pi \sin(\pi t) \underline{j}$
 $\underline{\dot{r}}\left(\frac{1}{2}\right) = 0 \underline{i} - \pi \underline{j}$
 $\underline{\dot{s}}(t) = -\pi \sin(\pi t) \underline{i} + \sqrt{3}\pi \cos(\pi t) \underline{j}$
 $\underline{\dot{s}}\left(\frac{1}{2}\right) = -\pi \underline{i} + 0 \underline{j}$

(1 mark)

$$\underline{\dot{r}}\left(\frac{1}{2}\right) \cdot \underline{\dot{s}}\left(\frac{1}{2}\right) = 0 \times -\pi - \pi \times 0 = 0$$

(1 mark)

Since $\underline{\dot{r}}\left(\frac{1}{2}\right) \cdot \underline{\dot{s}}\left(\frac{1}{2}\right) = 0$, the drones must be travelling in directions that are perpendicular to each other.

Question 4 (10 marks)

a. When $t=0$, $\tan\left(\frac{N-50\pi}{100}\right)=-4$

Solve for N using CAS

$$N = 100\left(k\pi + \tan^{-1}\left(\frac{1}{4}\right)\right)$$

$$N = 24.4978... \text{ for } k = 0$$

The number of pre-sold apartments is 24 (to the nearest integer).

(1 mark)

b. As $t \rightarrow \infty$, $\tan\left(\frac{N-50\pi}{100}\right) \rightarrow \infty$

so, $\left(\frac{N-50\pi}{100}\right) \rightarrow \frac{\pi}{2}$

and so $N \rightarrow 100\pi$

The limiting number is 314 (to the nearest integer).

(1 mark)

c. Differentiate the equation

$$\tan\left(\frac{N-50\pi}{100}\right) = \frac{t}{30} - 4 \quad \text{with respect to } t$$

$$\frac{d}{dN}\left(\tan\left(\frac{N-50\pi}{100}\right)\right)\frac{dN}{dt} = \frac{1}{30} \quad (\text{Chain rule})$$

$$\frac{1}{100}\sec^2\left(\frac{N-50\pi}{100}\right)\frac{dN}{dt} = \frac{1}{30}$$

$$\frac{1}{\cos^2\left(\frac{N-50\pi}{100}\right)}\frac{dN}{dt} = \frac{100}{30}$$

$$\frac{dN}{dt} = \frac{10}{3}\cos^2\left(\frac{N-50\pi}{100}\right)$$

(1 mark)

$$\text{Now, } \cos^2\left(\frac{N-50\pi}{100}\right) - 0.3\frac{dN}{dt} = 0$$

$$LS = \cos^2\left(\frac{N-50\pi}{100}\right) - 0.3\frac{dN}{dt}$$

$$= \cos^2\left(\frac{N-50\pi}{100}\right) - \frac{3}{10} \times \frac{10}{3}\cos^2\left(\frac{N-50\pi}{100}\right)$$

$$= \cos^2\left(\frac{N-50\pi}{100}\right) - \cos^2\left(\frac{N-50\pi}{100}\right)$$

$$= 0$$

$$= RS$$

(1 mark)

d. Method 1 – using the given expression

$$\begin{aligned}
 \frac{d^2N}{dt^2} &= \frac{d}{dt} \left(\frac{dN}{dt} \right) \\
 &= \frac{d}{dN} \left(\frac{dN}{dt} \right) \times \frac{dN}{dt} \\
 &= \frac{\sin\left(\frac{N}{100}\right) \cos\left(\frac{N}{100}\right)}{15} \times \frac{10}{3 \left(\tan^2\left(\frac{N-50\pi}{100}\right) + 1 \right)} \\
 &= \frac{2 \sin\left(\frac{N}{100}\right) \cos\left(\frac{N}{100}\right)}{9 \left(\tan^2\left(\frac{N-50\pi}{100}\right) + 1 \right)} \quad (1 \text{ mark}) \\
 &= \frac{\sin\left(\frac{N}{50}\right)}{9 \left(\tan^2\left(\frac{N-50\pi}{100}\right) + 1 \right)}
 \end{aligned}$$

We are told that the graph has a point of inflection so this occurs when $\frac{d^2N}{dt^2} = 0$.

Solve $\sin\left(\frac{N}{50}\right) = 0$ for N . (1 mark)

$$\frac{N}{50} = 0, \pi, 2\pi, \dots \quad (\text{note that } N \text{ is positive})$$

$$N = 0, 50\pi, 100\pi, \dots$$

$N = 50\pi$ is the only answer within the range of the function N .

So $N = 50\pi$

Substitute this into

$$\tan\left(\frac{N-50\pi}{100}\right) = \frac{t}{30} - 4$$

$$t = 120$$

So $a = 120$ and $b = 50\pi$

(1 mark) for a

(1 mark) for b

Method 2 – otherwise

From part c., $\cos^2\left(\frac{N-50\pi}{100}\right) - 0.3\frac{dN}{dt} = 0$

So $\frac{dN}{dt} = \frac{10}{3}\cos^2\left(\frac{N-50\pi}{100}\right)$

$$\begin{aligned} \text{so, } \frac{d^2N}{dt^2} &= \frac{d}{dt}\left(\frac{dN}{dt}\right) \\ &= \frac{d}{dN}\left(\frac{dN}{dt}\right) \times \frac{dN}{dt} \\ &= \frac{10}{3} \times \frac{\sin\left(\frac{N}{100}\right)\cos\left(\frac{N}{100}\right)}{50} \times \frac{dN}{dt} \\ &= \frac{\sin\left(\frac{N}{100}\right)\cos\left(\frac{N}{100}\right)}{15} \times \frac{10\cos^2\left(\frac{N-50\pi}{100}\right)}{3} \\ &= \frac{2\sin\left(\frac{N}{100}\right)\cos\left(\frac{N}{100}\right)\cos^2\left(\frac{N-50\pi}{100}\right)}{9} \end{aligned} \quad (1 \text{ mark})$$

We are told that the graph has a point of inflection. This occurs when $\frac{d^2N}{dt^2} = 0$.

Solve $\sin\left(\frac{N}{100}\right)\cos\left(\frac{N}{100}\right)\cos^2\left(\frac{N-50\pi}{100}\right) = 0$ for N (1 mark)

$N = 50(2k-1)\pi$ or $N = 100k\pi$

For $k = 0$, $N = -50\pi$ or $N = 0$

For $k = 1$, $N = 50\pi$ or $N = 100\pi$

Note that N is positive and $N = 50\pi$ is the only answer within the range of the function N .

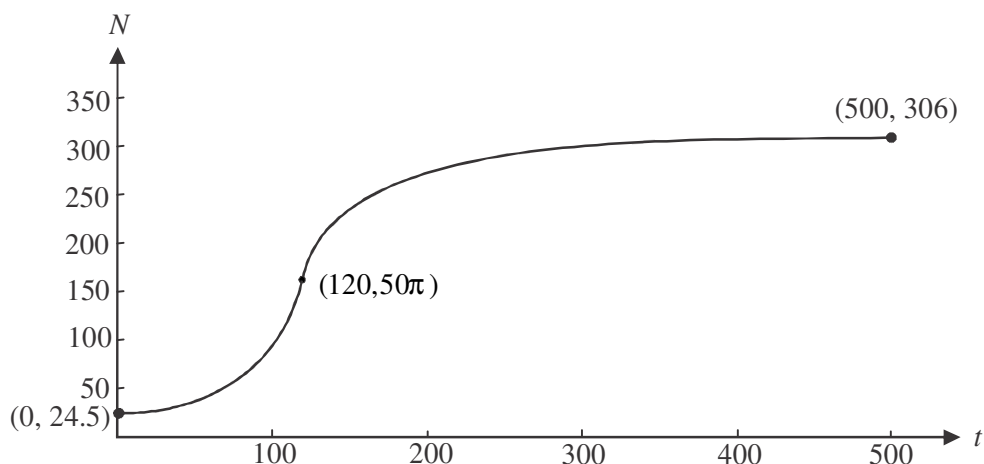
Substitute $N = 50\pi$ into $\tan\left(\frac{N-50\pi}{100}\right) = \frac{t}{30} - 4$, so $t = 120$.

So $a = 120$ and $b = 50\pi$ as required.

(1 mark) for a

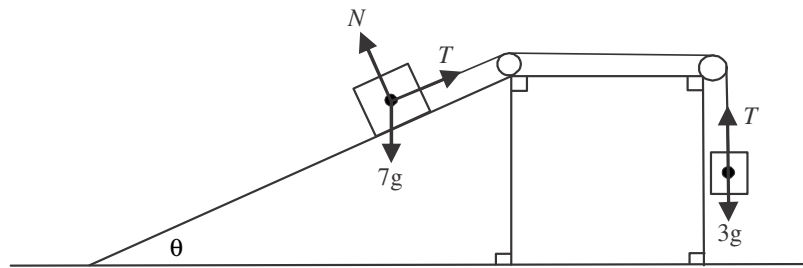
(1 mark) for b

e.



(1 mark) – correct endpoints i.e. (0,24.5) and (500,306)

(1 mark) – correct shape including point of inflection located at (120,50\pi)

Question 5 (12 marks)**a.**

i. Around the 7kg mass:
 $7g \sin(\theta) - T = 7a$ (1 mark) -(1)

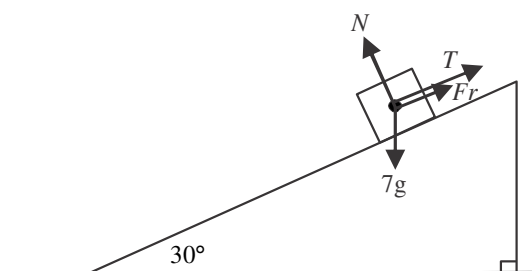
ii. Around the 3kg mass:
 $T - 3g = 3a$ (1 mark) -(2)

iii. (1) gives $T = 7g \sin(\theta) - 7a$
 (2) gives $T = 3g + 3a$
 So $7g \sin(\theta) - 7a = 3g + 3a$
 $-10a = 3g - 7g \sin(\theta)$
 $a = \frac{g(3 - 7 \sin(\theta))}{-10}$
 $= \frac{g(7 \sin(\theta) - 3)}{10}$
 as required. (1 mark)

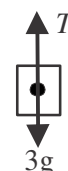
iv. If the system is in equilibrium then $a = 0$.
 So $\frac{g(7 \sin(\theta) - 3)}{10} = 0$
 $\sin(\theta) = \frac{3}{7}$
 $\theta = 25.4^\circ$
 (correct to 1 decimal place) (1 mark)

v. When the 2kg mass is moving vertically upwards, $a > 0$.
 So $\frac{g(7 \sin(\theta) - 3)}{10} > 0$
 $7 \sin(\theta) - 3 > 0$
 $\sin(\theta) > \frac{3}{7}$
 $25.4^\circ < \theta < 90^\circ$ (1 mark)

- b. Around the 7kg mass:
 $7g \sin(30^\circ) = T + Fr$
 $Fr = 3.5g - T \quad \text{--- (1)}$
 and $N = 7g \cos(30^\circ) = 59.4093\dots$



- Around the 3kg mass:
 $T = 3g \quad \text{--- (2)}$
 In (1) gives $Fr = 3.5g - 3g = 4.9$



(1 mark)

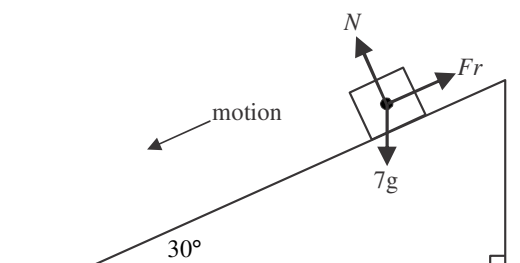
If the 7kg mass is on the point of moving then $Fr = \mu N$.

$$\text{Now } \mu N = 0.106 \times 59.4093\dots \\ = 6.2973\dots$$

Since $4.9 < 6.2973\dots$, then $Fr < \mu N$ and the mass is not at the point of moving.

(1 mark)

- c. i. $7g \sin(30^\circ) - Fr = 7a$
 $3.5g - Fr = 7a$
 $a = \frac{3.5g - Fr}{7}$
 Also $N = 7g \cos(30^\circ)$
 $= \frac{7\sqrt{3}g}{2}$
 So $a = \frac{3.5g - \mu N}{7}$
 $= 4.00037\dots$
 $= 4.00 \text{ m/s}^2 \text{ (to the nearest } 0.01 \text{ m/s}^2 \text{)}$



(1 mark)

- ii. $3g = 3a$
 $a = g \text{ m/s}^2$
 The 3kg mass is subject only to gravitational force so its acceleration is $g \text{ m/s}^2$ or 9.8 m/s^2 .



(1 mark)

(1 mark)

- d. Both masses are travelling with constant acceleration.

So, using $v^2 = u^2 + 2as$

$$\text{for the 3kg mass, } v^2 = 0 + 2 \times 9.8 \times s \\ = 19.6s$$

(1 mark)

$$\text{for the 7kg mass, } v^2 = 0 + 2 \times 4.00 \times 3s \\ = 24s$$

So ratio of speeds is $\sqrt{19.6s} : \sqrt{24s}$

$$\sqrt{196} : \sqrt{240}$$

$$\sqrt{49} : \sqrt{60}$$

$$7 : 2\sqrt{15}$$

(1 mark)