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Students Name:.....

SPECIALIST MATHEMATICS

TRIAL EXAMINATION 1

2015

Reading Time: 15 minutes

Writing time: 1 hour

Instructions to students

This exam consists of 9 questions.
All questions should be answered.
There is a total of 40 marks available.
The marks allocated to each of the questions are indicated throughout.
Students may not bring any notes or calculators into the exam.
Where more than one mark is allocated to a question, appropriate working must be shown.
Where an exact answer is required to a question, a decimal approximation will not be accepted.
Unless otherwise indicated, diagrams in this exam are not drawn to scale.
The acceleration due to gravity should be taken to have magnitude $g \text{ m/s}^2$ where $g = 9.8$
Formula sheets can be found on pages 11-13 of this exam.

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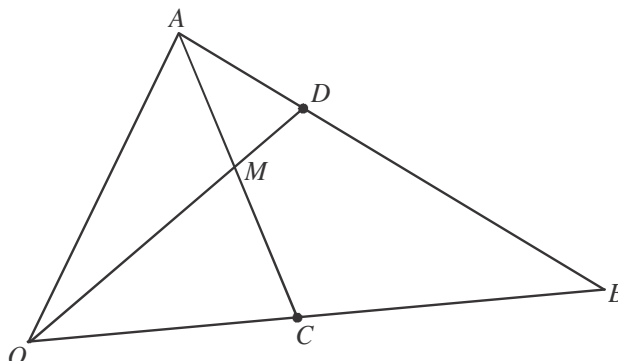
Question 1 (3 marks)

Evaluate $\int_0^{\sqrt{6}} \frac{x-5}{x^2+2} dx$

Question 2 (5 marks)

OAB is a triangle where C is the midpoint of $\overline{OB}$ and $3\overline{AD} = \overline{AB}$.

$\overline{OD}$ and $\overline{AC}$ intersect at point M .



Let $\vec{OA} = \underline{a}$, $\vec{OB} = \underline{b}$, $\vec{CM} = \alpha \vec{CA}$ and $\vec{OM} = \beta \vec{OD}$.

a. Express $\vec{CM}$ in terms of

i. $\underline{a}$, $\underline{b}$ and α .

1 mark

ii. $\underline{a}$, $\underline{b}$ and β

2 marks

b. Hence find the values of α and β .

2 marks

Question 3 (3 marks)

Let $\underline{a} = \underline{i} - 4\underline{j} + 2\underline{k}$ and $\underline{b} = 2\underline{i} + \underline{j} - 2\underline{k}$.

Resolve $\underline{a}$ into two vector resolutes, one parallel to $\underline{b}$ and the other perpendicular to $\underline{b}$.

Question 4 (4 marks)

Let $f(z) = z^4 - 2z^3 + 8z^2 - 8z + 16$, $z \in \mathbb{C}$. Given that $z = 2i$ is a solution of $f(z) = 0$, find all the other solutions in Cartesian form.

Question 5 (5 marks)

A box of mass 2kg rests on a rough horizontal floor. It is given a one-off push which causes it to travel across the floor with an initial velocity of 10m/s. The frictional force F shown in the diagram below, exerted by the floor on the box, causes the box to come to rest 4 seconds later.



- a.** Find the acceleration of the box across the floor. 2 marks

- b.** How far does the box travel before coming to rest? 1 mark

- c.** The coefficient of friction between the box and the floor equals $\frac{r}{sg}$, where r and s are positive integers and $\frac{r}{s}$ is a fraction expressed in simplest form.
Find the values of r and s . 2 marks

Question 6 (4 marks)

Find the gradient of the tangent to the curve defined by $2xy - \arctan\left(\frac{x}{2}\right) + y^2 = 5 - \frac{\pi}{4}$ at the point $(2,1)$.

Question 7 (4 marks)

Calculate the area of the region enclosed by the graphs of $y = \frac{1}{x^2 - 2x - 3}$, $x = 2$ and the x and y axes.

Question 8 (5 marks)

A particle moves in a straight line with acceleration a m/s², velocity v m/s and position x m, relative to a fixed point on the line. The relationship between the position and the velocity of the particle at time t seconds, $t \geq 0$, is given by $v = \sqrt{2x + 4}$. The particle is initially at the fixed point.

- a.** Show that the acceleration of the particle is constant. 2 marks

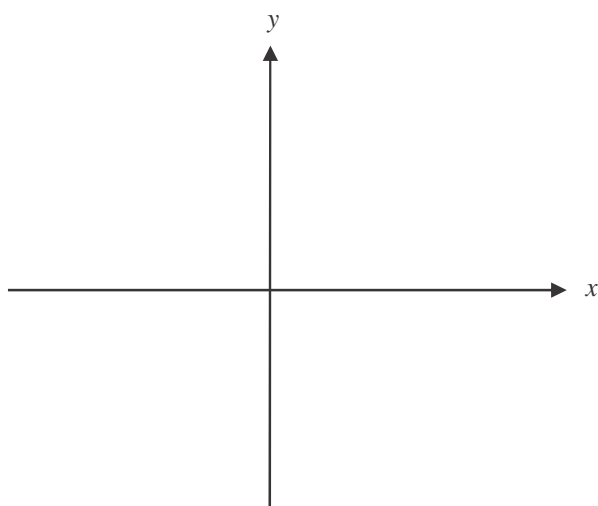
- b.** Find the value of x when $t = 3$. 3 marks

Question 9 (7 marks)

Consider the function $f(x) = 6\arccos(2x)$.

- a.** Write down the maximal domain and range of f . 2 marks

- b.** Sketch the graph of $y = f(x)$ on the set of axes below. Label any endpoints and axes intercepts with their coordinates. 2 marks



Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of a triangle:	$\frac{1}{2}bc \sin A$
sine rule:	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
cosine rule:	$c^2 = a^2 + b^2 - 2ab \cos C$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	hyperbola: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$
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Circular (trigonometric) functions

$\cos^2(x) + \sin^2(x) = 1$ $1 + \tan^2(x) = \sec^2(x)$ $\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$ $\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$ $\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$ $\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$ $\sin(2x) = 2\sin(x)\cos(x)$	$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$ $\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$ $\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$ $\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$ $\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$
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function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (Complex numbers)

$z = x + yi = r(\cos \theta + i \sin \theta) = r \operatorname{cis} \theta$ $ z = \sqrt{x^2 + y^2} = r$ $z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$ $z^n = r^n \operatorname{cis}(n\theta)$ (de Moivre's theorem)	$-\pi < \operatorname{Arg} z \leq \pi$ $\frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$
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Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$$

$$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$$

$$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$$

$$\frac{d}{dx}(\tan(ax)) = a \sec^2(ax)$$

$$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$$

$$\int \frac{1}{x} dx = \log_e|x| + c$$

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$$

$$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$$

$$\int \sec^2(ax) dx = \frac{1}{a} \tan(ax) + c$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c, a > 0$$

$$\int \frac{-1}{\sqrt{a^2 - x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, a > 0$$

$$\int \frac{a}{a^2 + x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$$

product rule:

$$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

quotient rule:

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

chain rule:

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

Euler's method:

$$\text{If } \frac{dy}{dx} = f(x), x_0 = a \text{ and } y_0 = b,$$

$$\text{then } x_{n+1} = x_n + h \text{ and } y_{n+1} = y_n + hf(x_n)$$

acceleration:

$$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$\text{constant (uniform) acceleration: } v = u + at \quad s = ut + \frac{1}{2}at^2 \quad v^2 = u^2 + 2as \quad s = \frac{1}{2}(u+v)t$$

Vectors in two and three dimensions

$$\underline{r} = x \underline{i} + y \underline{j} + z \underline{k}$$

$$|\underline{r}| = \sqrt{x^2 + y^2 + z^2} = r$$

$$\underline{r}_1 \cdot \underline{r}_2 = r_1 r_2 \cos \theta = x_1 x_2 + y_1 y_2 + z_1 z_2$$

$$\dot{\underline{r}} = \frac{d \underline{r}}{dt} = \frac{dx}{dt} \underline{i} + \frac{dy}{dt} \underline{j} + \frac{dz}{dt} \underline{k}$$

Mechanics

momentum:

$$\underline{p} = m \underline{v}$$

equation of motion:

$$\underline{R} = m \underline{a}$$

friction:

$$F \leq \mu N$$