E

Free Exam
for 2006-15 VCE study design

Engage
Education
Foundation

Units 3 and 4 Specialist Maths: Exam 2

Practice Exam Solutions

Stop!

Don’t look at these solutions until you have attempted the exam.

Any questions?

Check the Engage website for updated solutions, then email practiceexams@ee.org.au.

Section A – Multiple-choice questions

Question 1

The correct answer is E.

$$\frac{x-5}{3} = (\cos(2t))^2 \text{ and } y = 3(2(\cos(2t))^2 - 1)$$
$$\Rightarrow y = 2x - 13$$

From the parametric equation,

$x_{\min} = 5 + 3(0) = 5$

$x_{\max} = 5 + 3(\pm 1)^2 = 8$

$\Rightarrow x \in [5, 8]$

Question 2

The correct answer is D.

Asymptotes are $y = \pm\frac{3}{2}(x-3) - 2$.

The relation is a hyperbola.

The relation has gradient $\frac{dy}{dx} = \frac{9(x-3)}{4(y+2)}$.

The parametric equation can be rearranged:

$\csc(t) = \frac{x-3}{2}$ and $\cot(t) = \frac{y+2}{3}$

Since $(\csc(t))^2 - (\cot(t))^2 = 1$, $\frac{(x-3)^2}{4} - \frac{(y+2)^2}{9} = 1$ as required.

Question 3

The correct answer is A.

$$-1 \le 3x - 5 \le 1$$
$$4 \le 3x \le 6$$
$$\frac{4}{3} \le x \le 2$$
$$\Rightarrow x \in \left[\frac{4}{3},\ 2\right]$$

Question 4

The correct answer is C.

Asymptote $y = -kx \Rightarrow k = -1$.

Let $g(x) = ax^2 + bx + c$ and $f(x) = -x + \frac{1}{g(x)}$

Use CAS to solve using the given information:

Define $g(x)=a\cdot x^2+b\cdot x+c$	*Done*
Define $f(x)=^-x+\frac{1}{g(x)}$	*Done*
solve$\left(g\left(\frac{^-5}{2}\right)=0 \text{ and } g(4)=0 \text{ and } \frac{d}{dx}(f(x))=0 \mid x=^-2.777,a,b,c\right)$	$a=2.00171$ and $b=^-3.00256$ and $c=^-20.0171$

Question 5

The correct answer is B.

The circles are centred at (2,-3) with radii 2 and 3. Hence the desired region is where the distance from $2-3i$ to z is between 2 and 3.

Question 6

The correct answer is A.

$z = 5cis\left(\frac{7\pi}{9}\right)$

$\Rightarrow z^3 = 5^3 cis\left(\frac{7\pi}{3}\right)$

The angle $\frac{7\pi}{3}$ is equivalent to $\frac{\pi}{3}$.

Question 7

The correct answer is D.

Trial and error may be best method:

$\text{solve}\left(a=(\sin(2\cdot x))^3,x\right)\mid z=cis(x) \text{ and } a=\frac{-(z^4-1)}{2\cdot z^2}\cdot i$	$x=\frac{(2\cdot n1-1)\cdot\pi}{4}$ or $x=\frac{n2\cdot\pi}{2}$
$\text{solve}\left(a=(\sin(2\cdot x))^3,x\right)\mid z=cis(x) \text{ and } a=\left(\frac{z^4-1}{2\cdot z^2}\right)^3$	$x=\frac{n3\cdot\pi}{2}$ and $\left(\sin\left(\frac{n3\cdot\pi}{2}\right)\right)^3\cdot\left(\cos\left(\frac{n3\cdot\pi}{2}\right)\right)^3=0$
$\text{solve}\left(a=(\sin(2\cdot x))^3,x\right)\mid z=cis(x) \text{ and } a=\frac{(z^4-1)^3}{8\cdot z^6}\cdot i$	true

Alternatively:

$$(\sin(2\theta))^3 = \left(\frac{cis(2\theta)-cis(-2\theta)}{2i}\right)^3$$

$$= \left(\frac{z^2-z^{-2}}{2i}\right)^3$$

Inputting this on the calculator gives:

⚠ $\left(\frac{z^2-z^{-2}}{2\cdot i}\right)^3$	$\frac{(z^4-1)^3}{8\cdot z^6}\cdot i$

Question 8

The correct answer is C.

Solving on the calculator:

$$\text{cSolve}\left(z^3=-4\cdot\sqrt{2}+4\cdot\sqrt{2}\cdot i, z\right) \quad z=\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}+\left(\frac{-\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\cdot i \text{ or } z=\frac{-\sqrt{6}}{2}-\frac{\sqrt{2}}{2}+\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)\cdot i \text{ or } z=\sqrt{2}+\sqrt{2}\cdot i$$

$$\left(\text{cSolve}\left(z^3=-4\cdot\sqrt{2}+4\cdot\sqrt{2}\cdot i, z\right)\right)\blacktriangleright\text{Polar} \quad z=e^{-i\cdot\tan^{-1}\left(\frac{\sqrt{12}+4}{2}\right)}\cdot 2 \text{ or } z=e^{i\cdot\left(\tan^{-1}\left(\frac{\sqrt{12}+4}{2}\right)+\frac{\pi}{2}\right)}\cdot 2 \text{ or } z=e^{\frac{i\cdot\pi}{4}}\cdot$$

This shows that A, B, D and E are incorrect. To find nicer expressions for the other two solutions, use the fact that the three solutions are evenly spaced in a circle. Hence if

$$\theta_1=\frac{\pi}{4}$$
$$\theta_2=\frac{\pi}{4}+\frac{2\pi}{3}=\frac{11\pi}{12}$$
$$\theta_3=\frac{\pi}{4}-\frac{2\pi}{3}=-\frac{5\pi}{12}$$

Question 9

The correct answer is C.

$$\int_0^1 \frac{1}{x\cdot\left((\ln(x))^2+2\right)}\,dx \quad \frac{\pi\cdot\sqrt{2}}{4}$$

$$\int_{-\infty}^0 \frac{1}{u^2+2}\,du \quad \frac{\pi\cdot\sqrt{2}}{4}$$

Question 10

The correct answer is E.

The relation $x=y^2(y-2)$ is the cubic function $y=x^2(x-2)$ reflected in the line $y=x$. The question is asking for the volume generated by rotating the area bounded by this function and the x axis around the x axis.

$$A=\pi\int_0^2 y^2\,dx$$
$$A=\pi\int_0^2 \left(x^2(x-2)\right)^2\,dx$$

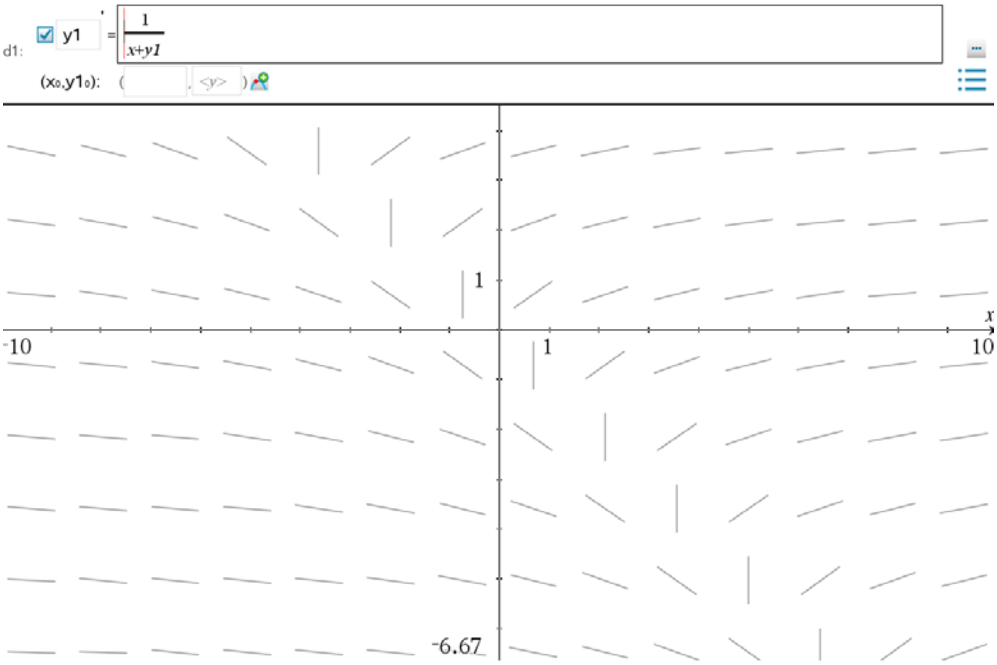
Question 11
The correct answer is B.

$$\text{euler}\left(\frac{x \cdot y}{x^2+y^2}, x, y, \{0, 0.3\}, 1, 0.1\right) \qquad \begin{bmatrix} 0. & 0.1 & 0.2 & 0.3 \\ 1. & 1. & 1.0099 & 1.02896 \end{bmatrix}$$

Question 12
The correct answer is A.

$$\frac{dx}{dy} = x + y$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{x+y}$$



Question 13
The correct answer is E.

$$\frac{dQ}{dt} = \frac{dQ_{in}}{dt} - \frac{dQ_{out}}{dt}$$

$$\frac{dQ}{dt} = \frac{dQ_{in}}{dV_{in}} \cdot \frac{dV_{in}}{dt} - \frac{dQ_{out}}{dV_{out}} \cdot \frac{dV_{out}}{dt}$$

$$\frac{dQ}{dt} = 0.5 \cdot 15 - \frac{Q}{100 + (15-5)t} \cdot 5$$

$$\frac{dQ}{dt} = 7.5 - \frac{Q}{20 + 2t}$$

Question 14
The correct answer is C.

For the vectors to be linearly dependent, the following equation has infinitely many solutions:

$$x\boldsymbol{a} + y\boldsymbol{b} + z\boldsymbol{c} = \boldsymbol{0}$$
$$\begin{bmatrix} 1 & 2 & m \\ 2 & m & 1 \\ m & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

For the system of equations to have infinitely many solutions, the determinant of the square matrix must be zero. By solving on the calculator:

$$\text{solve}\left(\det\left(\begin{bmatrix} 1 & 2 & m \\ 2 & m & 1 \\ m & 1 & 2 \end{bmatrix}\right)=0, m\right) \qquad m=-3$$

Question 15
The correct answer is A.

$\overrightarrow{OA} = \overrightarrow{OB} + \overrightarrow{BA}$
$\overrightarrow{OA} = \overrightarrow{OB} - \overrightarrow{AB}$

On the calculator:

$$\text{norm}\left(\begin{bmatrix} 3 & 4 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 2 \end{bmatrix}\right) \qquad 3$$

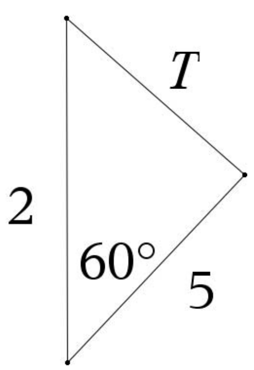
Question 16
The correct answer is E.

On the calculator:

$$\text{solve}\left(|x+y\cdot \boldsymbol{i}-(1+2\cdot \boldsymbol{i})|=|x+y\cdot \boldsymbol{i}-(5-2\cdot \boldsymbol{i})|, y\right) \qquad y=x-3$$

Question 17
The correct answer is B.

Since the particle is in equilibrium, the forces sum to zero and so the diagram can be rearranged to:



T is found by applying the cosine rule.

Question 18
The correct answer is B.

$$\Sigma F = 4v^2 + 2v = ma$$
$$4v^2 + 2v = 2v \cdot \frac{dv}{dx}$$

On the calculator:

deSolve$\left(4\cdot v^2+2\cdot v=2\cdot v\cdot v' \text{ and } v(0)=1,x,v\right)$	$v=\frac{3\cdot e^{2\cdot x}}{2}-\frac{1}{2}$

Question 19
The correct answer is B.

$$u = 5$$
$$a = -g$$
$$x = 0 = ut + \frac{at^2}{2}$$
$$0 = 5t - \frac{gt^2}{2}$$
$$\Rightarrow t = 0 \text{ or } t = \frac{10}{g}$$

Question 20
The correct answer is B.

$$\Sigma F = N - W = ma$$
$$N - mg = -3m$$
$$N = m(g - 3)$$

Question 21
The correct answer is D.

Since the particle slows at a constant rate, the net force should have no time dependence.

To check:
$$t = 3$$
$$u = 8$$
$$v = 2 = u + at$$
$$2 = 8 + 3a$$
$$\Rightarrow a = -2$$
$$F = ma = 5(-2) = -10$$

Question 22
The correct answer is A.

$$F_1 = -kx$$
$$F_2 = -bv^2$$
$$\Sigma F = F_1 + F_2 = ma$$
$$-(kx + bv^2) = mv\frac{dv}{dx}$$
$$\frac{-(kx + bv^2)}{mv} = \frac{dv}{dx}$$

Section B – Short-answer questions

Marks are indicated by either Mx (for method marks) or Ax (for answer marks), where x is the number of marks allocated for that line.

Question 1a i

$$\frac{d(x^2)}{dx}+\frac{d(xy)}{dx}+\frac{d(y^2)}{dx}=0 \text{ ...M1}$$

$$\frac{dy}{dx}=\frac{-(2x+y)}{x+2y} \text{ ...A1}$$

$$\text{impDif}\left(x^2+x\cdot y+y^2-6=0,x,y\right) \quad \frac{-(2\cdot x+y)}{x+2\cdot y}$$

Question 1a ii

$x^2+xy+y^2-6=0$

$$\frac{dy}{dx}=0 \Rightarrow \left(-\sqrt{2},\ 2\sqrt{2}\right) \text{ or } \left(\sqrt{2},\ 2\sqrt{2}\right) \text{ ...A1}$$

$$\frac{dx}{dy}=0 \Rightarrow \left(-2\sqrt{2},\ \sqrt{2}\right) \text{ or } \left(2\sqrt{2},\ \sqrt{2}\right) \text{ ...A1}$$

Some indication of needing to solve simultaneous equations …M1

$$\text{solve}\left(x^2+x\cdot y+y^2-6=0 \text{ and } \frac{-(2\cdot x+y)}{x+2\cdot y}=0,x,y\right)$$

$$x=-\sqrt{2} \text{ and } y=2\cdot\sqrt{2} \text{ or } x=\sqrt{2} \text{ and } y=-2\cdot\sqrt{2}$$

$$\text{solve}\left(x^2+x\cdot y+y^2-6=0 \text{ and } \frac{-(x+2\cdot y)}{2\cdot x+y}=0,x,y\right)$$

$$x=-2\cdot\sqrt{2} \text{ and } y=\sqrt{2} \text{ or } x=2\cdot\sqrt{2} \text{ and } y=-\sqrt{2}$$

Question 1b

$x^2 + xy + y^2 - 6 = 0$

$$x = \frac{1}{\sqrt{2}}(u - v)$$

$$y = \frac{1}{\sqrt{2}}(u + v)$$

$$\Rightarrow 3u^2 + v^2 = 12 \text{ ...A1}$$

$$\frac{u^2}{4} + \frac{v^2}{12} = 1$$

This is an ellipse …A1

Some indication of needing to solve system of equations …M1

$$x^2+x\cdot y+y^2-6=0|x=\frac{1}{\sqrt{2}}\cdot(u-v) \text{ and } y=\frac{1}{\sqrt{2}}\cdot(u+v)$$

$$\frac{3\cdot u^2}{2}+\frac{v^2}{2}-6=0$$

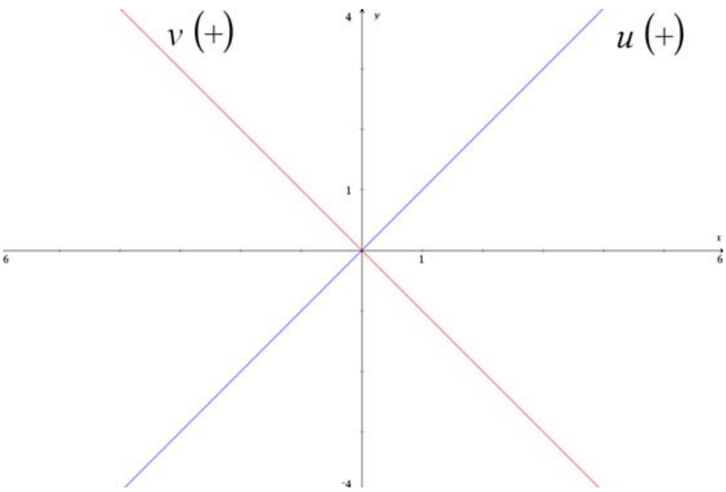
Question 1c i

$$x = \frac{1}{\sqrt{2}}(u - v)$$

$$y = \frac{1}{\sqrt{2}}(u + v)$$

$$\Rightarrow u = \frac{1}{\sqrt{2}}(x + y) \text{ and } v = \frac{1}{\sqrt{2}}(y - x) \text{ ...A1}$$

Some indication of need to solve the system of equations …M1



Half mark for each of:

- Correctly drawing perpendicular lines $y = \pm x$
- Indicating which is u, v
- Indicating positive directions for u and v
- Indicating the scale – 1 unit along the u, v axes measured with a ruler is the same length as one unit along the x, y axes.

Question 1c ii

Note that the relation can be expressed as:

$x^2 + xy + y^2 - 6 = 0$ or $\frac{u^2}{4} + \frac{v^2}{12} = 1$

x-intercept:

$y = 0$ ...M1/2

$\Rightarrow x = \pm\sqrt{6}$ ...A1/2

y-intercept:

$x = 0$ ...M1/2

$\Rightarrow y = \pm\sqrt{6}$ ...A1/2

u-intercept:

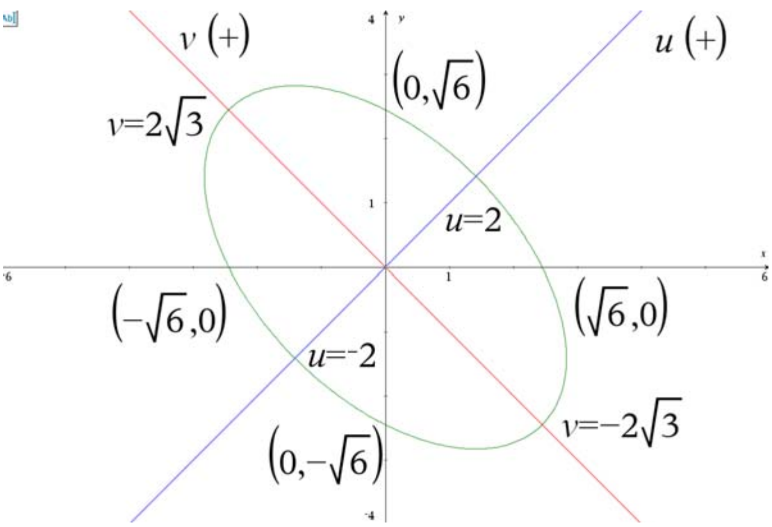
$v = 0$ ...M1/2

$\Rightarrow u = \pm 2$ ...A1/2

v-intercept:

$u = 0$ ...M1/2

$v = \pm 2\sqrt{3}$ ...A1/2



[1] for correct shape.

Question 1d

Area can be found by integrating along the u (or v) axis. (It can also be found by the formula for the area of an ellipse.)

$$3u^2 + v^2 = 12$$

$$\Rightarrow v = \pm\sqrt{12 - 3u^2}$$

$$A = 2 \cdot \int_{-2}^{2} v\, du \quad \text{...M1}$$

$$A = 2 \cdot \int_{-2}^{2} \sqrt{12 - 3u^2}\, du$$

$$A = 4\pi\sqrt{3} \text{ square units ...A1}$$

$2 \cdot \int_{-2}^{2} \sqrt{12-3 \cdot u^2}\, du$	$4 \cdot \pi \cdot \sqrt{3}$

Question 1e i

$3u^2 + v^2 = 2c$ ...A1

$x^2+x \cdot y+y^2-c=0 \mid x=\frac{1}{\sqrt{2}} \cdot (u-v)$ and $y=\frac{1}{\sqrt{2}} \cdot (u+v)$	$\frac{3 \cdot u^2}{2}+\frac{v^2}{2}-c=0$

Question 1e ii

The volumes of the solids of revolution are found in the normal way, as one would find the volumes when rotating around the x or y axes.

$$3u^2 + v^2 = 2c$$

$$\Rightarrow u^2 = \frac{2c - v^2}{3} \text{ and } v^2 = 2c - 3u^2$$

$$V_u = \pi \int_{-2}^{2} v^2\, du \quad \text{...M1/2}$$

$$= \pi \int_{-2}^{2} (2c - 3u^2)\, du \text{ ...A1}$$

$$V_v = \pi \int_{-2\sqrt{3}}^{2\sqrt{3}} u^2\, dv \quad \text{...M1/2}$$

$$= \pi \int_{-2\sqrt{3}}^{2\sqrt{3}} \frac{2c - v^2}{3}\, dv \quad \text{...A1}$$

$$k = \frac{V_u}{V_v}$$

$$= \frac{\pi \int_{-2}^{2} (2c - 3u^2)\, du}{\pi \int_{-2\sqrt{3}}^{2\sqrt{3}} \frac{2c - v^2}{3}\, dv}$$

$$= \sqrt{3} \text{ ...A1}$$

$$\frac{\int_{-2}^{2} \left(2\cdot c - 3\cdot u^2\right) du}{\int_{-2\cdot\sqrt{3}}^{2\cdot\sqrt{3}} \frac{2\cdot c - v^2}{3}\, dv} \qquad \sqrt{3}$$

Question 2a i

$[-3\ \ 2\ \ 2] \to a$	$[-3\ \ 2\ \ 2]$
$[-3\ \ 0\ \ 0] \to b$	$[-3\ \ 0\ \ 0]$
$[3\ \ w\ \ 4] \to c$	$[3\ \ w\ \ 4]$

The vectors are linearly depended when the following equation has infinitely many solutions. One technique is to convert to a matrix equation and find when the determinant is zero. (M1)

$$\alpha_1 \boldsymbol{a} + \alpha_2 \boldsymbol{b} + \alpha_3 \boldsymbol{c} = \boldsymbol{0}$$
$$\begin{bmatrix} -3 & -3 & 3 \\ 2 & 0 & w \\ 2 & 0 & 4 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$
$$\det\left(\begin{bmatrix} -3 & -3 & 3 \\ 2 & 0 & w \\ 2 & 0 & 4 \end{bmatrix}\right) = 0 \text{ ...M1}$$
$$\Rightarrow w = 4 \text{ ...A1}$$

$\text{solve}\left(\det\left(\begin{bmatrix} -3 & -3 & -3 \\ 2 & 0 & w \\ 2 & 0 & 4 \end{bmatrix}\right)=0, w\right)$	$w=4$

Question 2a ii

$$\boldsymbol{c} = [3\quad 4\quad 4] = k[-3\quad 2\quad 2] + h[-3\quad 0\quad 0] \text{ ...M1}$$
$$\Rightarrow k = 2 \text{ and } h = -3$$
$$\Rightarrow \boldsymbol{c} = 2\boldsymbol{a} - 3\boldsymbol{b} \text{ ...A1}$$

$\text{solve}(c=k\cdot a+h\cdot b,k,h)\vert w=4$	$k=2$ and $h=-3$

Question 2b i

$$\boldsymbol{b}_{\text{parallel to }\mathbf{a}} = (\boldsymbol{b}\cdot\hat{\boldsymbol{a}})\hat{\boldsymbol{a}} \text{ ...M1}$$
$$= \frac{1}{17}(-27\boldsymbol{i} + 18\boldsymbol{j} + 18\boldsymbol{k}) \text{ ...A1}$$

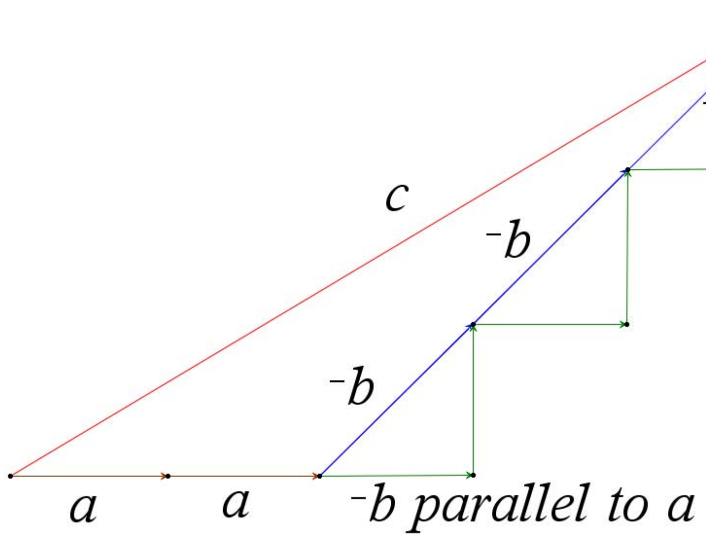
$vctpll(b,a)$	$\begin{bmatrix} \frac{-27}{17} & \frac{18}{17} & \frac{18}{17} \end{bmatrix}$

Question 2b ii

$\boldsymbol{c} = 2\boldsymbol{a} - 3\boldsymbol{b}$

$= 2\boldsymbol{a} - 3(\boldsymbol{b}_{\text{parallel to }\mathbf{a}} + \boldsymbol{b}_{\text{perpendicular to }\mathbf{a}})$

Consider the diagram below, which shows $\boldsymbol{c}$ in terms of $\boldsymbol{a}$ and $-\boldsymbol{b}$.



From the diagram it can be seen that:

$\boldsymbol{c}_{\text{parallel to }\mathbf{a}} = 2\boldsymbol{a} - 3\boldsymbol{b}_{\text{parallel to }\mathbf{a}}$ ...M1

$= \frac{1}{17}(-21\boldsymbol{i} + 14\boldsymbol{j} + 14\boldsymbol{k})$ ...A1

$2\cdot a-3\cdot\begin{bmatrix}\frac{-27}{17} & \frac{18}{17} & \frac{18}{17}\end{bmatrix}$	$\begin{bmatrix}\frac{-21}{17} & \frac{14}{17} & \frac{14}{17}\end{bmatrix}$

To check:

$vctpll(c,a)\vert w=4$	$\begin{bmatrix}\frac{-21}{17} & \frac{14}{17} & \frac{14}{17}\end{bmatrix}$

Question 2c i

$$\boldsymbol{a} \cdot \boldsymbol{c} = |\boldsymbol{a}||\boldsymbol{c}| \cdot \cos(\theta)$$

$$\theta = \cos^{-1}\left(\frac{\boldsymbol{a} \cdot \boldsymbol{c}}{|\boldsymbol{a}||\boldsymbol{c}|}\right) \text{ ...M1}$$

$$\theta = \cos^{-1}\left(\frac{(2w-1) \cdot \sqrt{17}}{17 \cdot \sqrt{w^2+25}}\right) \text{ ...M1}$$

θ_{min} occurs when $w \to \infty$ ...M1/2

$$\theta_{\text{min}} = 60.98°$$

θ_{max} occurs when $w = -50$ ...M1/2

$$\theta_{\text{max}} = 119.18°$$

$$\Rightarrow \theta \in (60.98°, 119.18°] \text{ ...A1}$$

$angvct(a,c)$	$\cos^{-1}\left(\frac{\sqrt{17} \cdot (2 \cdot w-1)}{17 \cdot \sqrt{w^2+25}}\right)$
$\text{fMin}\left(\cos^{-1}\left(\frac{\sqrt{17} \cdot (2 \cdot w-1)}{17 \cdot \sqrt{w^2+25}}\right), w\right)$	$w=\infty$
$\dfrac{\lim_{w \to \infty}\left(\cos^{-1}\left(\frac{\sqrt{17} \cdot (2 \cdot w-1)}{17 \cdot \sqrt{w^2+25}}\right)\right)}{rr}$	60.9829
$\text{fMax}\left(\cos^{-1}\left(\frac{\sqrt{17} \cdot (2 \cdot w-1)}{17 \cdot \sqrt{w^2+25}}\right), w\right)$	$w=-50$
$\dfrac{\cos^{-1}\left(\frac{\sqrt{17} \cdot (2 \cdot w-1)}{17 \cdot \sqrt{w^2+25}}\right)}{rr} \mid w=-50$	119.176

The final answers are divided by rr to convert them to degrees (see explanation on page Error! Bookmark not defined. of these solutions for further information).

Question 2c ii

The forces are perpendicular $\Rightarrow \boldsymbol{a} \cdot \boldsymbol{c} = 0$ ⋯M1

$\Rightarrow w = \frac{1}{2}$ ...A1

solve(dotP(a,c)=0,w)	$w=\frac{1}{2}$

Or, one can solve for the angle between the vectors to be 90 degrees:

solve$\left(angvct(a,c)=\frac{\pi}{2},w\right)$	$w=\frac{1}{2}$

Question 2d i

$[x \ \ y \ \ z] \rightarrow v$	$[x \ \ y \ \ z]$

$\hat{\boldsymbol{v}} = \frac{\boldsymbol{v}}{|\boldsymbol{v}|}$ ...M1

$= \frac{1}{\sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}}(\dot{x}\boldsymbol{i} + \dot{y}\boldsymbol{j} + \dot{z}\boldsymbol{k})$ ...A1

unitV(v)	$\left[\frac{x}{\sqrt{x^2+y^2+z^2}} \ \ \frac{y}{\sqrt{x^2+y^2+z^2}} \ \ \frac{z}{\sqrt{x^2+y^2+z^2}}\right]$

Question 2d ii

magnitude $\propto |\boldsymbol{v}|$

Hence magnitude $= k\sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}$ ...M1

Direction is $-\hat{\boldsymbol{v}}$ ...M1

$\Rightarrow \boldsymbol{F} = -k(\dot{x}\boldsymbol{i} + \dot{y}\boldsymbol{j} + \dot{z}\boldsymbol{k})$ ...A1

⚠ $-k \cdot$ norm(v) $\cdot$ unitV(v)	$[-k \cdot x \ \ -k \cdot y \ \ -k \cdot z]$

Question 2e

$\Sigma \boldsymbol{F} = \boldsymbol{a} + \boldsymbol{c} + \boldsymbol{F}$

$= -k\dot{x}\boldsymbol{i} + \left(\frac{5-2k\dot{y}}{2}\right)\boldsymbol{j} + (6-k\dot{z})\boldsymbol{k}$ ...A1

$a+c+\left[-k\cdot x \quad -k\cdot y \quad -k\cdot z\right] \mid w=\frac{1}{2}$	$\left[-k\cdot x \quad \frac{5}{2}-k\cdot y \quad 6-k\cdot z\right]$

Question 2f i

$\Sigma \boldsymbol{F} = m\boldsymbol{a}$ ...M1/2

$\boldsymbol{a} = \frac{d(\dot{\boldsymbol{r}}(t))}{dt} = \frac{\Sigma \boldsymbol{F}}{m}$ ...M1

$\Rightarrow -\frac{k\dot{x}}{m} = \frac{d\dot{x}}{dt}$ and $\frac{5-2k\dot{y}}{2m} = \frac{d\dot{y}}{dt}$ and $\frac{6-k\dot{z}}{m} = \frac{d\dot{z}}{dt}$ ...M1

Since $\dot{\boldsymbol{r}}(0) = 0$ ...M1/2

$\Rightarrow \dot{x} = 0$ and $\dot{y} = \frac{5\left(1-e^{\frac{-kt}{m}}\right)}{2k}$ and $\dot{z} = \frac{6\left(1-e^{\frac{-kt}{m}}\right)}{k}$

$\Rightarrow \dot{\boldsymbol{r}}(t) = \frac{5\left(1-e^{\frac{-kt}{m}}\right)}{2k}\boldsymbol{j} + \frac{6\left(1-e^{\frac{-kt}{m}}\right)}{k}\boldsymbol{k}$ ...A1

$\text{deSolve}\left(\frac{-k\cdot x}{m}=x' \text{ and } x(0)=0,t,x\right)$	$x=0$
$\text{deSolve}\left(\frac{5-2\cdot k\cdot y}{2\cdot m}=y' \text{ and } y(0)=0,t,y\right)$	$y=\frac{5}{2\cdot k}-\frac{5\cdot e^{\frac{-k\cdot t}{m}}}{2\cdot k}$
$\text{deSolve}\left(\frac{6-k\cdot z}{m}=z' \text{ and } z(0)=0,t,z\right)$	$z=\frac{6}{k}-\frac{6\cdot e^{\frac{-k\cdot t}{m}}}{k}$

Define $r(t)=\left[x \quad y \quad z\right] \mid x=0 \text{ and } y=\frac{5}{2\cdot k}-\frac{5\cdot e^{\frac{-k\cdot t}{m}}}{2\cdot k} \text{ and } z=\frac{6}{k}-\frac{6\cdot e^{\frac{-k\cdot t}{m}}}{k}$	*Done*

Question 2f ii

speed $= |\dot{\boldsymbol{r}}(t)|$ ...M1

terminal speed $= \lim_{t\to\infty}|\dot{\boldsymbol{r}}(t)|$ ...M1

$= \frac{13}{2k}$ ...A1

$$\lim_{t \to \infty} \left(\text{norm}(r(t))\right)|k>0 \text{ and } m>0 \qquad \frac{13}{2\cdot k}$$

Question 2g

The direction of the net force doesn't change with time, so the gradient of the position-time curve should not be time dependent. (M1)

$$\frac{dz}{dy} = \frac{dz}{dt}\cdot\frac{dt}{dy} \text{ ...M1/2}$$

$$= \frac{\dot{z}}{\dot{y}}$$

$$= \frac{12}{5}$$

$$\Rightarrow z = \int \frac{12}{5} dy \text{ ...M1/2}$$

$$\Rightarrow z = \frac{12y}{5}, y \geq 0 \text{ (since } \dot{\boldsymbol{r}}(0) = 0) \text{ ...A1}$$

$$\frac{z}{y}|y=\frac{5}{2\cdot k}-\frac{5\cdot e^{\frac{-k\cdot t}{m}}}{2\cdot k} \text{ and } z=\frac{6}{k}-\frac{6\cdot e^{\frac{-k\cdot t}{m}}}{k} \qquad \frac{12}{5}$$

$$\text{deSolve}\left(z'=\frac{12}{5} \text{ and } z(0)=0,y,z\right) \qquad z=\frac{12\cdot y}{5}$$

Question 2h

$$\frac{dD}{dt} = \text{speed} = |\dot{\boldsymbol{r}}(t)| \text{ ...M1}$$

$$\Rightarrow \Delta D = \int_0^t |\dot{\boldsymbol{r}}(t)|\ dt = 20 \text{ ...M1}$$

$$\Rightarrow t = 8.57\text{seconds ...A1}$$

$$\text{solve}\left(\int_0^t \text{norm}(r(t))\ dt=20,t\right)|k=2 \text{ and } m=5 \qquad t=8.57281$$