

SPECIALIST MATHEMATICS

Units 3 & 4 – Written examination 1



(TSSM's 2014 trial exam updated for the current study design)

SOLUTIONS

Question 1

$$\frac{3-2x}{x^2-4x+3} = \frac{A}{x-3} + \frac{B}{x-1}$$

$$3-2x = A(x-1) + B(x-3)$$

$$x=1 \Rightarrow B = -\frac{1}{2} \quad \text{and} \quad x=3 \Rightarrow A = -\frac{3}{2}$$

$$\int_4^6 \frac{3-2x}{x^2-4x+3} dx = -\frac{3}{2} \int_4^6 \frac{1}{x-3} dx - \frac{1}{2} \int_4^6 \frac{1}{x-1} dx$$

$$\int_4^6 \frac{3-2x}{x^2-4x+3} dx = \left(-\frac{3}{2} \log_e |x-3| \right)_4^6 - \left(\frac{1}{2} \log_e |x-1| \right)_4^6$$

$$\int_4^6 \frac{3-2x}{x^2-4x+3} dx = -\frac{3}{2} (\log_e 3 - \log_e 1) - \frac{1}{2} (\log_e 5 - \log_e 3)$$

$$\int_4^6 \frac{3-2x}{x^2-4x+3} dx = -\frac{3}{2} \log_e (3) - \frac{1}{2} \log_e \left(\frac{5}{3} \right)$$

$$\int_4^6 \frac{3-2x}{x^2-4x+3} dx = -\frac{1}{2} \log_e (45)$$

M3+A1

Question 2

a. $\left| \vec{AB} \right| = \left| \vec{OB} - \vec{OA} \right| = \left| 2\vec{i} - 2\vec{j} + \vec{k} \right| = \sqrt{4+4+1} = 3 \text{ units}$

M1+A1

b. $\vec{BA} \bullet \vec{CA} = \left(-2\vec{i} + 2\vec{j} - \vec{k} \right) \bullet \left(-4\vec{i} - 3\vec{j} + 2\vec{k} \right) = 8 - 6 - 2 = 0$
 $\vec{BA} \perp \vec{CA}$

M2

c. $Area = \frac{1}{2} \times \sqrt{16+9+4} \times 3 = \frac{3}{2} \sqrt{29} \text{ sq units}$

A1

Question 3

a.

$$-1 \leq 2 - 3x \leq 1$$

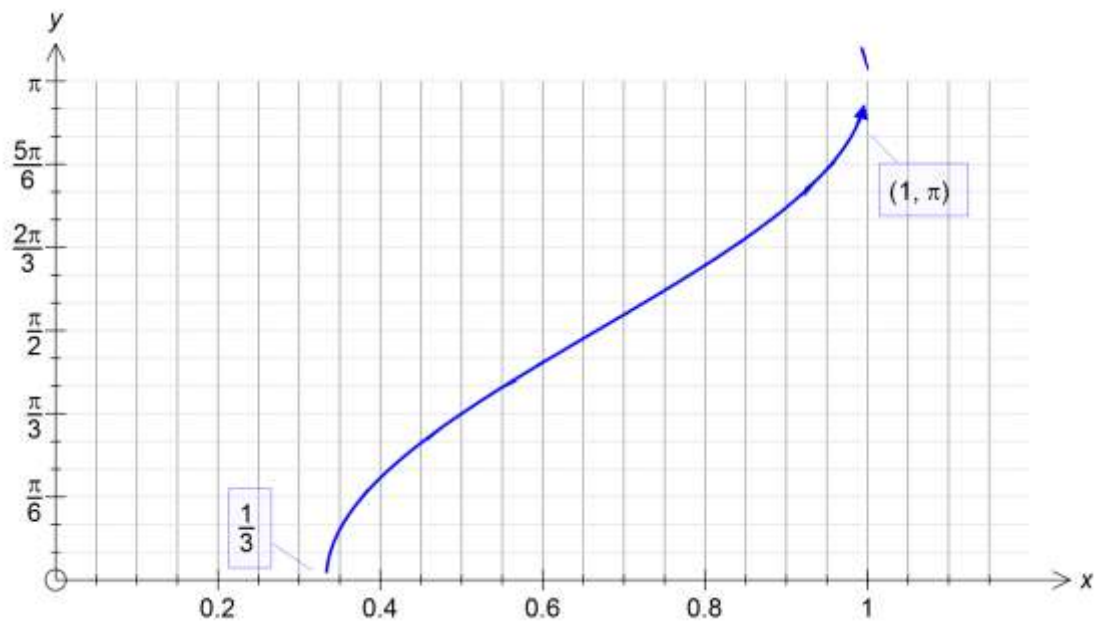
$$-3 \leq -3x \leq -1$$

$$\frac{1}{3} \leq x \leq 1$$

$$\text{Max domain is } \left[\frac{1}{3}, 1 \right]$$

M1+A1

b.



1 mark for shape and 1 mark for end-points

c.

$$y = \frac{\pi}{3} \Rightarrow x = \frac{1}{2}$$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1-(2-3x)^2}} \times -3$$

$$m_T = 2\sqrt{3}$$

$$m_N = \frac{-\sqrt{3}}{6}$$

M2+A1

Question 4

a. $(3i)^3 - 2(3i)^2 + 9(3i) - 18 = -27i + 18 + 27i - 18 = 0$

M1

b. $z = -3i$ is another solution for the equation.

$$(z - 3i)(z + 3i)(z - k) = z^3 - 2z^2 + 9z - 18$$

$$(z^2 + 9)(z - k) = z^3 - 2z^2 + 9z - 18$$

$$z^3 - kz^2 + 9z - 9k = z^3 - 2z^2 + 9z - 18$$

$$k = 2$$

Solutions are $3i, -3i, 2$

M2+A1

Question 5

a.

$$2y - xy^2 + 5x = -6$$

$$2y' - (y^2 + 2xyy') + 5 = 0$$

$$y'(2 - 2xy) = y^2 - 5$$

$$\frac{dy}{dx} = \frac{y^2 - 5}{2(1 - xy)}$$

M2+A1

b.

$$y = 1 \Rightarrow x = -2$$

$$\frac{dy}{dx}(-2, -1) = \frac{1 - 5}{2(1 + 2)} = -\frac{2}{3}$$

M1+A1

Question 6

$$Volume = \pi \int_0^1 (x^2) dy$$

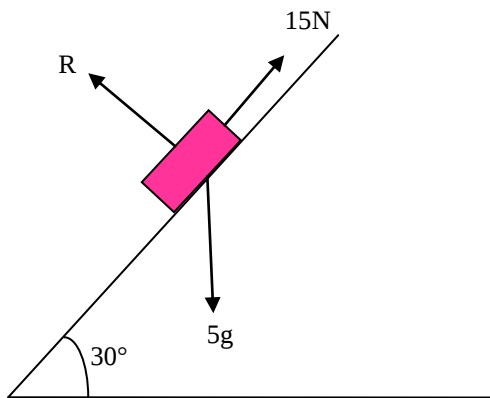
$$Volume = \pi \int_0^1 \left(\frac{4}{y+1} - 9 \right) dy$$

$$Volume = \pi (4 \log_e(y+1) - 9y)_0^1 = \pi (4 \log_e(2) - 9)$$

M3+A1

Question 7

a.



A2

b. $5g \sin(30^\circ) - 15 = 5a$

$$5a = \frac{19}{2}$$

$$a = 1.9 \text{ m/s}^2$$

M1+A1

c. $R + 5g \cos(150^\circ) = 0$

$$R = \frac{5\sqrt{3}}{2} g$$

M1+A1

Question 8

a. $\frac{dT}{dt} = -k(T - 20)$

$$t = -\frac{1}{k} \log_e (T - 20) + c$$

$$t = 0, T = 80 \Rightarrow c = \frac{1}{k} \log_e 60$$

$$t = -\frac{1}{k} \log_e (T - 20) + \frac{1}{k} \log_e 60$$

$$t = 5, T = 70 \Rightarrow 5 = \frac{1}{k} \log_e \left(\frac{60}{50} \right)$$

$$k = \frac{1}{5} \log_e \left(\frac{6}{5} \right)$$

M2+A1

b. $10 = \frac{1}{k} \log_e \left(\frac{60}{T - 20} \right)$

$$\frac{60}{T - 20} = e^{10k}$$

$$T = 60e^{-10k} + 20$$

$$T = 60 \times \left(\frac{5}{6} \right)^2 + 20 = 61 \frac{2}{3} ^\circ\text{C}$$

M1+A1