



Trial Examination 2014

VCE Specialist Mathematics Units 3&4

Written Examination 1

Suggested Solutions

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Question 1 (2 marks)

$$\begin{aligned}
 P(-i) &= (-i)^3 - (-i)^2 - 5 && \text{M1} \\
 &= i + 1 - 5 \\
 &= -4 + i && \text{A1}
 \end{aligned}$$

Question 2 (3 marks)

$$\begin{aligned}
 x_1 &= \frac{1}{2} \times 30 \times 5 \\
 &= 75 \text{ (m)} && \text{A1}
 \end{aligned}$$

$$\begin{aligned}
 x_2 &= \frac{1}{2} \times 20 \times 4 \\
 &= 40 \text{ (m)} && \text{A1}
 \end{aligned}$$

So the particle's displacement is 35 metres to the right of O . A1

Question 3 (4 marks)

$$\begin{aligned}
 \text{a.} \quad \Sigma \vec{F} &= (-\vec{i} - 2\vec{j}) + (4\vec{i} - \vec{j}) + (3\vec{i} + 11\vec{j}) && \text{M1} \\
 &= 6\vec{i} + 8\vec{j} \\
 |\Sigma \vec{F}| &= \sqrt{6^2 + 8^2} \\
 &= 10 \text{ (N)} && \text{A1}
 \end{aligned}$$

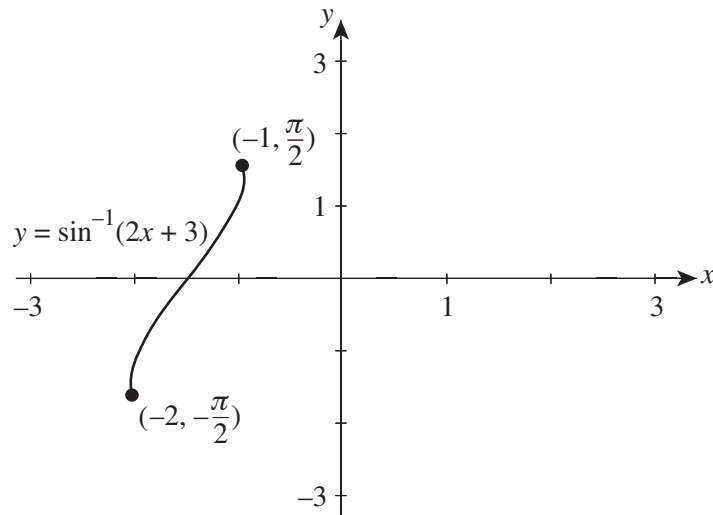
$$\begin{aligned}
 \text{b.} \quad |\vec{a}| &= \sqrt{(1.5)^2 + (2)^2} && \text{M1} \\
 &= 2.5 \text{ (m/s}^2\text{)} \\
 10 &= 2.5m \\
 m &= 4 \text{ (kg)} && \text{A1}
 \end{aligned}$$

Question 4 (7 marks)

$$\text{a.} \quad \text{Solving } -1 \leq 2x + 3 \leq 1 \text{ for } x \text{ we obtain } -2 \leq x \leq -1. \quad \text{A1}$$

$$\text{The range is } -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}. \quad \text{A1}$$

b.



correct shape

A1

endpoints labelled with correct coordinates

A1

c. attempting the chain rule

M1

Let $u = 2x + 3$ and so $\frac{du}{dx} = 2$.

$y = \arcsin(u)$ and so $\frac{dy}{du} = \frac{1}{\sqrt{1-u^2}}$

$$\frac{dy}{dx} = \frac{2}{\sqrt{1-(2x+3)^2}} \text{ (or equivalent)}$$

A1

When $x = -\frac{5}{4}$, $\frac{dy}{dx} = \frac{4\sqrt{3}}{3}$ (or equivalent).

A1

Question 5 (4 marks)

$$\cos(3x) = \cos(2x + x)$$

A1

$$= \cos(2x)\cos(x) - \sin(2x)\sin(x)$$

$$= (2\cos^2(x) - 1)\cos(x) - (2\sin(x)\cos(x))\sin(x)$$

M1

$$= 2\cos^3(x) - \cos(x) - 2\sin^2(x)\cos(x)$$

$$= 2\cos^3(x) - \cos(x) - 2\cos(x)(1 - \cos^2(x))$$

M1

$$= 2\cos^3(x) - \cos(x) - 2\cos(x) + 2\cos^3(x)$$

A1

$$= 4\cos^3(x) - 3\cos(x)$$

Question 6 (4 marks)

Let $u = 4x - 1$ and so $du = 4dx$ (or equivalent).

$$\int \frac{x^2}{4x-1} dx = \int \frac{(u+1)^2}{16u} \frac{du}{4} \quad \text{M1}$$

$$= \frac{1}{64} \int \left(\frac{u^2 + 2u + 1}{u} \right) du \quad \text{A1}$$

$$= \frac{1}{64} \int \left(u + 2 + \frac{1}{u} \right) du$$

$$= \frac{1}{64} \left(\frac{u^2}{2} + 2u + \log_e |u| \right) \quad \text{M1}$$

$$= \frac{1}{64} \left(\frac{(4x-1)^2}{2} + 2(4x-1) + \log_e |4x-1| \right) + c \quad \text{A1}$$

Alternatively:

$$\int \frac{x^2}{4x-1} dx = \int \frac{x}{4} + \frac{1}{16} + \frac{1}{16(4x-1)} dx \quad \text{M1 A1}$$

$$= \frac{x^2}{8} + \frac{x}{16} + \frac{1}{64} \log_e |4x-1| + C \quad \text{M1 A1}$$

Question 7 (4 marks)

$$-2 \sin(2x) - 3 \sin(3y) \frac{dy}{dx} = 0 \quad \text{M1}$$

$$\frac{dy}{dx} = -\frac{2 \sin(2x)}{3 \sin(3y)} \quad \text{A1}$$

At $P\left(\frac{\pi}{6}, \frac{\pi}{9}\right)$, $\frac{dy}{dx} = -\frac{2}{3}$, and so the gradient of the normal is $\frac{3}{2}$. M1

$$y - \frac{\pi}{9} = \frac{3}{2} \left(x - \frac{\pi}{6} \right)$$

$$y = \frac{3}{2}x - \frac{5\pi}{36} \quad \text{A1}$$

Question 8 (5 marks)

$$-\int_{25}^T \frac{dT}{T-T_0} = k \int_0^t dt \quad \text{M1}$$

$$[\log_e |T - T_0|]_T^{25} = kt \text{ (or equivalent)}$$

$$\log_e \left(\frac{25 - T_0}{T - T_0} \right) = kt \quad \text{A1}$$

$$\text{When } t = 5, \log_e \left(\frac{25 - T_0}{15 - T_0} \right) = 5k \quad (1)$$

$$\text{When } t = 10, \log_e \left(\frac{25 - T_0}{10 - T_0} \right) = 10k \quad (2) \quad \text{A1}$$

Note: Award A1 for these two equations.

$$(1) \times 2 \text{ gives } 2\log_e \left(\frac{25 - T_0}{15 - T_0} \right) = 10k$$

$$\log_e \left(\frac{25 - T_0}{15 - T_0} \right)^2 = \log_e \left(\frac{25 - T_0}{10 - T_0} \right) \text{ and so } \left(\frac{25 - T_0}{15 - T_0} \right)^2 = \frac{25 - T_0}{10 - T_0} \quad \text{M1}$$

$$\begin{aligned} (25 - T_0)(10 - T_0) &= (15 - T_0)^2 \\ 250 - 35T_0 + T_0^2 &= 225 - 30T_0 + T_0^2 \\ 5T_0 &= 25 \end{aligned}$$

$$T_0 = 5 \text{ and so the outside temperature is } 5^\circ\text{C}. \quad \text{A1}$$

Question 9 (7 marks)

$$\text{a. } (x + yi)(x - yi) + (a - bi)(x + yi) + (a + bi)(x - yi) + 1 = 0 \quad \text{M1}$$

$$(x^2 + y^2) + (ax + by) + (ay - bx)i + (ax + by) + (bx - ay)i + 1 = 0 \quad \text{A1}$$

$$(x^2 + y^2) + 2ax + 2by + 1 = 0 \quad \text{A1}$$

$$(x + a)^2 + (y + b)^2 = a^2 + b^2 - 1 \text{ or } (x + a)^2 + (y + b)^2 = |u|^2 - 1 \quad \text{M1 A1}$$

$$\text{b. centre is } (-a, -b) \quad \text{A1}$$

$$\text{radius is } \sqrt{|u|^2 - 1} \text{ (or equivalent)} \quad \text{A1}$$