

Trial Exam 2014
SPECIALIST MATHEMATICS
Written Examination 2

STUDENT NAME _____

Reading time: 15 minutes

Writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of Book

<i>Section</i>	<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
1	22	22	22
2	5	5	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 25 pages with a detachable sheet of miscellaneous formulas in the centrefold.
- Answer sheet for multiple-choice questions.

Instructions

- Detach the formula sheet from the centre of this book during reading time.
- Write your **name** in the space provided above on this page and on the answer sheet for multiple-choice questions.
- All written responses must be in English.

At the end of the examination

- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION 1**Instructions for Section 1**

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Take the **acceleration due to gravity** to have magnitude $g \text{ m/s}^2$, where $g = 9.8$.

Question 1

If $\operatorname{cosec}(\phi) = \frac{4}{3}$, then $\cot(\phi)$ could equal

- A. $-\frac{3}{5}$
- B. $\frac{4}{7}$
- C. $-\frac{4}{7}$
- D. $-\frac{\sqrt{7}}{3}$
- E. $\frac{\sqrt{7}}{4}$

Question 2

Consider the function $f : D \rightarrow R$, $f(x) = \frac{x^2}{x^2 - 4}$, where D is the maximal domain of f , and let

$g(x) = f(x - 1)$. The maximal domain and range of g are

- A. Domain $R \setminus \{-2, 2\}$, Range $R \setminus \{0\}$
- B. Domain $R \setminus \{-3, 1\}$, Range $R \setminus \{0\}$
- C. Domain $R \setminus \{-2, 2\}$, Range $R \setminus \{1\}$
- D. Domain $R \setminus \{-1, 3\}$, Range $R \setminus \{1\}$
- E. Domain $R \setminus \{-1, 3\}$, Range $R \setminus \{0\}$

Question 3

The value of $\sin\left(2 \tan^{-1}\left(\frac{1}{2}\right)\right)$ is

- A. $\frac{4}{5}$
- B. $\frac{3}{5}$
- C. $\frac{2}{\sqrt{5}}$
- D. $\frac{\sqrt{5}}{5}$
- E. $\frac{\sqrt{2}}{2}$

Question 4

If $u = 3 + 4i$, then $\frac{u}{6 - 2u}$ is equal to

- A. $\frac{i}{8}$
- B. $-\frac{i}{8}$
- C. $\frac{1 - 3i}{8}$
- D. $\frac{3 + 4i}{8}$
- E. $\frac{4 - 3i}{8}$

Question 5

Which one of the following is **not** a fourth root of $1 + i\sqrt{3}$?

- A. $\sqrt[4]{2} \operatorname{cis}\left(-\frac{11\pi}{12}\right)$
- B. $\sqrt[4]{2} \operatorname{cis}\left(-\frac{5\pi}{12}\right)$
- C. $\sqrt[4]{2} \operatorname{cis}\left(\frac{\pi}{12}\right)$
- D. $\sqrt[4]{2} \operatorname{cis}\left(\frac{7\pi}{12}\right)$
- E. $\sqrt[4]{2} \operatorname{cis}\left(\frac{11\pi}{12}\right)$

Question 6

Let $M = \{z : |z - 1| = |z + i|\}$ and $P = \{z : |z + i| = |z - 4 + i|\}$ be subsets of the complex plane. $M \cap P$ is

- A. $\{z : z = 1 - i\}$
- B. $\{z : z = -1 + i\}$
- C. $\{z : z = -2 + 2i\}$
- D. $\{z : z = 2 - 2i\}$
- E. $\{z : -4\operatorname{Re}(z) + \operatorname{Im}(z) = 0\}$

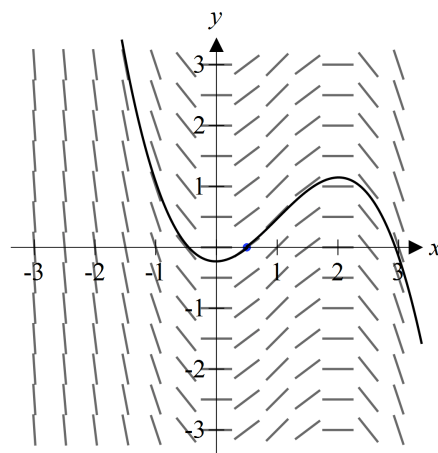
Question 7

Let p be a polynomial function with integer coefficients. If -5 , $\sqrt{3}$ and $1 - 2i$ are roots of p , then the **minimum** degree of the polynomial is

- A. 2
- B. 3
- C. 4
- D. 5
- E. 6

Question 8

The direction (slope) field of a particular differential equation is shown below.



The differential equation could be

- A. $\frac{dy}{dx} = -x^2$
- B. $\frac{dy}{dx} = 2x - x^2$
- C. $\frac{dy}{dx} = x^2 - 2x$
- D. $\frac{dy}{dx} = x^2 - x^3$
- E. $\frac{dy}{dx} = -x(x+2)^2$

Question 9

Using a suitable substitution, $\int_{-\frac{\pi}{6}}^{\frac{\pi}{3}} (\tan(x) \log_e(\sec(x))) dx$ can be expressed completely in terms of u as

- A. $\int_{-\frac{\pi}{6}}^{\frac{\pi}{3}} (u) du$
- B. $\int_{-\log\left(\frac{\sqrt{3}}{2}\right)}^{\log_e(2)} (u) du$
- C. $-\int_{-\frac{\sqrt{3}}{2}}^{\frac{1}{2}} (u) du$
- D. $\int_{-\frac{\pi}{6}}^{\frac{\pi}{3}} (\log_e(u)) du$
- E. $\int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} (\log_e(u)) du$

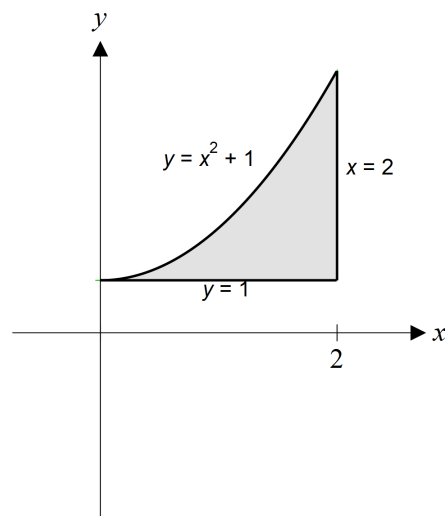
Question 10

Euler's method is used to solve the differential equation $\frac{dy}{dx} = \log_e \left(\frac{1}{\sqrt{x}} \right)$, with step size of $\frac{1}{10}$ and initial values $x = 1$ and $y = 2$. The value of y when $x = \frac{6}{5}$ is given by

- A. $\frac{1}{4} \log_e \left(\frac{6}{5} \right) + 2$
- B. $\frac{1}{4} \log_e \left(\frac{10}{11} \right) + 2$
- C. $2 - \frac{1}{20} \log_e \left(\frac{5}{6} \right)$
- D. $2 + \frac{1}{20} \log_e \left(\frac{11}{10} \right)$
- E. $2 + \frac{1}{20} \log_e \left(\frac{10}{11} \right)$

Question 11

The shaded region shown below, which is enclosed by the graph of $y = x^2 + 1$ and the lines $y = 1$ and $x = 2$, is rotated about the x -axis to form a solid of revolution.

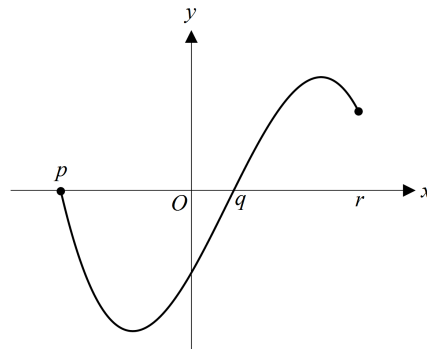


The volume of the solid is given by

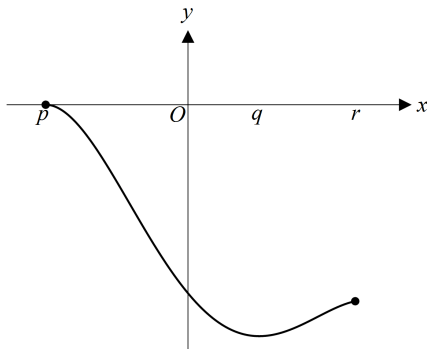
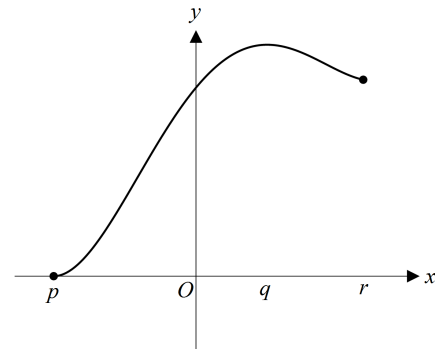
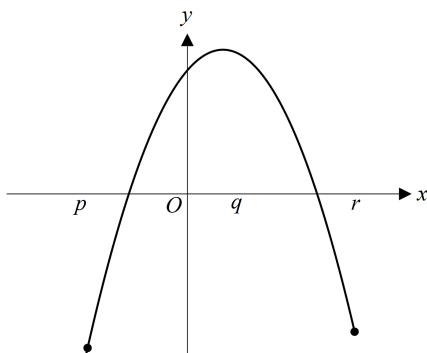
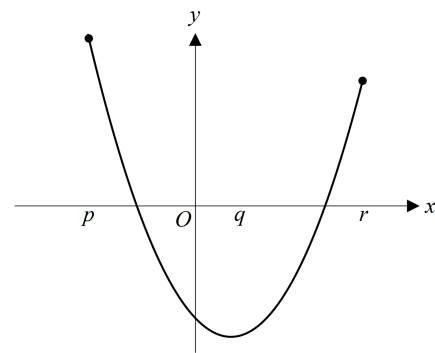
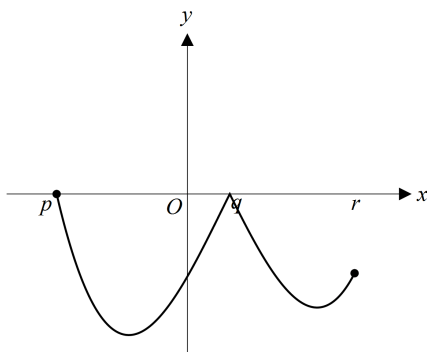
- A. $\pi \int_1^5 x^4 dx$
- B. $\pi \int_0^2 (x^4 + 2x^2) dx$
- C. $\pi \int_0^2 (x^4 + 2x^2 + 1) dx$
- D. $\pi \int_0^2 (x^2 + 1) dx$
- E. $\pi \int_1^5 (x^2 + 1) dx$

Question 12

The graph of a function f is shown below.



If $g(x) = \int_p^x f(t) dt$, then the graph of g could be

A.**B.****C.****D.****E.**

Question 13

The gradient of the **normal** to the graph of $x^2 - y^2 + xy - 1 = 0$, at the points where $y = 0$, is equal to

- A. 1
- B. 2
- C. $\frac{1}{2}$
- D. -1
- E. $-\frac{1}{2}$

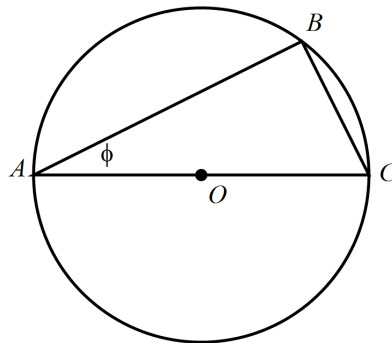
Question 14

The angle between the vectors $\underline{p} = \underline{i} + 2\underline{j} - \underline{k}$ and $\underline{q} = -2\underline{i} - \underline{j} - \underline{k}$ is

- A. 120°
- B. 90°
- C. 60°
- D. 45°
- E. 30°

Question 15

In the diagram below, the vertices of triangle ABC are on the circumference of the circle and the centre of the circle, O , is on side AC .



Which one of the following is **false**?

- A. $\vec{AC} \cdot \vec{BC} = |\vec{AC}| |\vec{BC}| \sin(\phi)$
- B. $\vec{AC} \cdot \vec{AB} = |\vec{AC}| |\vec{AB}| \cos(\phi)$
- C. $(\vec{AC} - \vec{AB}) \cdot \vec{AB} = |\vec{BC}|^2$
- D. $(\vec{AC} - \vec{BC}) \cdot \vec{AB} = |\vec{AB}|^2$
- E. $|\vec{CA} - \vec{BA}|^2 = |\vec{CA}|^2 - |\vec{AB}|^2$

Question 16

The position vector of a particle moving along a curve is $\underline{r} = \cos(2t)\underline{i} - 2\sin(t)\underline{j}$.

The cartesian equation of the curve is

- A. $x^2 - 2y + 2 = 0$
- B. $y^2 + 2x - 2 = 0$
- C. $x^2 - y + 1 = 0$
- D. $y^2 + x - 1 = 0$
- E. $y^2 + 2x - 1 = 0$

Question 17

At all points of a particular curve, $\frac{d^2y}{dx^2} = -12x$. The point P with coordinates $(-1, 4)$ is on the curve and the gradient of the tangent to the curve at P is -1 . The equation of the curve is

- A. $y = -2x^3 + 5x + 7$
- B. $y = 2x^3 - 5x^2 + 7$
- C. $y = -2x^3 + 7x + 5$
- D. $y = 2x^3 - 7x^2 + 5$
- E. $y = -12x^3 - x + 4$

Question 18

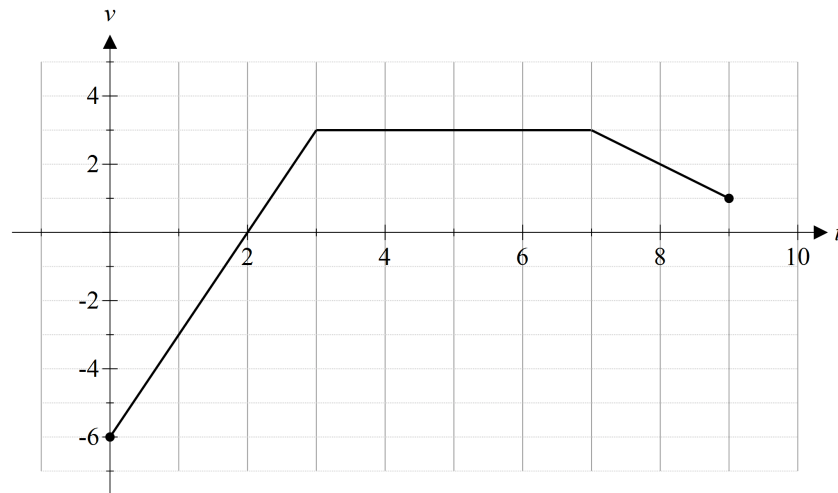
A golf ball is hit from an origin, O , on horizontal ground. The position vector of the ball is given by $\underline{r} = 35t\underline{i} + (20t - 5t^2)\underline{j}$, where $\underline{i}$ and $\underline{j}$ are unit vectors in the forward and upward directions, respectively.

The ball strikes the ground at a point P . The length of the line segment OP is

- A. 20
- B. 35
- C. 80
- D. 140
- E. 175

Question 19

The velocity-time graph of a particle travelling **from the origin** along the x -axis on the cartesian plane is shown.



The coordinates of the final position of the particle are

- A. $(9, 1)$
- B. $(9, 0)$
- C. $\left(\frac{23}{2}, 0\right)$
- D. $\left(\frac{25}{2}, 1\right)$
- E. $\left(\frac{29}{2}, 0\right)$

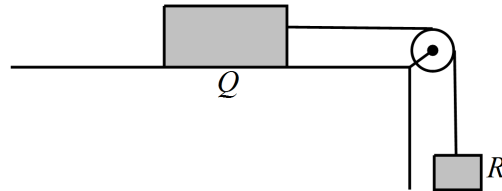
Question 20

A particle moves in a straight line such that its acceleration, a m/s, is given by $a = \frac{9}{x^2}$, where x metres is its displacement from a fixed origin. If $v = -2$ m/s at $x = 9$, where v is the velocity of the particle, then the relationship between v and x is

- A. $v = 3 - \frac{9}{x}$
- B. $v = \frac{9}{x} + 1$
- C. $v = \sqrt{6 - \frac{18}{x}}$
- D. $v = -\sqrt{\frac{6x - 18}{x}}$
- E. $v = -\sqrt{\frac{9 + 4x}{x}}$

Question 21

A body Q of mass $5m$ is on a rough horizontal plane. The coefficient of friction between body Q and the plane is $\frac{1}{10}$. Body Q is connected to body R , of mass m , by means of an inextensible string, over a smooth pulley. R is suspended over the edge of the plane, as shown below.



When the system is released from rest, the acceleration of the system is

- A. $6mg$
- B. $6g$
- C. $\frac{g}{6}$
- D. $\frac{mg}{12}$
- E. $\frac{g}{12}$

Question 22

A particle of mass m is projected vertically upwards from the ground with an initial speed of $\sqrt{\frac{g}{k}}$, where k is a positive real constant. When the speed of the particle is v , air resistance exerts a force on the particle of magnitude mkv^2 , so that the equation of motion is $-mg - mkv^2 = ma$, where a is the acceleration of the particle. The maximum height reached by the particle is given by

- A. $\frac{1}{2g} \sqrt{\frac{g}{k}}$
- B. $\frac{\log_e(2)}{2k}$
- C. $\frac{m}{2g} \sqrt{\frac{g}{k}}$
- D. $\frac{m \log_e(2)}{2k}$
- E. $\frac{\tan^{-1}\left(\sqrt{\frac{k}{g}}\right)}{\sqrt{gk}}$

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.

Unless otherwise specified an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ m/s}^2$, where $g = 9.8$.

Question 1 (13 marks)

Consider the complex number $w = 4 - 4i$.

a. i. Show that $|w| = 4\sqrt{2}$.

1 mark

ii. Show that $\text{Arg}(w) = -\frac{\pi}{4}$.

1 mark

b. Find $\{z : z^5 = w, z \in \mathbb{C}\}$. Express each answer in the form $a \text{cis}(\theta)$, where a and θ are real numbers with $-\pi < \theta \leq \pi$.

3 marks

- e. If u is a root of the equation $z^9 + 16(1+i)z^3 + c + id = 0$, find the values of the real constants c and d .

3 marks

End of Question 1

Question 2 (12 marks)

Consider the vectors $\underline{m} = -6\underline{i} + 2\underline{j} + 3\underline{k}$ and $\underline{n} = 2\underline{i} - 3\underline{j} + 6\underline{k}$.

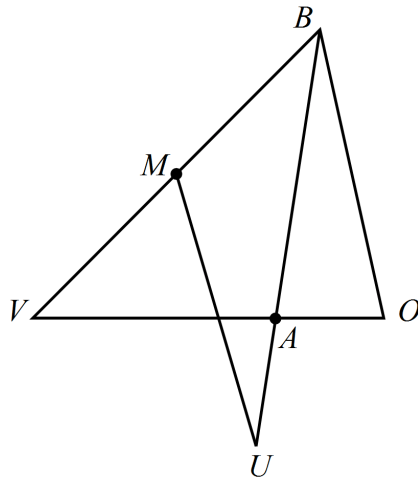
- a. If ϕ is the angle that $\underline{m}$ makes with the y -axis, find the value of $\cos(\phi)$.

1 mark

- b. Let $\hat{\underline{t}} = x\underline{i} + y\underline{j} + z\underline{k}$, where x , y and z are real numbers, be a unit vector perpendicular to both $\underline{m}$ and $\underline{n}$. Find the possible values of x , y and z .

3 marks

In the diagram shown below, A is a point on side OV of triangle $OV B$, such that $|\vec{OV}| = 3|\vec{OA}|$. M is the midpoint of side VB .



Let $\vec{OA} = \underline{a}$ and $\vec{OB} = \underline{b}$.

c. i. Express $\vec{BA}$ in terms of $\underline{a}$ and $\underline{b}$.

1 mark

ii. Express $\vec{BM}$ in terms of $\underline{a}$ and $\underline{b}$.

1 mark

d. The points B , A and U are collinear. If $\vec{BU} = p\vec{BA}$, where p is a scalar, express $\vec{MU}$ in terms of $\underline{a}$, $\underline{b}$ and p .

2 marks

- e. If the line segment MU bisects the line segment OV , use a vector method to show that MU is parallel to BO .

2 marks

- f. Given that MU is parallel to BO , find the value of p .

2 marks

End of Question 2

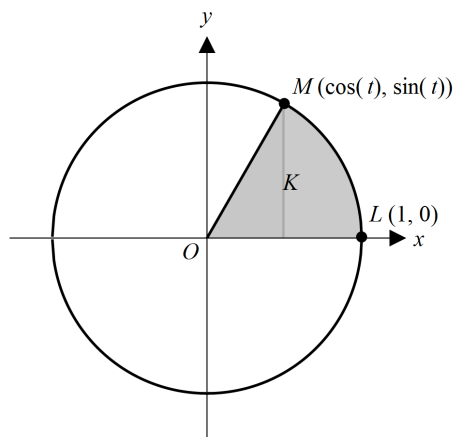
Question 3 (12 marks)

The position vector of a point M is $\underline{r} = \cos(t)\underline{i} + \sin(t)\underline{j}$, where t is a real variable such that $t \in [0, 2\pi]$.

a. Show that the locus of M is a unit circle.

1 mark

Let K be the area of the shaded region shown below.



b. Find an expression for K in terms of t .

2 marks

A point N has position vector $\underline{r} = \frac{1}{2}(e^t + e^{-t})\underline{i} + \frac{1}{2}(e^t - e^{-t})\underline{j}$, where t is a non-negative real variable.

c. Show that the locus of N has cartesian equation $x^2 - y^2 = 1$.

2 marks

- e. Determine the cartesian coordinates of N for which the rate of change in the x -coordinate with respect to t is equal to $\frac{3}{4}$.

2 marks

[illegible]

- f.** When $t = a$ the gradient of the normal to the hyperbola with equation $x^2 - y^2 = 1$ is $-\frac{3}{5}$.
Find the value of a .

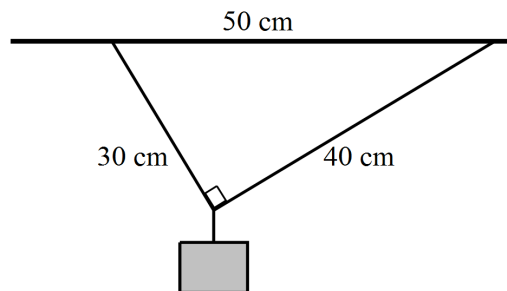
2 marks

[illegible]

End of Question 3

Question 4 (10 marks)

A block, A , is suspended by two light inextensible strings of lengths 30 cm and 40 cm, attached to two points on a horizontal beam, 50 cm apart. The tension in the shorter string is $4g$ newtons.



a. Determine the mass of block A .

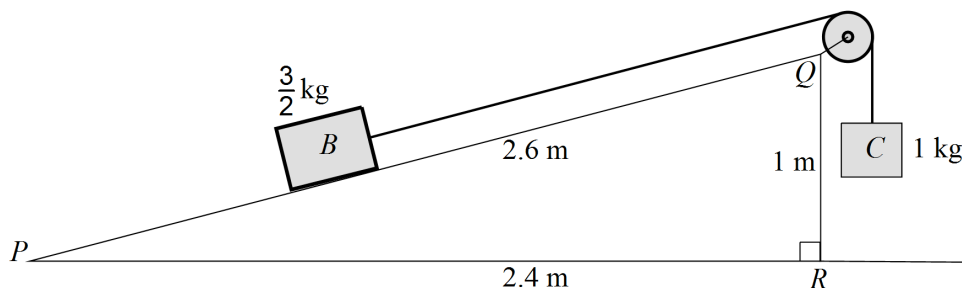
2 marks

b. Determine the tension in the longer string.

1 mark

A different block, B , is placed on a smooth inclined plane. The cross section of the plane forms a right-angled triangle with height, QR , of 1 m and hypotenuse, PQ , of length 2.6 m.

The block is connected by a light inextensible string which passes over a frictionless pulley, to a particle, C , of mass of 1 kg which is hanging freely, as shown below.



Particle C is 1 m above the ground when the system is released from rest.

- c. Show that when the system is released, the magnitude of the acceleration of block B up the plane is $\frac{11g}{65} \text{ ms}^{-2}$.

2 marks

- d. When particle C has travelled vertically a distance of 1 metre, it hits the ground and the string becomes slack.
- i. Find the speed of block B at this instant when C hits the ground. Express the answer in ms^{-1} , correct to two decimal places.

2 marks

- ii. Find the magnitude of the acceleration of the block B immediately after C hits the ground. Express the answer in ms^{-2} , correct to two decimal places.

1 mark

- iii. Find the total distance travelled by block *B* **up the plane** before it stops and starts sliding down. Express the answer in metres, correct to two decimal places.

2 marks

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

End of Question 4

Question 5 (11 marks)

Lin, a skydiver, drops vertically from a hot-air balloon that is at rest, so that her velocity immediately before dropping is zero. Lin falls freely from the hot-air balloon for 10 seconds before her parachute is opened. Let v_0 be Lin's speed immediately before the parachute is opened.

- a. Assuming that air resistance is negligible during free fall, find the value of v_0 in ms^{-1} , correct to the nearest integer.

1 mark

Lin is using a new type of parachute that introduces a resistive force as soon as it is opened. As a result, Lin's speed, $v \text{ ms}^{-1}$, t seconds after her parachute is opened, is given by

$v(t) = (v_0 - 4)e^{-\frac{g}{4}t} + 4$. Lin hits the ground 6 seconds after her parachute is opened.

- b. What was Lin's speed immediately before she hits the ground? Express the answer in ms^{-1} , correct to one decimal place.

1 mark

- c. What was Lin's altitude above the ground when the parachute was opened? Give the answer in metres, correct to one decimal place.

2 marks

- d. How high above the ground was the hot-air balloon when Lin dropped from it? Express the answer in metres, correct to one decimal place.

1 mark
