

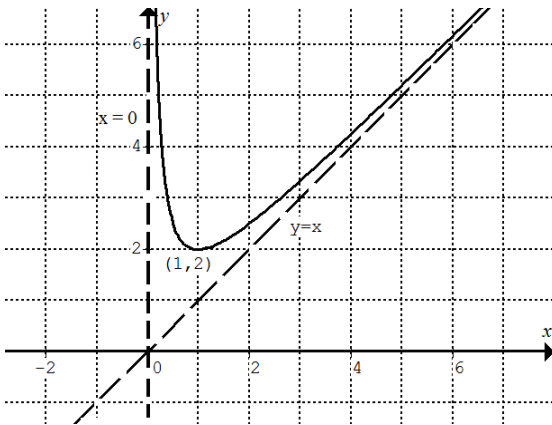
2014 Specialist Mathematics Trial Exam 1 Solutions

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Q1a $f(x) = \left(\frac{1}{\sqrt{x}} - \sqrt{x}\right)^2 + 2 = \left(\frac{1}{\sqrt{x}}\right)^2 - 2\sqrt{x}\left(\frac{1}{\sqrt{x}}\right) + (\sqrt{x}) + 2$
 $\therefore f(x) = \frac{1}{x} + x$, where $x > 0$

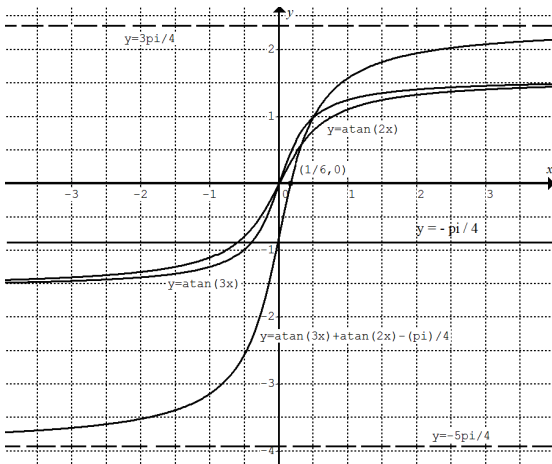
Q1b Let $f'(x) = -\frac{1}{x^2} + 1 = 0$, $x = 1$ and $y = 2$
 $\therefore f(x)$ has a local minimum $(1, 2)$. The range is $[2, \infty)$.

Q1c



Q2a $g(x) = 0$, $\tan^{-1}(3x) + \tan^{-1}(2x) - \frac{\pi}{4} = 0$,
 $\tan^{-1}(3x) + \tan^{-1}(2x) = \frac{\pi}{4}$, $\tan(\tan^{-1}(3x) + \tan^{-1}(2x)) = \tan \frac{\pi}{4}$
 $\frac{3x + 2x}{1 - (3x)(2x)} = 1$, $6x^2 + 5x - 1 = 0$, $(x + 1)(6x - 1) = 0$
 $\therefore x = \frac{1}{6}$. Note: $x = -1$ does not satisfy $g(x) = 0$.

Q2b



Q3a $\sqrt{3}z - \sqrt{2}i = \sqrt{2}iz + \sqrt{3}$, $\sqrt{3}z - \sqrt{2}iz = \sqrt{2}i + \sqrt{3}$
 $(\sqrt{3} - \sqrt{2}i)z = \sqrt{2}i + \sqrt{3}$, $z = \frac{\sqrt{2}i + \sqrt{3}}{\sqrt{3} - \sqrt{2}i}$
 $z = \frac{(\sqrt{2}i + \sqrt{3})(\sqrt{3} + \sqrt{2}i)}{(\sqrt{3} - \sqrt{2}i)(\sqrt{3} + \sqrt{2}i)} = \frac{1 + 2\sqrt{6}i}{5} = \frac{1}{5} + \frac{2\sqrt{6}}{5}i$

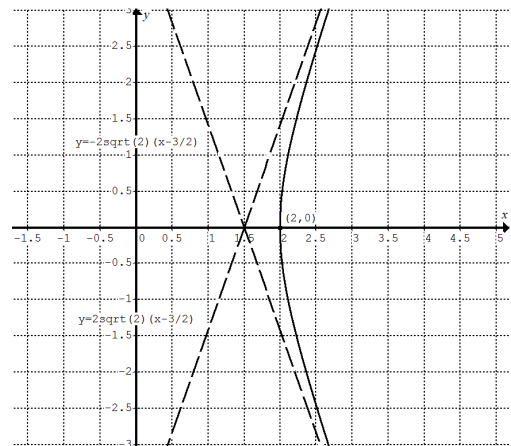
Q3b $P(z) = (z - i)Q(z) + 1$ and $P(z) = (2z - 1)T(z) + 1$
 $\therefore P(z) - 1 = (z - i)Q(z)$ and $P(z) - 1 = (2z - 1)T(z)$
 $\therefore P(z) - 1 = (z - i)(2z - 1)W(z)$
 Since $P(z)$ has real coefficients, $\therefore W(z) = z + i$
 $\therefore P(z) - 1 = (z - i)(z + i)(2z - 1) = 2z^3 - z^2 + 2z - 1$
 $P(z) = 2z^3 - z^2 + 2z$

When $P(z) = 0$, $2z^3 - z^2 + 2z = z(2z^2 - z + 2) = 0$

$\therefore z = 0$ or $z = \frac{1}{4} \pm \frac{\sqrt{15}}{4}i$

Q4a $|z| - |3 - z| = 1$, $|x + yi| - 1 = |3 - (x + yi)|$
 $\sqrt{x^2 + y^2} - 1 = \sqrt{(3 - x)^2 + (-y)^2}$
 $x^2 + y^2 - 2\sqrt{x^2 + y^2} + 1 = (3 - x)^2 + y^2$
 $3x - 4 = \sqrt{x^2 + y^2}$, $(3x - 4)^2 = x^2 + y^2$, $8x^2 - 24x - y^2 = -16$
 $8(x^2 - 3x) - y^2 = -16$, $8\left(x^2 - 3x + \frac{9}{4}\right) - y^2 = -16 + 18$
 $8\left(x - \frac{3}{2}\right)^2 - y^2 = 2$, $4\left(x - \frac{3}{2}\right)^2 - \frac{y^2}{2} = 1$

Q4b



Q5 $\tilde{p} = \tilde{i} - \tilde{j}$, $\tilde{q} = 2\tilde{i} + \tilde{j}$, $\tilde{r} = \tilde{i} + 2\tilde{j}$ and $\tilde{s} = 3\tilde{i} - 2\tilde{j}$
 $\therefore \tilde{p} + \tilde{q} = 3\tilde{i}$ and $\tilde{r} + \tilde{s} = 4\tilde{i}$
 $\therefore \frac{\tilde{p} + \tilde{q}}{3} = \frac{\tilde{r} + \tilde{s}}{4}$, $\therefore 4\tilde{p} + 4\tilde{q} - 3\tilde{r} - 3\tilde{s} = \tilde{0}$

Hence $\tilde{p}$, $\tilde{q}$, $\tilde{r}$ and $\tilde{s}$ are linearly dependent.

Q6 Let $\vec{h} = \vec{AH} = \sqrt{10}\vec{i} + 3\vec{j} + \sqrt{6}\vec{k}$, $|\vec{AH}| = \sqrt{10+9+6} = 5$

$\therefore \hat{h} = \frac{1}{5}(\sqrt{10}\vec{i} + 3\vec{j} + \sqrt{6}\vec{k})$

Let $\vec{g} = \vec{AG} = \sqrt{10}\vec{i} + \sqrt{6}\vec{k}$

$\vec{g} \cdot \hat{h} = \frac{1}{5}(10+6) = \frac{16}{5}$

$\therefore (\vec{g} \cdot \hat{h})\hat{h} = \frac{16}{5} \times \frac{1}{5}(\sqrt{10}\vec{i} + 3\vec{j} + \sqrt{6}\vec{k})$

$= \frac{16}{25}(\sqrt{10}\vec{i} + 3\vec{j} + \sqrt{6}\vec{k})$

$\therefore \vec{g} - (\vec{g} \cdot \hat{h})\hat{h} = \sqrt{10}\vec{i} + \sqrt{6}\vec{k} - \frac{16}{25}(\sqrt{10}\vec{i} + 3\vec{j} + \sqrt{6}\vec{k})$

$= \frac{9\sqrt{10}}{25}\vec{i} - \frac{48}{25}\vec{j} + \frac{9\sqrt{6}}{25}\vec{k}$

$\therefore |\vec{g} - (\vec{g} \cdot \hat{h})\hat{h}| = \frac{12}{5}$ is the shortest distance from G to AH .

Q7a $\frac{dy}{dx} = -\frac{y}{x}$

$x=1 \quad y=2$

$\frac{dy}{dx} = -\frac{2}{1} = -2$

$x=1.5 \quad y=2-2 \times 0.5 = 1$

$\frac{dy}{dx} = -\frac{1}{1.5} = -\frac{2}{3}$

$x=2 \quad y=1-\frac{2}{3} \times 0.5 = \frac{2}{3}$

$\frac{dy}{dx} = -\frac{2/3}{2} = -\frac{1}{3}$

$x=2.5 \quad y=\frac{2}{3}-\frac{1}{3} \times 0.5 = \frac{1}{2}$

Q7b $xy=2$, $y=\frac{2}{x}$, $\frac{dy}{dx} = -\frac{2}{x^2}$

$LHS = \frac{dy}{dx} + \frac{y}{x} = -\frac{2}{x^2} + \frac{2}{x^2} = 0 = RHS$

Q7c $\frac{dy}{d\lambda} = \frac{dy}{dx} \times \frac{dx}{d\lambda}$

$\therefore \frac{dy}{d\lambda} = -\frac{y}{x} \times \frac{dx}{d\lambda}$, $-1 = -\frac{2}{1} \times \frac{dx}{d\lambda}$, $\therefore \frac{dx}{d\lambda} = \frac{1}{2}$

Q8a $\int_1^6 f(x)dx = -10 + 7 - 2 = -5$

Q8b $y = \int_1^6 f(x)dx + 5 = -5 + 5 = 0$

Q9a The gradient of the graph is positive at $t=120$ s, $\therefore$ the direction of motion is north.

Q9b Total distance = $60 + 60 + 60 = 180$ m

Average speed = $\frac{180}{160} = \frac{9}{8} \text{ m s}^{-1}$

Q9c The velocity is negative at $t=120$ s, $\therefore$ the direction of motion is south.

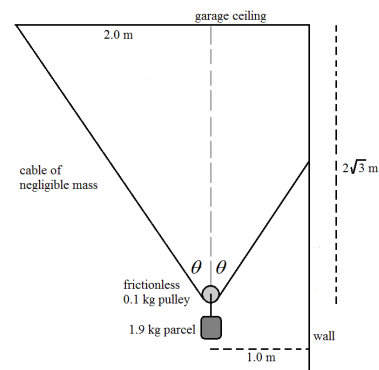
Q9d Total distance = area bounded by the graph and the t -axis

$= \frac{1}{2}(110+20) \times 60 + \frac{1}{2} \times 30 \times 60 = 4800 \text{ km h}^{-1} \text{ s}$

Average speed = $\frac{4800}{160} = 30 \text{ km h}^{-1}$

Q9e Once only while the car travels southwards starting from 500 m north of the street sign.

Q10a

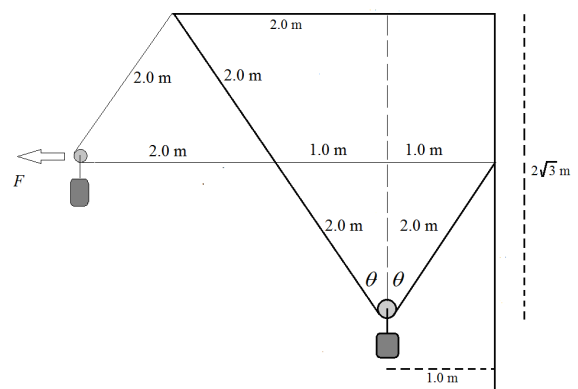


When the pulley stops, $\tan \theta = \frac{2}{2\sqrt{3}}$, $\theta = \frac{\pi}{6}$

Let T newtons be the tension in the cable.

$2T \cos \frac{\pi}{6} = 2.0 \times 10$, $T = \frac{20\sqrt{3}}{3}$

Q10b The length of the cable can be found to be 6.0 m.



Let T_n newtons be the tension in the cable.

$T_n \cos \frac{\pi}{6} = 2.0 \times 10$, $T_n = \frac{40\sqrt{3}}{3}$, $\therefore F = T_n + T_n \cos \frac{\pi}{3} = 20\sqrt{3}$

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