

insight™
Year 12 Trial Exam Paper
2014
SPECIALIST MATHEMATICS

Written examination 2

STUDENT NAME:

Reading time: 15 minutes

Writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

<i>Section</i>	<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
1	22	22	22
2	5	5	58
			Total 80

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT have to be cleared.
- Students are NOT permitted to bring sheets of paper, notes of any kind or white out liquid/tape into the examination.

Materials provided

- The question and answer book of 31 pages, a formula sheet, and an answer sheet for the multiple-choice questions.
- Working space is provided throughout this book.

Instructions

- Write your **name** in the box provided.
- Remove the formula sheet during reading time.
- You must answer the questions in English.

At the end of the examination

- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones or any other electronic devices into the examination.

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SECTION 1**Instructions for Section 1**

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Take the **acceleration due to gravity** to have magnitude $g \text{ m/s}^2$, where $g = 9.8$

Question 1

The equation of an ellipse with domain $\left[-\frac{9}{2}, \frac{3}{2}\right]$ and range $\left[\frac{-7}{2}, \frac{1}{2}\right]$ is:

A. $\frac{(2x+3)^2}{16} + \frac{(2y+3)^2}{36} = 1$

B. $\frac{\left(x + \frac{3}{2}\right)^2}{9} - \frac{\left(y + \frac{3}{2}\right)^2}{4} = 1$

C. $\frac{(2x+3)^2}{36} + \frac{(2y+3)^2}{16} = 1$

D. $\frac{(2x+3)^2}{9} + \frac{(2y+3)^2}{4} = 1$

E. $\frac{\left(x + \frac{3}{2}\right)^2}{36} + \frac{\left(y + \frac{3}{2}\right)^2}{16} = 1$

Question 2

The Cartesian equation of the graph specified by the parametric equations

$$x = 2 \sec(t) + 1 \text{ and } y = 3 \tan(t), \text{ where } t \in \left(-\frac{\pi}{2}, \frac{\pi}{2} \right) \text{ is}$$

- A. $\frac{(x-1)^2}{9} - \frac{y^2}{4} = 1, x \in [3, \infty) \text{ and } y \in R$
- B. $\frac{(x-1)^2}{9} - \frac{y^2}{4} = 1, x \in [1, \infty) \text{ and } y \in R$
- C. $\frac{(x-1)^2}{4} - \frac{y^2}{9} = 1, x \in [3, \infty) \text{ and } y \in R$
- D. $\frac{(x-1)^2}{4} - \frac{y^2}{9} = 1, x \in R \setminus [-3, 3) \text{ and } y \in R$
- E. $\frac{(x-1)^2}{4} - \frac{y^2}{9} = 1, x \in [3, \infty) \text{ and } y \in [0, \infty)$

Question 3

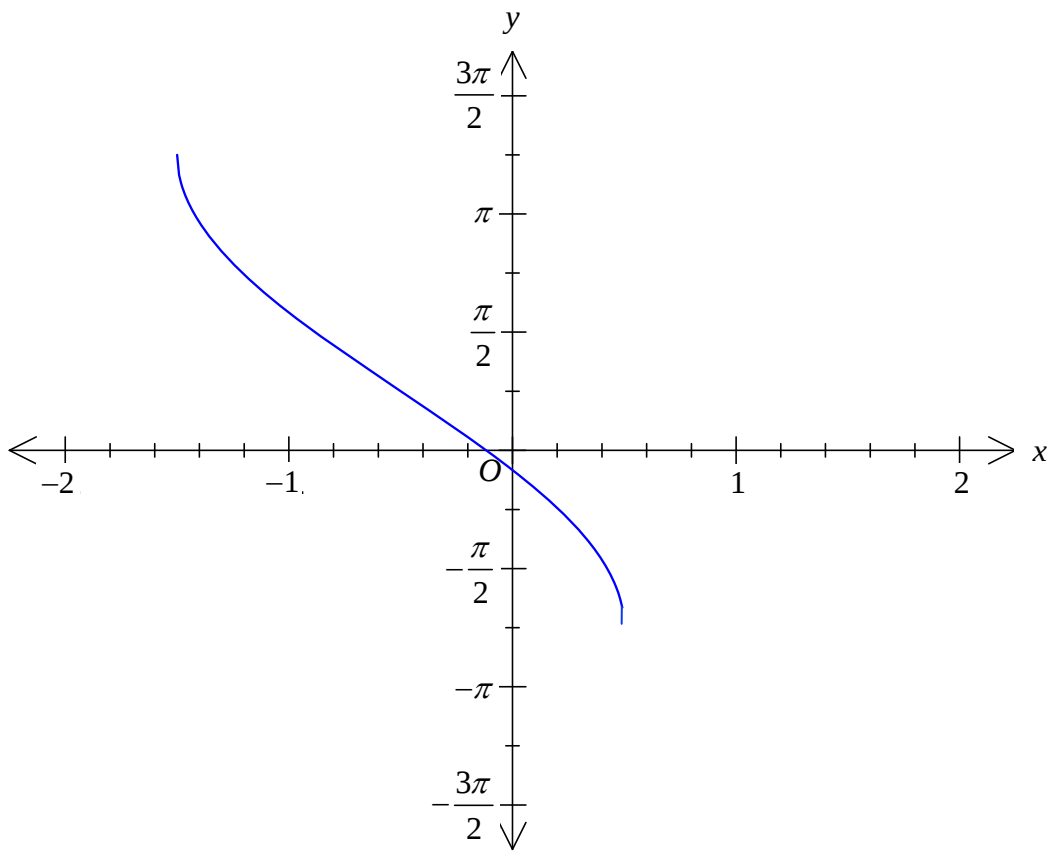
The graph of $y = \frac{1}{ax^2 + bx + c}$ has vertical asymptotes and a local maximum when

- A. $a > 0$ and $b^2 < 4ac$
- B. $a < 0$ and $b^2 < 4ac$
- C. $a > 0$ and $b^2 > 4ac$
- D. $a < 0$ and $b^2 = 4ac$
- E. $a < 0$ and $b^2 > 4ac$

Question 4

If $\cos(x) = a$ and $\operatorname{cosec}(x) = -b$, where $a > 0$ and $b > 0$, then $\cot(-x)$ equals

- A. $-ab$
- B. $\frac{-1}{ab}$
- C. ab
- D. $\frac{b}{a}$
- E. $\frac{a}{b}$

Question 5

The graph of $y = a + b \sin^{-1}(x + c)$ with endpoints $\left(-\frac{3}{2}, \frac{5\pi}{4}\right)$ and $\left(\frac{1}{2}, -\frac{3\pi}{4}\right)$ is shown above.

The values of a , b and c are

- A. $a = \frac{\pi}{4}$, $b = 2$ and $c = \frac{1}{2}$
- B. $a = \frac{\pi}{4}$, $b = -2$ and $c = -\frac{1}{2}$
- C. $a = \frac{\pi}{2}$, $b = -2$ and $c = \frac{1}{2}$
- D. $a = \frac{\pi}{2}$, $b = -2$ and $c = -\frac{1}{2}$
- E. $a = \frac{\pi}{4}$, $b = -2$ and $c = \frac{1}{2}$

Question 6

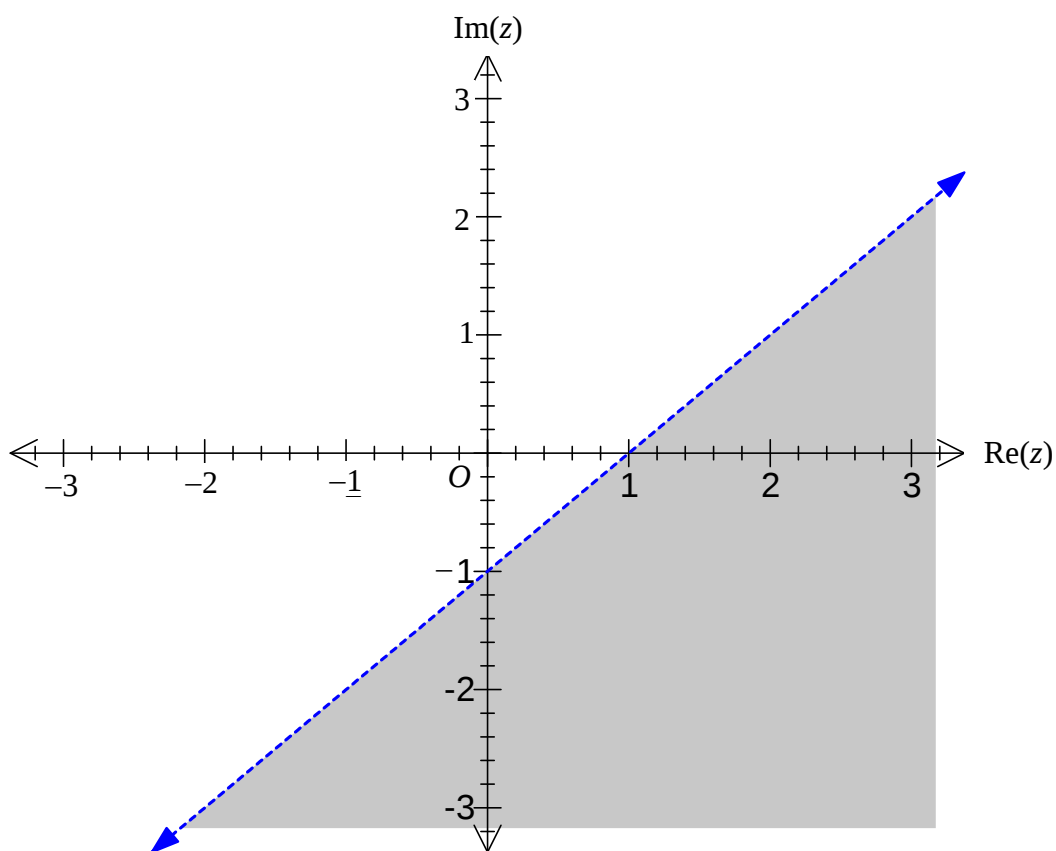
If $z = a + ai$, where $a < 0$, then the location of the complex number iz^8 on an Argand diagram is found

- A. in quadrant 1
- B. on the real axis
- C. in quadrant 3
- D. on the imaginary axis
- E. in quadrant 4

Question 7

One of the solutions of the equation $z^3 - z^2 + iz^2 - z + 1 - i = 0$ is

- A. $z - 1 + i$
- B. $1 - i$
- C. $1 + i$
- D. $-1 - i$
- E. $z - 1 - i$

Question 8

Not including boundaries, the shaded region shown above on the Argand diagram could be described by

- A. $|z| > |z - 1 + i|$
- B. $|z - 1 - i| < |z|$
- C. $\text{Arg}(z - 1) < \frac{\pi}{4}$
- D. $\text{Arg}(z + 1) > \frac{-3\pi}{4}$
- E. $|z + 1 - i| < |z|$

Question 9

If $z = \frac{-2}{i-1}$ and $\bar{z}$ has modulus a and argument θ , then the values of a and θ are

- A. $a = \sqrt{2}$ and $\theta = \frac{\pi}{4}$
- B. $a = \sqrt{2}$ and $\theta = \frac{-\pi}{4}$
- C. $a = \sqrt{2}$ and $\theta = \frac{3\pi}{4}$
- D. $a = 2$ and $\theta = \frac{-\pi}{4}$
- E. $a = 1$ and $\theta = \frac{\pi}{4}$

Question 10

If $\frac{dx}{dy} = \frac{1}{15}(13 + x^2)$, then

- A. $y = \int \frac{15\sqrt{13}}{13+x^2} dx$
- B. $y = \frac{15\sqrt{13}}{13} \int \frac{\sqrt{13}}{13+x^2} dx$
- C. $y = \int \frac{13+x^2}{15} dx$
- D. $y = \frac{\sqrt{13}}{15} \int \frac{\sqrt{13}}{13+x^2} dx$
- E. $y = \frac{15}{13} \tan^{-1}\left(\frac{x}{\sqrt{13}}\right) + c$

Question 11

Using a suitable substitution, $\int_3^5 \left(\frac{2-3x}{\sqrt{1-x}} \right) dx$ can be expressed as

A. $\int_{-2}^{-4} (3u^{\frac{1}{2}} - u^{\frac{-1}{2}}) du$

B. $-\int_3^5 (3u^{\frac{1}{2}} - u^{\frac{-1}{2}}) du$

C. $-\int_{-4}^{-2} (3u^{\frac{1}{2}} - u^{\frac{-1}{2}}) du$

D. $\int_{-4}^{-2} (3u^{\frac{1}{2}} - u^{\frac{-1}{2}}) du$

E. $\int_3^5 (3u^{\frac{1}{2}} - u^{\frac{-1}{2}}) du$

Question 12

A container has the shape of an inverted cone with the diameter equal to half the height.

Water is pouring in at 0.5 L/min. The rate at which the water level is rising, in cm/min, when the diameter of the water is 10 cm is

A. $\frac{80}{\pi}$

B. 80π

C. 20π

D. $\frac{20}{\pi}$

E. $\frac{5}{4\pi}$

Question 13

The expression $\int \frac{2x-1}{(x+2)^2} dx$ can be written as

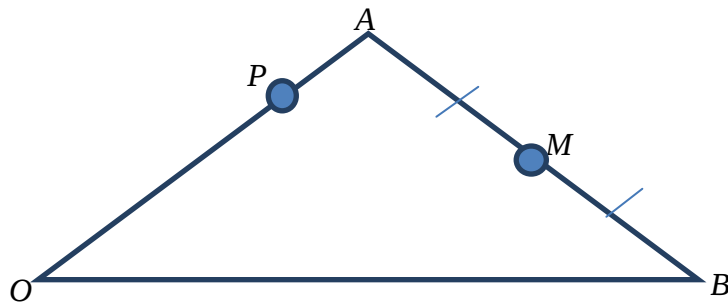
A. $\int \frac{5}{(x+2)^2} - \frac{2}{(x+2)} dx$

B. $\int \frac{2}{(x+2)} - \frac{5}{(x+2)^2} dx$

C. $\int \frac{2}{(x+2)} + \frac{5}{(x+2)^2} dx$

D. $\int \frac{5}{(x+2)} - \frac{2}{(x+2)^2} dx$

E. $\int \frac{5}{(x+2)^2} + \frac{2}{(x+2)} dx$

Question 14

In the triangle shown, M is the midpoint of AB and P is a point on OA such that $OP = \frac{4}{5} OA$.

If $OA = a$ and $OB = b$, then MP equals

- A. $\frac{1}{2}b + \frac{7}{10}a$
- B. $\frac{1}{2}a - \frac{7}{10}b$
- C. $\frac{1}{2}a + \frac{7}{10}b$
- D. $\frac{3}{10}a - \frac{1}{2}b$
- E. $\frac{1}{2}b - \frac{7}{10}a$

Question 15

If $\vec{a} = 2\vec{j} - \vec{k}$ and $\vec{b} = -\vec{j} + m\vec{k}$, then $\vec{a}$ and $\vec{b}$ are linearly dependent when

- A. $m = \frac{1}{2}$
- B. $m = -2$
- C. $m = 2$
- D. $m = 0$
- E. $m = -\frac{1}{2}$

Question 16

A particle moves in a straight line with velocity $v = e^{3x}$ metres per second with displacement x metres from a fixed point, O .

The acceleration of the particle is given by

- A. $a = 3e^{9x}$
- B. $a = 3e^{3x}$
- C. $a = \frac{e^{3x}}{3}$
- D. $a = 3e^{6x}$
- E. $a = \frac{3}{e^{3x}}$

Question 17

If $\vec{a} = 6\vec{i} - 3\vec{j} + 2\vec{k}$ and $\vec{b} = \vec{i} - \vec{j} + \vec{k}$, the vector resolute of $\vec{b}$ in the direction of $\vec{a}$ is

A. $\frac{11}{3}(\vec{i} - \vec{j} + \vec{k})$

B. $\frac{11}{49}(6\vec{i} - 3\vec{j} + 2\vec{k})$

C. $\frac{5}{49}(\vec{i} - \vec{j} + \vec{k})$

D. $\frac{11}{49}(\vec{i} - \vec{j} + \vec{k})$

E. $\frac{11}{7}(\vec{i} - \vec{j} + \vec{k})$

Question 18

The position vector of a particle at time t is given by $\vec{r}(t) = \cos(2t)\vec{i} + \cos^2(t)\vec{j}$, $t \geq 0$.

The equation of the particle's path is

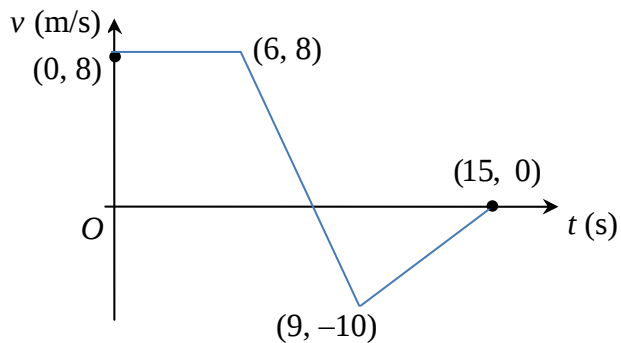
A. $y = \frac{1}{2}(x+1), -1 \leq x \leq 1$

B. $y^2 = \frac{1}{2}(x+1), x \geq 0$

C. $y = \frac{1}{2}(x+1)^2, x \geq 0$

D. $y^2 = \frac{1}{2}(x+1), -1 \leq x \leq 1$

E. $y^2 = \frac{1}{2}(x-1), -1 \leq x \leq 1$

Question 19

The velocity–time graph of a particle moving in a straight line starting from a fixed position O is shown above. Initially the particle moves in an easterly direction.

Where is the particle located 15 seconds after it started?

- A. The particle is located at O .
- B. The particle is located 15 m east of O .
- C. The particle is located $\frac{275}{3}$ m east of O .
- D. The particle is located $\frac{275}{3}$ m west of O .
- E. The particle is located 15 m west of O .

Question 20

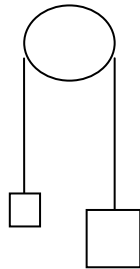
An approximate solution to the differential equation $\frac{dy}{dx} = x - y$ is found using Euler's method with an increment size of 0.5, given that $y = 0$ when $x = 2$.

The value obtained for $x = 3$, to three decimal places, would be

- A. 1.750
- B. 2.375
- C. 2.938
- D. 1.000
- E. 2.735

Question 21

The diagram shows a smooth pulley with two objects attached to each end of an inextensible string. The mass of the smaller object is one-third the mass of the larger object.



The magnitude of the acceleration of the larger object is

- A. $g \text{ m/s}^2$
- B. $\frac{g}{2} \text{ m/s}^2$
- C. $\frac{g}{4} \text{ m/s}^2$
- D. $\frac{1}{2} \text{ m/s}^2$
- E. $2g \text{ m/s}^2$

Question 22

A person travelling in a lift that is accelerating downwards at 3 m/s^2 has an apparent weight of 30 kg wt. When the lift is stationary, the person's weight is closest to

- A. 44 kg
- B. 37 kg
- C. 43 kg
- D. 38 kg
- E. 39 kg

END OF SECTION 1

SECTION 2**Instructions for Section 2**

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

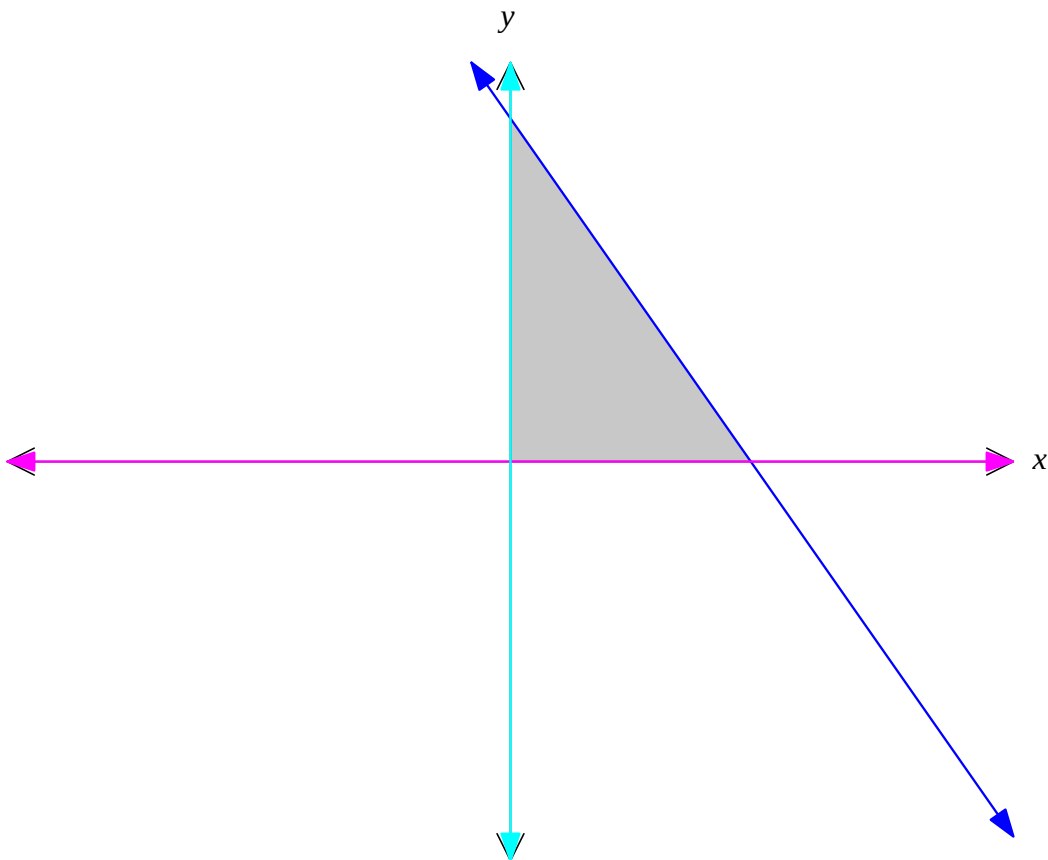
In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ m/s}^2$, where $g = 9.8$.

Question 1 (11 marks)

The area enclosed by the graph of $y \leq ax + b$, $x \geq 0$ and $y \geq 0$ is shown below.



- a. Find the coordinates of the x-intercept and y-intercept.

1 mark

SECTION 2 – Question 1 – continued
TURN OVER

- b.** The area enclosed by the graph of $y \leq ax + b$, $x \geq 0$ and $y \geq 0$ is rotated around the y -axis to form a solid of revolution. Find the volume of this solid in terms of a and b .

2 marks

- c.** Find $\frac{dy}{dx}$ for the relationship $3y \sin(x) + \frac{2}{y^2} = \frac{1}{y}$, $y \neq 0$.

4 marks

- Find the equation of this tangent and clearly state the exact values of a and b .

[illegible]

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-
-
-

Question 2 (11 marks)

Tarquin is preparing some pre-packaged soup for his friend Harry. The soup is taken from the fridge and placed in a microwave oven. Let $T^{\circ}\text{C}$ be the temperature of the soup t minutes after it is placed in the microwave oven.

A differential equation that models the temperature of the soup is $\frac{dT}{dt} = b(100 - T)$, $b > 0$.

- a.** The temperature of the soup is 3°C when removed from the fridge.
Use calculus to solve the differential equation to show that
 $T = 100 - 97e^{-bt}$ for $0 \leq t \leq t_1$, where t_1 minutes is the time when Tarquin removed the soup from the microwave oven.

4 marks

- $$T = Be^{-kt} + 20, \quad k > 0 \text{ and } B > 0 \text{ for } t \geq t_1.$$

2 marks

- c. If the soup takes twice the time to cool down as it takes to heat up and Harry drinks the soup when it reaches a temperature of 40°C , **show** that $B = 40\sqrt{2}$.

3 marks

[illegible]

- d.** If $k = \frac{1}{4}$, show that the total time taken from when Tarquin began preparing the soup to the time that Harry started to drink it took less than 5 minutes. 2 marks

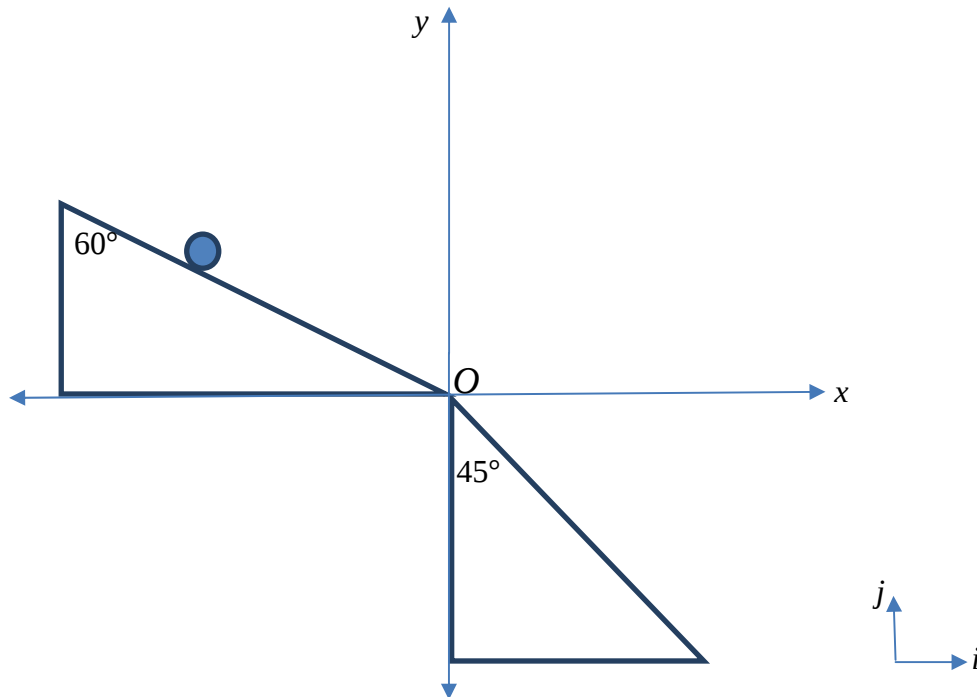
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Question 3 (12 marks)

A construction is comprised of two connected inclined planes, where the first is inclined at 60° to the vertical and the second is inclined at 45° to the vertical.

The coefficient of friction is $\frac{1}{2}$ for both planes.

A 10 kg bowling ball is released from rest from a point on the upper plane, becomes airborne at the point at which the planes are connected and lands on some point on the lower plane.



Let the origin O of a Cartesian coordinate system be the point at which the ball leaves the upper inclined plane with $\hat{i}$ being a unit vector in the positive x direction and $\hat{j}$ being a unit vector in the positive y direction.

Note that displacement is measured in metres and time is measured in seconds.

- a. On the diagram, use arrows to label weight (W), friction (F) and normal reaction (N) forces acting on the ball when it is rolling down the upper plane. 1 mark

- b. Show that the exact acceleration of the ball as it rolls down the upper plane is $\frac{g(2-\sqrt{3})}{4} \text{ m/s}^2$. 3 marks

- c. If the ball exits the upper plane at O with a speed of 5 m/s, find the exact distance travelled by the ball on the upper plane. 2 marks

When the ball exits the upper plane at a speed of 5 m/s, it is temporarily airborne, and subjected to an acceleration of $\vec{a}(t) = -0.4t \vec{i} - (g - 0.4t) \vec{j}$, $t \geq 0$ where t is time in seconds.

- d. Show that the velocity of the ball when it leaves the upper plane is 1 mark

$$\vec{v} = \left(\frac{5\sqrt{3}}{2} \vec{i} - \frac{5}{2} \vec{j} \right) \text{ m/s}$$

- e. If the ball is airborne for exactly $\frac{a(\sqrt{b}-1)}{g}$ seconds, find the exact values of a and b .

5 marks

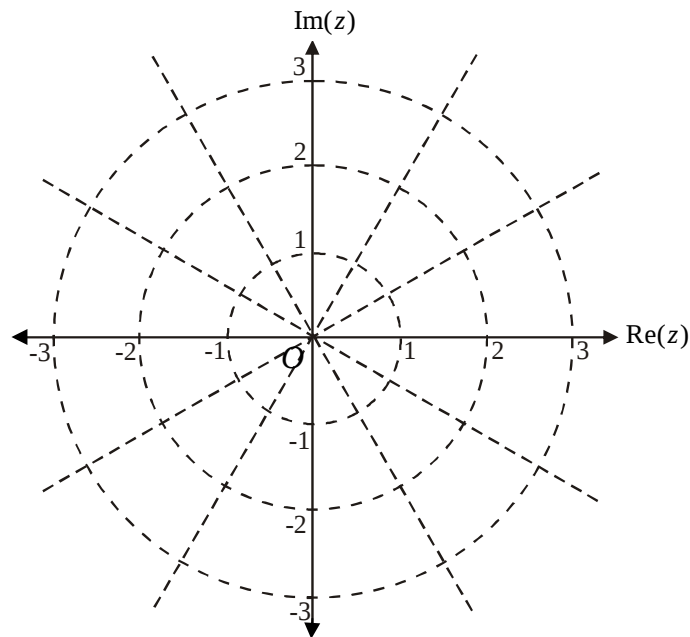
[illegible]

Question 4 (14 marks)

The solutions to the equation $z^2 + 2 = 0$ are z_1 and z_2 , where $z \in \mathbb{C}$, $\text{Arg}(z_1) > 0$ and $\text{Arg}(z_2) < 0$.

- a. Find z_1 and z_2 and plot them on an Argand diagram.

3 marks



SECTION 2 – Question 4 – continued
TURN OVER

- b.** Show that the Cartesian equation of the complex relationship given by $|z - z_1| - |z - z_2| = 1$ is the hyperbola $28y^2 - 4x^2 = 7$.

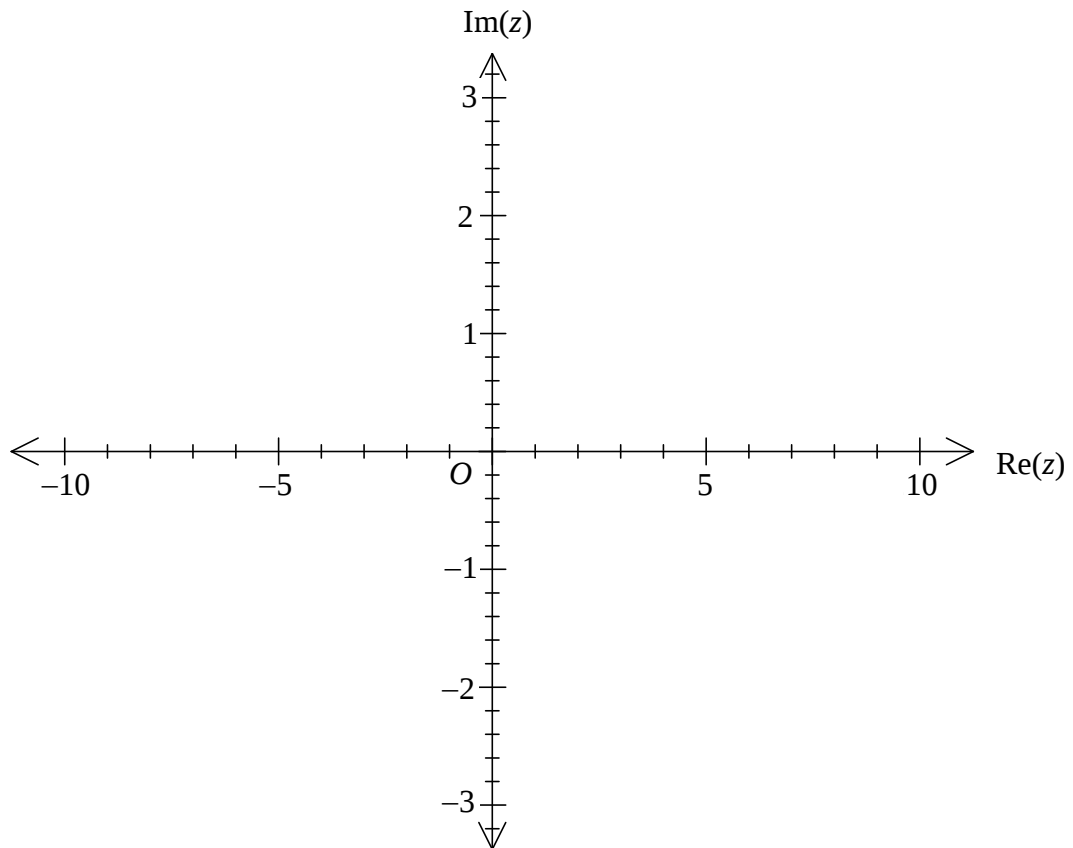
3 marks

- c.** State the coordinates of the vertices and the equation of the asymptotes for the hyperbola described by $|z - z_1| - |z - z_2| = 1$.

3 marks

- d. Let $S = \{z : |z - z_1| - |z - z_2| \leq 1\}$ and $T = \{z : |z - z_1| \leq |z - z_2|\}$ be subsets of the complex plane.
Sketch $S \cap T$ (Asymptotes are not required.)

2 marks



- e. For the equation $z^4 - 2az^3 + 2(3a + 1)z^2 - 4az + 12a = 0$, where $z \in \mathbb{C}$ and $a \in \mathbb{R} \setminus \{0\}$, two of the solutions are z_1 and z_2 . Another solution is $z = a + ai$.

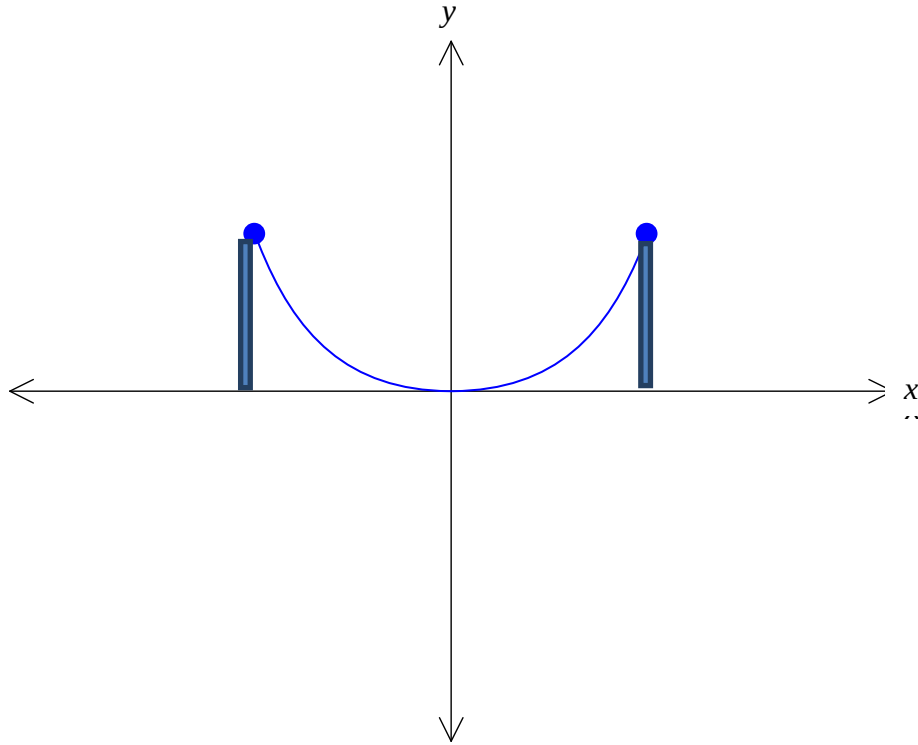
Find the value of a .

3 marks

Question 5 (10 marks)

A rope of length l metres hangs between two posts of equal height b metres situated $2a$ metres apart so that it sags to the ground, just touching it at its centre. If $(0, 0)$ is the point at which the rope touches the ground, the equation of the rope can be described by

$$y = \frac{1}{2k} (e^{kx} + e^{-kx} - 2), \quad k > 0.$$



- a.** Find the height of the posts in terms of a and k .

1 mark

The length of the rope (i.e. l metres) can be determined by the formula $l = 2 \int_0^a \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$.

- b.** Use calculus to show that $\left(\frac{dy}{dx}\right)^2 = \frac{1}{4}(e^{2kx} + e^{-2kx} - 2)$.

2 marks

- c. Use calculus to show that the length of the rope is $l = \frac{1}{k}(e^{ak} + e^{-ak})$ metres.

4 marks

- d. Find the length of the rope, to two decimal places, if $k = \frac{1}{8}$ and the height of the post is 5 metres.

3 marks

END OF QUESTION AND ANSWER BOOK