

**insight™**  
**Year 12 Trial Exam Paper**  
**2014**

**SPECIALIST MATHEMATICS**

**Written examination 1**

**STUDENT NAME:**

**Reading time: 15 minutes**

**Writing time: 1 hour**

**QUESTION AND ANSWER BOOK**

**Structure of book**

| <i>Number of questions</i> | <i>Number of questions to be answered</i> | <i>Number of marks</i> |
|----------------------------|-------------------------------------------|------------------------|
| 10                         | 10                                        | 40                     |

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring sheets of paper, notes of any kind or white out liquid/tape into the examination.
- Calculators are not permitted in this examination.

**Materials provided**

- The question and answer book of 15 pages with a separate sheet of miscellaneous formulas.
- Working space is provided throughout this book.

**Instructions**

- Write your **name** in the box provided.
- Remove the formula sheet during reading time.
- You must answer the questions in English.

**Students are NOT permitted to bring mobile phones and/or any other electronic devices into the examination.**

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**Instructions**

Answer **all** questions in the spaces provided.

Unless otherwise specified, an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

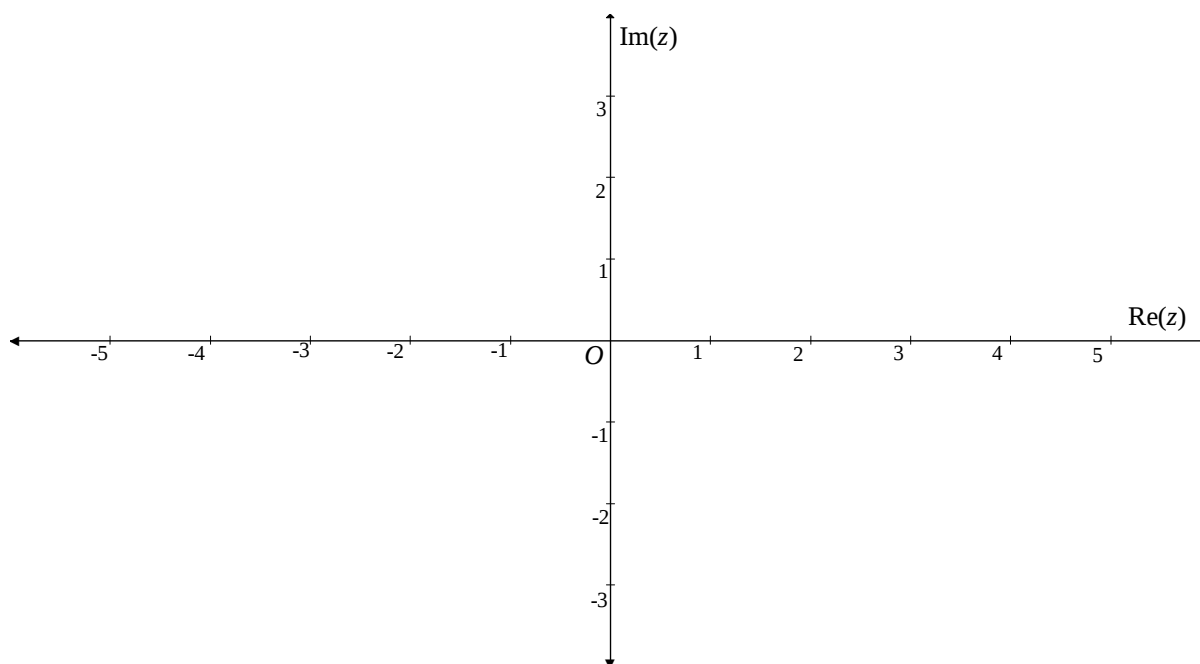
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude  $g \text{ m/s}^2$ , where  $g = 9.8$ .

**Question 1** (5 marks)

- a. Show the region represented by  $\{z : \text{Arg}(z - 2) = \text{Arg}(z + 1)\}$  on the Argand diagram below.

3 marks



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**QUESTION 1** – continued

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- b.**  $P(z) = z^3 - (4+i)z^2 + (5+4i)z - 5i$ , where  $z \in \mathbb{C}$ . Given that  $(z-i)$  is a factor of  $P(z)$ , find the other factors.

2 marks

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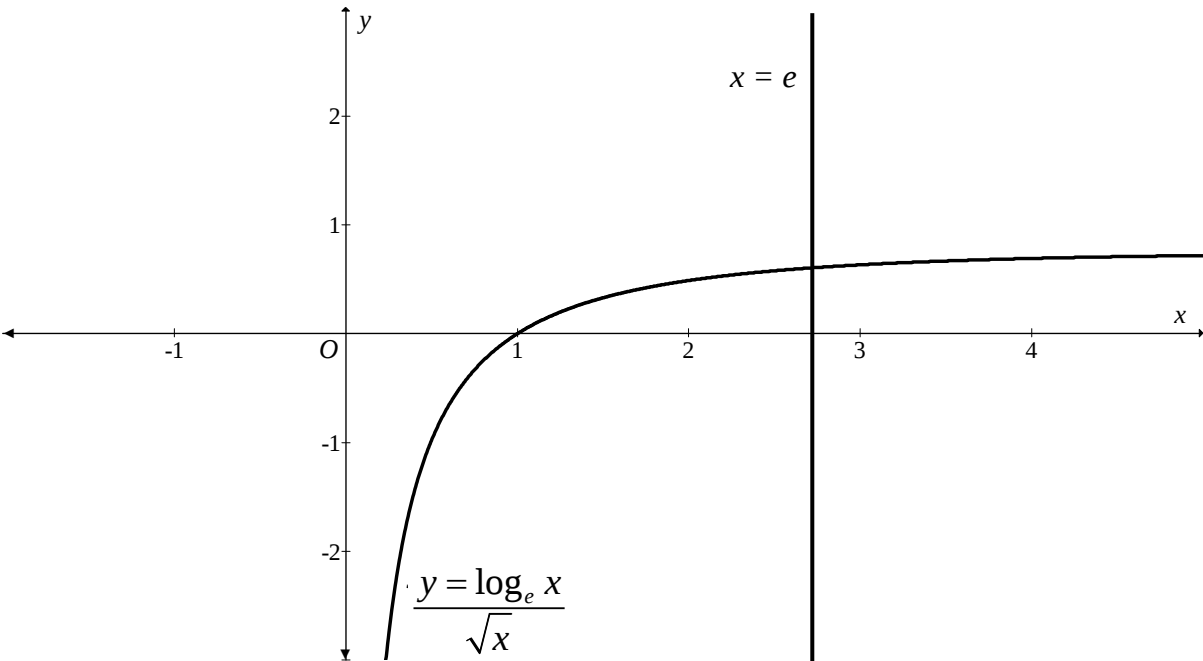
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**Question 2** (4 marks)

The area bounded by the curve  $y = \frac{\log_e x}{\sqrt{x}}$ , the  $x$ -axis and the straight line  $x = e$  is rotated about the  $x$ -axis. Calculate the volume.



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Find the equation of the normal to the curve with equation  $ye^{x^2+2x} + xe^y = 2x+1$  at the point  $(0, 1)$ .

[illegible]

**Question 4** (5 marks)

The position vector of a particle is given by  $r = \tan(t)i + tj$ , where  $t \in \left[0, \frac{\pi}{2}\right)$ .

- a.** Find the Cartesian equation of the path of this particle in the form  $y = f(x)$ .  
State the domain.

2 marks

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- b.** Find the rule for the gradient function  $y = f'(x)$ .

1 mark

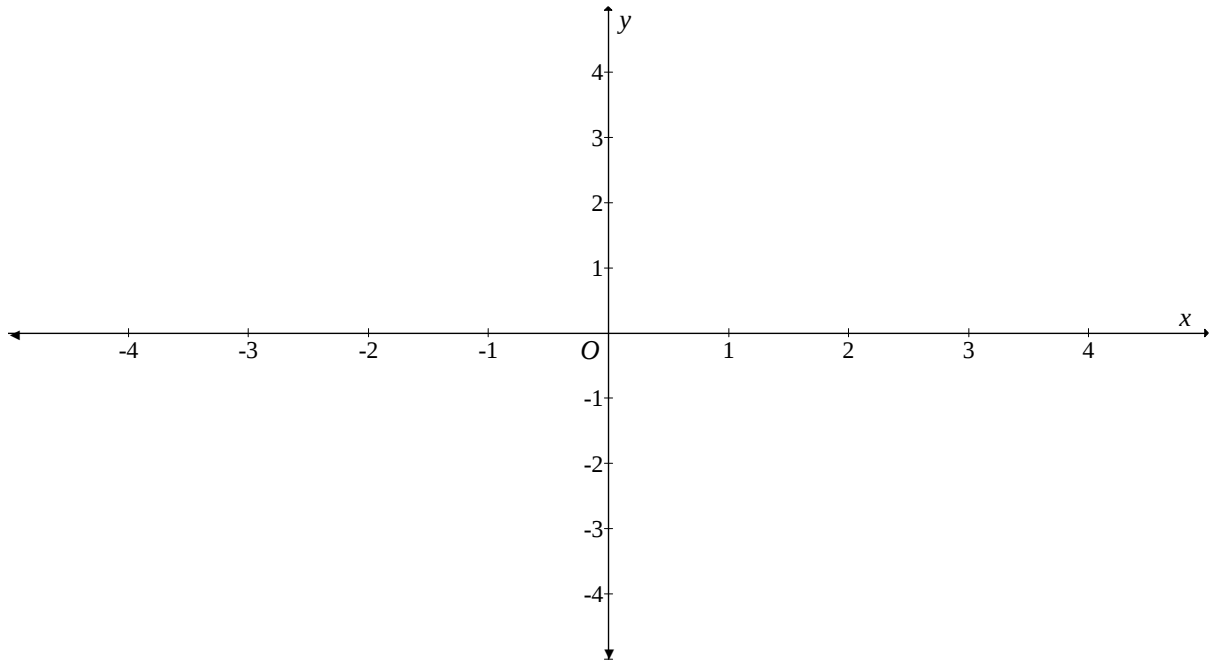
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- c. Sketch the graph of  $y = f'(x)$  on the set of axes below. Clearly indicate any axes intercepts, stationary points or asymptotes.

2 marks



**Question 5** (5 marks)

A person of mass 80 kg stands on a set of scales in a lift, which is moving up.

- a. i.** What is the reading on the scales, in kg, when it is accelerating at  $0.2 \text{ m/s}^2$ ?

Express the answer in the form  $\frac{a}{g}$ , where  $a \in \mathbb{Z}^+$  and  $g$  is the acceleration due to gravity.

1 mark

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- ii.** What is the acceleration when the reading on the scales is 60 kg?

1 mark

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The lift stops at a height of 50 m above the basement. The total mass of its container and passengers is 500 kg. At the same time, the cable supporting the lift snaps and the lift plummets towards the basement subject only to the force of gravity and combined resistive forces of  $F = 0.6x^2$ , where  $x$  represents the number of metres that the lift has fallen.

The lift impacts the basement at a speed of  $a\sqrt{g-b}$  m/s, where  $g$  is the acceleration due to gravity.

- b.** Find the values of  $a$  and  $b$ .

3 marks

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When left to itself, an electric current  $J$  (amps) drops to  $\frac{1}{10}$ th of its original strength in  $\frac{1}{10}$ th of a second.

[illegible]

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Given that  $\sec(4\theta) = 8$ ,  $0 \leq \theta \leq \frac{\pi}{8}$ , find the exact value of  $\tan(\theta)$ .

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Given that  $f(x) = \frac{x}{1+x^2}$ :

- a.** Show that  $f'(x) = \frac{(1-x^2)}{(1+x^2)^2}$ .

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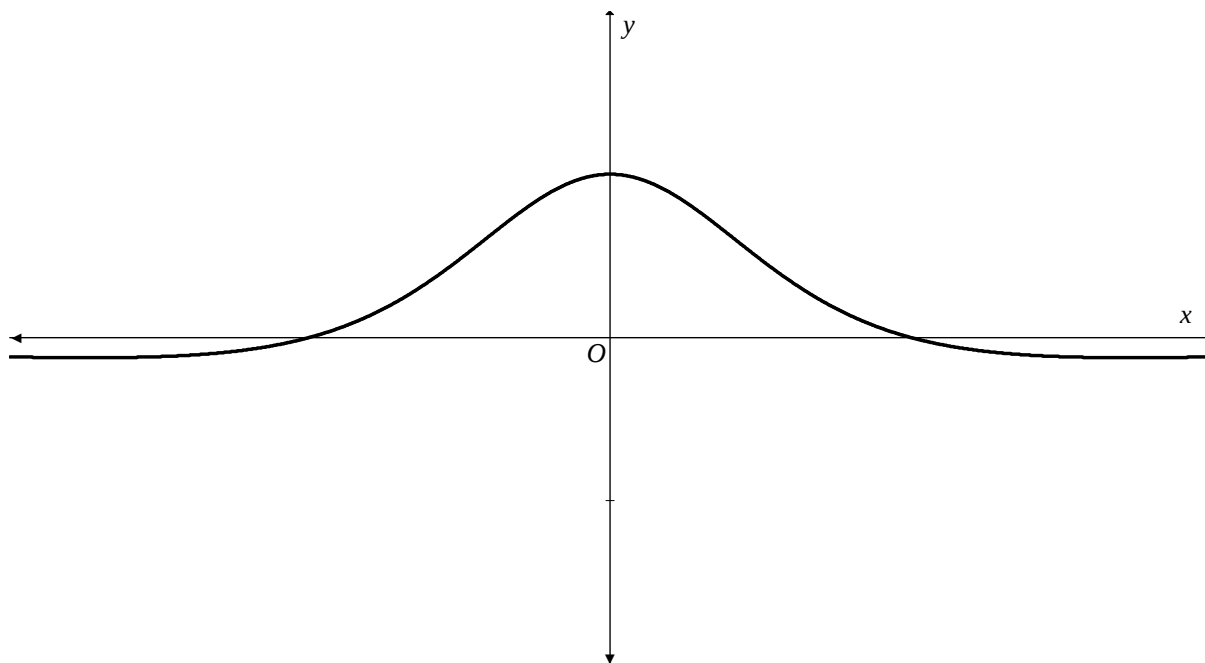
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- b.** Hence, calculate the area bounded by the curve with equation  $y = \frac{a(1-x^2)}{(1+x^2)^2}$  (shown below) and the  $x$ -axis, where  $a \in \mathbb{R}^+$ .



3 marks

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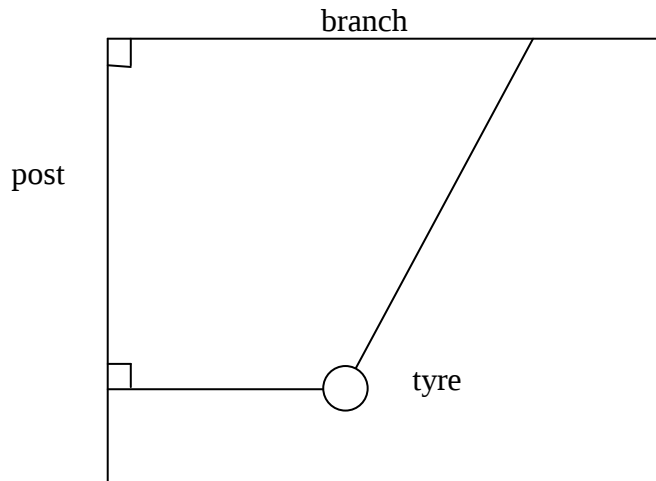
**Question 10** (3 marks)

A tyre swing of mass 8 kg is suspended from a tree branch by an inextensible rope. When it is not in use, the tyre is supported by a second, horizontal, inextensible rope that is attached to a post. Assume the masses of the ropes are negligible.

The tension in the rope attached to the branch is  $\sqrt{3}$  times the tension in the other rope.

- a.** Label all the forces, in newtons, acting on the tyre in the diagram.

1 mark



- b.** Calculate the tension in the rope attached to the post.  
Express the tension in the form  $a\sqrt{b}g$ , where  $a$  and  $b$  are positive integers.

2 marks

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**END OF QUESTION AND ANSWER BOOK**