TSSM
Creating VCE Success

'2016 Examination Package' - Trial Examination 3 of 5

THIS BOX IS FOR ILLUSTRATIVE PURPOSES ONLY

STUDENT NUMBER

									Letter
Figures									
Words									

SPECIALIST MATHEMATICS

Units 3 & 4 – Written examination 2

(TSSM's *2013 trial exam updated for the current study design)*

Reading time: 15 minutes
Writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	4	4	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved graphics calculator or approved CAS calculator and a scientific calculator.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 28 pages.(including a multiple choice answer sheet)

Instructions
- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other electronic devices into the examination room.

SECTION 1

Instructions for Section 1
Answer **all** questions on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks are **not** deducted for incorrect answers.
If more than 1 answer is completed for any question, no mark will be given.
Take the **acceleration due to gravity**, to have magnitude g m/ s^2, where g = 9.8.

Question 1

The number of asymptotes of the graph of $y = \dfrac{2x^3 - 8x^2 + 8x - 3}{x^2 - 4x + 4}$ is

A. 0

B. 1

C. 2

D. 3

E. 4

Question 2

Which of the equations below defines a circle with centre at (-2,3) and radius 5 units?

A. $x^2 - 4x + y^2 + 6y - 25 = 0$

B. $x^2 + 4x + y^2 - 6y - 12 = 0$

C. $x^2 + 4x + y^2 - 6y - 32 = 0$

D. $x^2 + 2x + y^2 - 3y - 1 = 0$

E. $x^2 - 2x + y^2 + 3y - 25 = 0$

SECTION 1 – continued

Question 3

If $\text{Cos}^{-1}(a-1)=b$ then $\cot\left(b-\frac{\pi}{2}\right)$ is equal to

A. $\dfrac{a-1}{\sqrt{a(2-a)}}$

B. $\dfrac{1-a}{\sqrt{a(2-a)}}$

C. $\dfrac{\sqrt{a(2-a)}}{a-1}$

D. $\dfrac{-1}{\sqrt{a(2-a)}}$

E. $\dfrac{\sqrt{a(2-a)}}{1-a}$

Question 4

A function is defined by the rule $f(x)=2-\frac{1}{2}\text{Sin}^{-1}(2x-1)$. The implied domain and range of the function $y=\frac{1}{2}f(x)$ are respectively

A. $\left[0,\frac{1}{2}\right]$ and $\left[\frac{8-\pi}{8},\frac{8+\pi}{8}\right]$

B. $(0,1]$ and $\left[\frac{8-\pi}{8},\frac{8+\pi}{8}\right]$

C. $[0,1]$ and $\left[\frac{8-\pi}{8},\frac{8+\pi}{8}\right]$

D. $\left(0,\frac{1}{2}\right)$ and $\left(\frac{8-\pi}{4},\frac{8+\pi}{4}\right)$

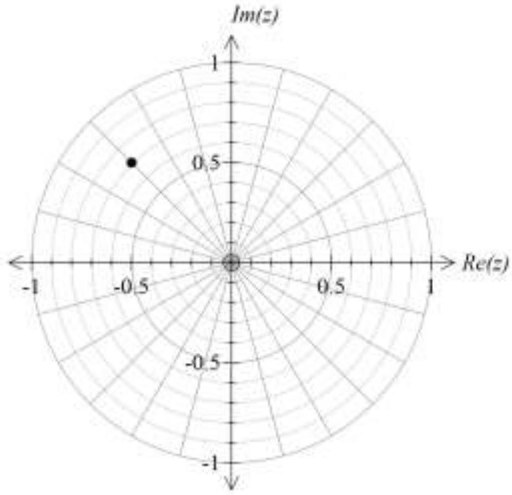
E. $(0,1)$ and $\left(\frac{8-\pi}{4},\frac{8+\pi}{4}\right)$

SECTION 1 – continued
TURN OVER

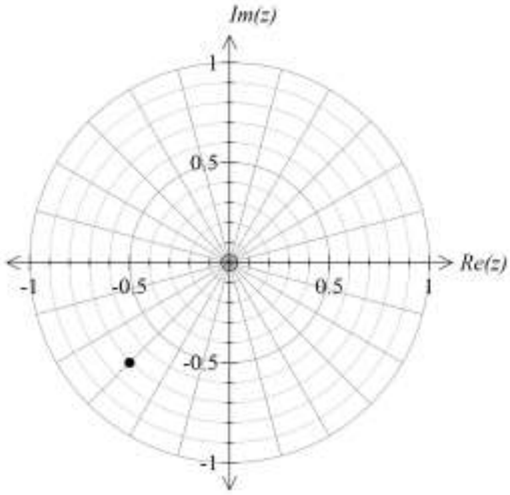
Question 5

The complex number $z = \frac{1+i}{(1-i)^2}$. The conjugate $\bar{z}$ is best represented by

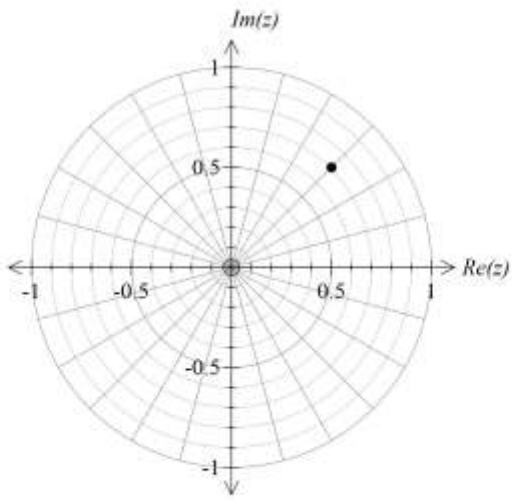
A.



B.



C.



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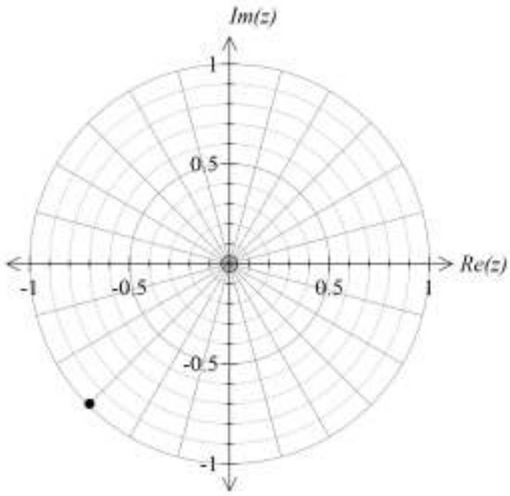
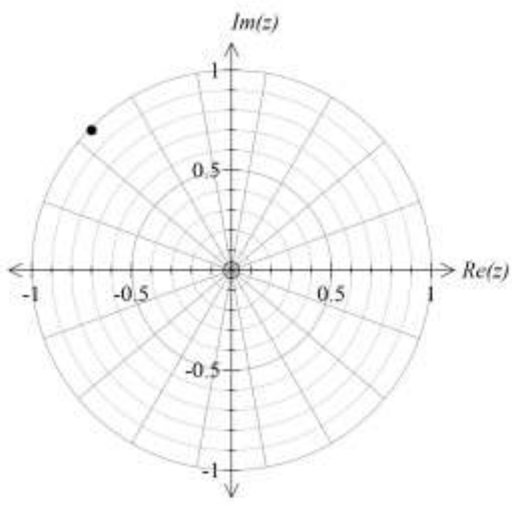


E.



SECTION 1 – continued

Question 6

A polynomial $P(z)$ is defined as $P(z)=z^3+bz^2+(2+i)z-2$. If $z+i$ is a factor, the coefficient b is equal to

A. $1+i$

B. $-1+i$

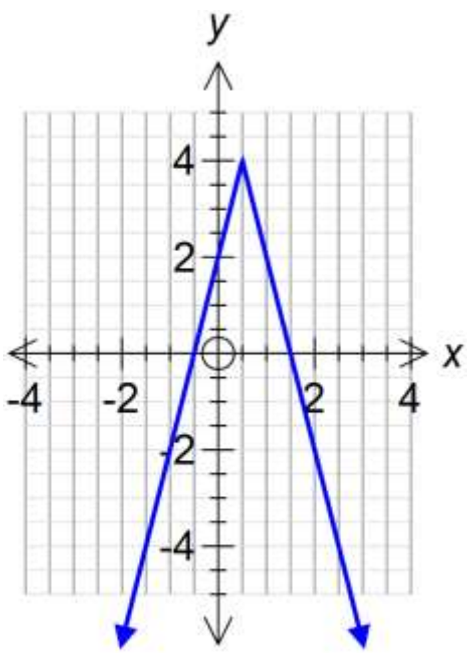
C. $1-i$

D. $-1-i$

E. $2i$

Question 7

The graph of the absolute value function $f: R \to R,\ f(x) = -2|1 + a\,x| + 4$ is shown below.



The value of a is:

A. 2

B. -2

C. 4

D. -4

E. 0.75

SECTION 1 – continued
TURN OVER

Question 8

The complex roots of the equation $z^3 - 64i = 0$ are equal to z_1, z_2 and z_3, where z_1 and z_2 are located in the first and second quadrants respectively.

$\text{Arg}\left(\dfrac{2z_1z_3}{z_2}\right)$ is equal to

A. $\dfrac{5\pi}{6}$

B. $-\dfrac{\pi}{3}$

C. $\dfrac{\pi}{2}$

D. $-\dfrac{7\pi}{6}$

E. $-\dfrac{\pi}{6}$

Question 9

Which of the equations below is equivalent to $\sec^2(2\pi x) = 2\tan(2\pi x)$?

A. $\tan(2\pi x) + 1 = 0$

B. $\sec(2\pi x) = 1$

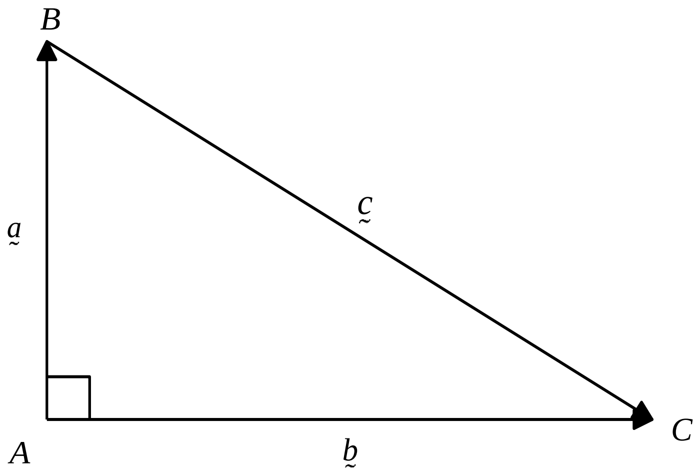
C. $2\tan(2\pi x) - 1 = 0$

D. $2\cot(2\pi x) + 1 = 0$

E. $\tan(2\pi x) = 1$

Question 10

The diagram below shows a right-angled triangle.



If vectors $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{BC}$ are denoted by $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ respectively, it follows that

A. $|\underset{\sim}{a}|^2 = (\underset{\sim}{c} - \underset{\sim}{b}).(\underset{\sim}{c} - \underset{\sim}{b})$

B. $\underset{\sim}{a} + \underset{\sim}{b} = \underset{\sim}{c}$

C. $\underset{\sim}{a} = \sqrt{\underset{\sim}{c}^2 - \underset{\sim}{b}^2}$

D. $|\underset{\sim}{c}|^2 = (\underset{\sim}{b} - \underset{\sim}{a}).(\underset{\sim}{b} - \underset{\sim}{a})$

E. $\underset{\sim}{c}.\underset{\sim}{c} = \underset{\sim}{b}.\underset{\sim}{b} - \underset{\sim}{a}.\underset{\sim}{a}$

SECTION 1 – continued
TURN OVER

Question 11

An object is kept in equilibrium by three forces, $\underset{\sim}{F}_1$, $\underset{\sim}{F}_2$ and $\underset{\sim}{F}_3$, where

$$\underset{\sim}{F}_1 = -5\underset{\sim}{i} + 4\underset{\sim}{j}\,,\ \underset{\sim}{F}_2 = 3\underset{\sim}{i} + 7\underset{\sim}{j} \text{ and } \underset{\sim}{F}_3 = a\underset{\sim}{i} + b\underset{\sim}{j}$$

The angle between $\underset{\sim}{F}_1$ and $\underset{\sim}{F}_3$ to the nearest degree, is equal to

A. $34°$

B. $139°$

C. $105°$

D. $41°$

E. $146°$

Question 12

The vector $a\underset{\sim}{i} + b\underset{\sim}{k}$ has magnitude 3 units. If $a\underset{\sim}{i} + b\underset{\sim}{k}$ is perpendicular to the vector $2\underset{\sim}{i} - 3\underset{\sim}{j} - \underset{\sim}{k}$. It follows that

A. $a = \frac{3\sqrt{5}}{5}$ and $b = -\frac{6\sqrt{5}}{5}$

B. $a = \frac{9\sqrt{13}}{13}$ and $b = \frac{6\sqrt{13}}{13}$

C. $a = -\frac{3\sqrt{5}}{5}$ and $b = -\frac{6\sqrt{5}}{5}$

D. $a = -\frac{9\sqrt{13}}{13}$ and $b = -\frac{6\sqrt{13}}{13}$

E. $a = -\frac{3\sqrt{5}}{5}$ and $b = \frac{6\sqrt{5}}{5}$

SECTION 1 – continued

Question 13

The position of a particle at time t seconds is given by $\underset{\sim}{r}(t) = (t^2 - 3)\underset{\sim}{i} + \sqrt{3t^2 + 1}\,\underset{\sim}{j}$ metres.
The speed of the particle after 4 seconds, correct to the nearest tenth of a second, is equal to

A. 14.8 ms^{-1}

B. 8.2 ms^{-1}

C. 5.2 ms^{-1}

D. 3.1 ms^{-1}

E. 10.1 ms^{-1}

Question 14

Given $x = 2\sin t$ and $y = 3\tan t$. When $t = \frac{5\pi}{6}$, $\frac{dy}{dx}$ is equal to

A. $-\frac{4\sqrt{3}}{3}$

B. $-\frac{\sqrt{3}}{4}$

C. -12

D. 12

E. $\frac{4\sqrt{3}}{3}$

SECTION 1 – continued
TURN OVER

Question 15

Using a suitable substitution, the definite integral $\int_4^8 \frac{2}{x\log_e(4x)}\,dx$ is equivalent to

A. $\int_4^8 \frac{1}{2u}\,du$

B. $\frac{1}{2}\int_{4\log_e 2}^{5\log_e 2} \frac{1}{u}\,du$

C. $\frac{1}{4}\int_{4\log_e 2}^{5\log_e 2} \frac{1}{u}\,du$

D. $\int_{4\log_e 2}^{5\log_e 2} \frac{2}{u}\,du$

E. $\int_4^8 \frac{2}{u}\,du$

Question 16

The volume of water in a tank, V is increasing at a rate that is inversely proportional to the square root of the volume of water in the tank at any time, t minutes after the tank has started to fill. Initially, the volume of water in the tank is $9\,\text{m}^3$. The volume of water in the tank after 5 minutes, in terms of the proportionality constant, k is

A. $\sqrt{\frac{15k-54}{2}}$

B. $\left(\frac{5k+6}{2}\right)^2$

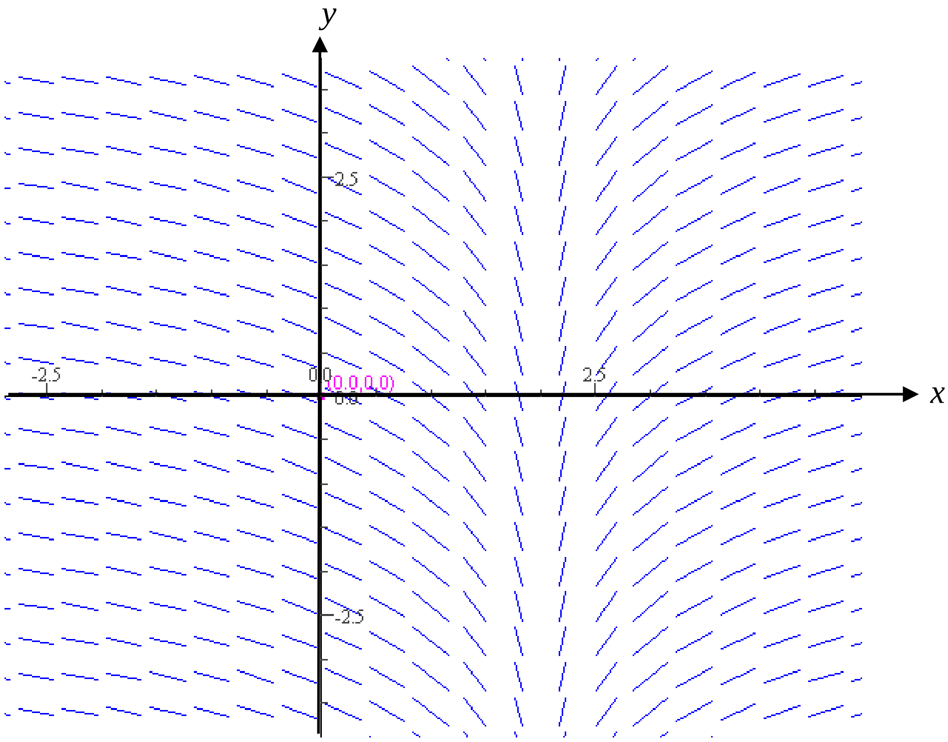
C. $\sqrt{10k+81}$

D. $\sqrt{\frac{5k+6}{2}}$

E. $\left(\frac{15k+54}{2}\right)^{\frac{2}{3}}$

SECTION 1 – continued

Question 17



The differential equation which best represents the above slope field could be

A. $\dfrac{dy}{dx} = \dfrac{1}{x-2}$

B. $\dfrac{dy}{dx} = \log_e|x-2|+1$

C. $\dfrac{dy}{dx} = \dfrac{2}{x+2}$

D. $\dfrac{dy}{dx} = \log_e\left(\dfrac{1}{x-2}\right)$

E. $\dfrac{dy}{dx} = \log_e|x+2|-2$

SECTION 1 – continued
TURN OVER

Question 18

A differential equation is defined as $\frac{dy}{dx} = x\log_e x$ where $y = 1$ when $x = 2$. Applying Euler's method with a step size of 0.2, an approximation for $y(2.4)$, correct to three decimal places, is found to be

A. 1.277

B. 1.624

C. 2.044

D. 2.101

E. 1.081

Question 19

An object moving at constant acceleration is found to travel 20 metres during its third second of motion and 10 metres during its fifth second of motion. The initial speed of the particle is equal to

A. $40.0\,\text{ms}^{-1}$

B. $32.5\,\text{ms}^{-1}$

C. $37.5\,\text{ms}^{-1}$

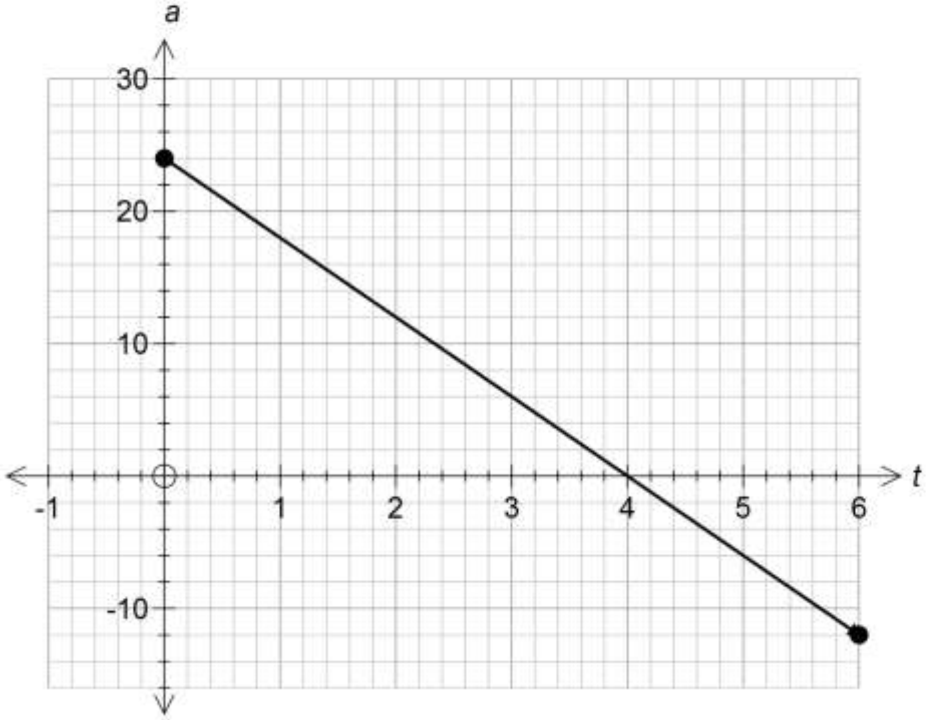
D. $42.5\,\text{ms}^{-1}$

E. $30.5\,\text{ms}^{-1}$

SECTION 1 – continued

Question 20

The motion of a particle is given by the acceleration-time graph below, where time t is in seconds and the acceleration of the particle is a metres per second per second.



If the initial velocity of the particle is $24\,\text{ms}^{-1}$, the distance travelled by the particle in the first 6 seconds is equal to

A. 36 metres

B. 60 metres

C. 360 metres

D. 48 metres

E. 224 metres

SECTION 1 – continued
TURN OVER

Question 21

A force, F acting on an object of mass 2 kg is equal to $F(v)=2v^2-1$ newtons. The object has a velocity of 1ms^{-1} when it is 1 metre to the right of the origin. The velocity of the object, in terms of the displacement, x is defined as

A. $\pm\sqrt{\frac{1}{2}\left(e^{2(x-1)}+1\right)}$

B. $\frac{1}{2}\log_e\left|2x^2-1\right|+1$

C. $\sqrt{\frac{1}{2}\left(e^{2(x-1)}+1\right)}$

D. $-\left(\frac{1}{2}\log_e\left|2x^2-1\right|+1\right)$

E. $-\sqrt{\frac{1}{2}\left(e^{2(x-1)}+1\right)}$

Question 22

A stone of mass 10 kg is released from rest from the top of a cliff. While falling, the stone experiences a wind resistance force that is proportional to its speed. If the constant of proportionality is 2.5, the time taken for the stone to reach a speed of $20\,\text{ms}^{-1}$, correct to two decimal places is equal to

A. 2.85 seconds

B. 1.65 seconds

C. 0.91 seconds

D. 1.18 seconds

E. 2.09 seconds

END OF SECTION 1

SECTION 2

Instructions for Section 2

Answer **all** questions.
A decimal approximation will not be accepted if the question specifically asks for an **exact** answer.
For questions worth more than one mark, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams are **not** drawn to scale.
Take the **acceleration due to gravity**, to have magnitude g m/s^2 , where g = 9.8.

Question 1

a. Write down the expansion of $(\cos\theta + i\sin\theta)^3$ in the form $a+ib$.

1 mark

b. Hence use De moivre's theorem to show that $\sin(3\theta) = 3\sin\theta - 4\sin^3\theta$.

2 marks

SECTION 2 – continued
TURN OVER

c. Write down the expansion of $(\cos\theta + i\sin\theta)^5$ in the form $a + ib$.

1 mark

d. Hence show that $\sin(5\theta) = 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta$.

3 marks

SECTION 2 – continued

e. Hence show that the solutions to the equation $\sin(5\theta)+\sin(3\theta)+\sin\theta=0$, where $-\frac{\pi}{2}\le\theta\le\frac{\pi}{2}$, are $\theta=0$ and $\theta=\frac{\pm\pi}{3}$.

3 marks

f. By considering the solutions of the equation $\sin(5\theta)=0$, show that $\sin\frac{2\pi}{5}=\sqrt{\frac{5+\sqrt{5}}{8}}$.

3 marks

Total 13 marks

SECTION 2 – continued

TURN OVER

Question 2

Consider the function, f with rule $f(x) = \dfrac{x^2 - 2}{2x^2 - 1}$.

a. For the curve $y = f(x)$, find the exact coordinates of any axes intercepts.

2 marks

b. Express f in the form $f(x) = A + \dfrac{B}{2x^2 - 1}$, where A and B are constants.

Hence state the equations of any asymptotes.

2 marks

SECTION 2 – continued

c. Use calculus techniques to locate and classify any stationary points.

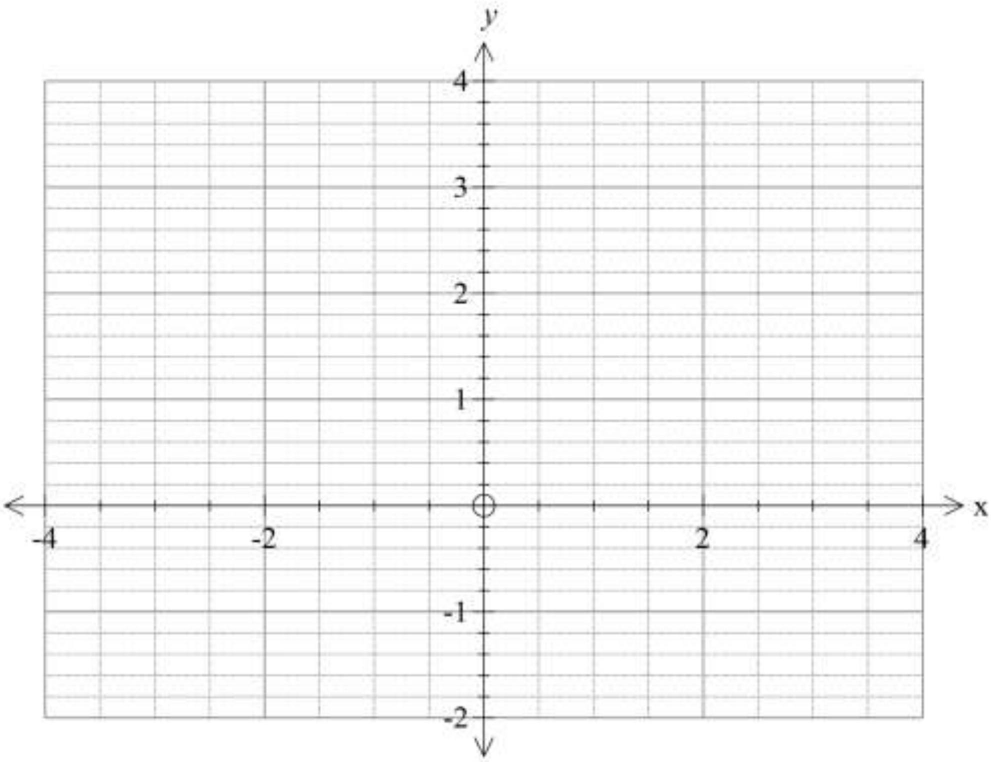
3 marks

d. Verify that the curve $y = f\left(x\right)$ has no points of inflection.

2 marks

SECTION 2 – continued
TURN OVER

e. On the set of axes below, sketch the graph of $y = f(x)$, showing any asymptotes and the coordinates of any axes intercepts.



2 marks

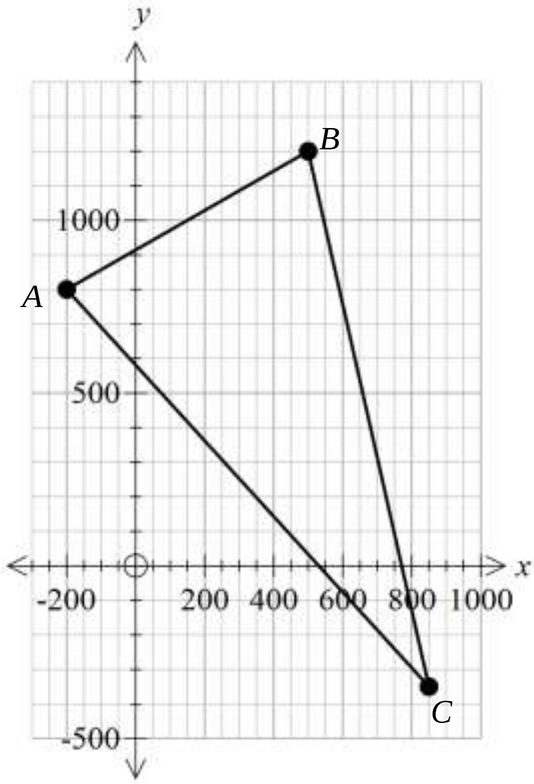
Consider the function g with rule $g(x) = 2 + \log_e(x+3)$.

f. Find, correct to two decimal places, the volume of the solid of revolution formed when the area enclosed by the curves $y = f(x)$ and $y = g(x)$ is rotated about the x-axis.

3 marks
Total 14 marks
SECTION 2 – continued

Question 3

The vertices of a large triangular paddock, ABC relative to a notable landmark have position vectors $-200\underset{\sim}{i}+800\underset{\sim}{j}$, $500\underset{\sim}{i}+1200\underset{\sim}{j}$ and $850\underset{\sim}{i}-350\underset{\sim}{j}$ respectively, as indicated below. The vectors $\underset{\sim}{i}$ and $\underset{\sim}{j}$ represent unit vectors in the easterly and northerly directions respectively, and measurements are in metres.



a. Define vectors $\overrightarrow{AB}$, $\overrightarrow{BC}$ and $\overrightarrow{AC}$ in terms of $\underset{\sim}{i}$ and $\underset{\sim}{j}$.

2 marks

SECTION 2 – continued
TURN OVER

b. Find the perimeter of the paddock, correct to the nearest metre.

2 marks

c. Find $\angle BAC$ correct to two decimal places.
Hence find the area of the paddock in hectares, correct to one decimal place.
(Note: $1 \text{ ha} = 10\ 000 \text{ m}^2$).

3 marks

d. Use vector methods to find the minimum distance from B to $[AC]$, correct to two decimal places.

3 marks

SECTION 2 – continued

e. Hence find the area of the paddock in hectares, correct to one decimal place and **confirm** your result in part **c.** above.

1 mark

f. A fence is to be constructed, which will extend from a point D along boundary $[AC]$ to a point E along boundary $[BC]$. Point D divides $[AC]$ in the ratio $2:1$ and the point E divides $[BC]$ in the ratio $2:1$.

Use vector methods to show that the fence $[DE]$ is parallel to the boundary $[AB]$.

3 marks

Total 14 marks

SECTION 2 – continued

TURN OVER

Question 4

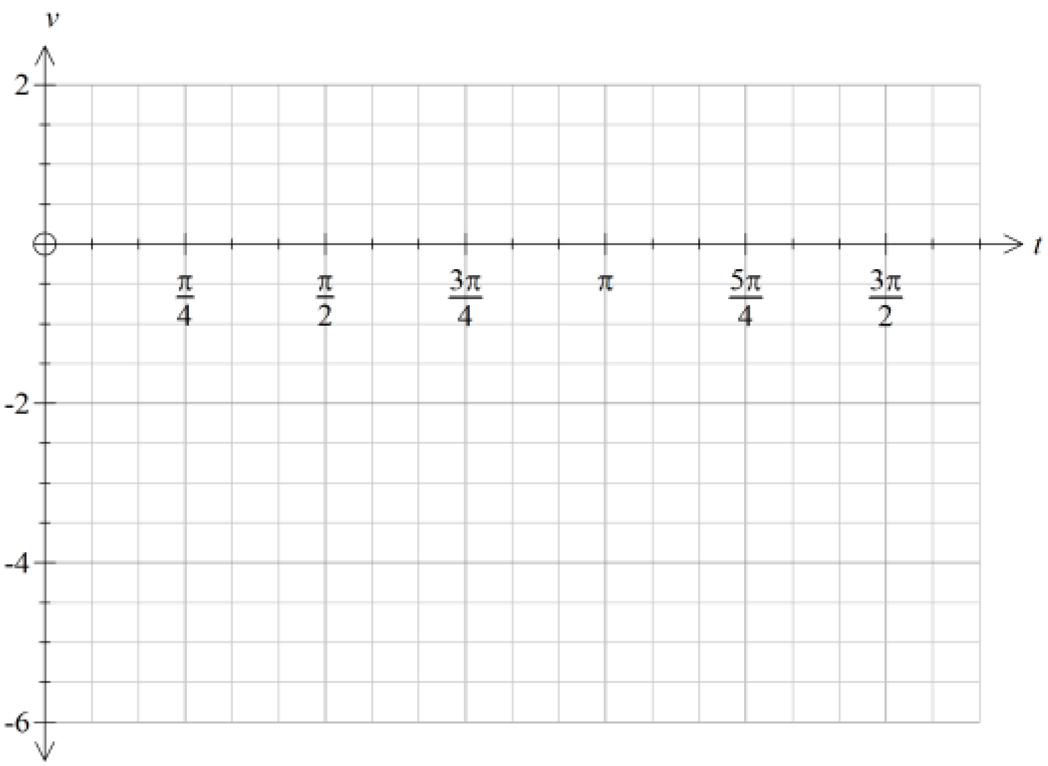
A particle is moving in a straight line with velocity v ms^{-1}. At any time, t seconds the velocity of the particle is given by the differential equation $2\frac{dv}{dt} + v^2 + 1 = 0$, where $0 \le t < a$.

a. If $v = 1$ when $t = 0$, use calculus methods to show that $v = \tan\left(\frac{\pi}{4} - \frac{t}{2}\right)$.

3 marks

SECTION 2 – continued

b. If $t = a$ is an asymptote, where $\pi \le a \le 2\pi$, find a.
Hence sketch a graph of v against t on the axes provided for $0 \le t < a$. Clearly show the coordinates of any intercepts, and the equation of the asymptote $t = a$.



3 marks

c. Find, correct to two decimal places, the distance travelled by the particle in the first $\frac{5\pi}{4}$ seconds.

1 mark

SECTION 2 – continued

TURN OVER

d. Find the time taken, correct to two decimal places, for the particle to travel a distance of 0.75 metres.

2 marks

e. Given that $x = 0$ when $t = 0$, use calculus methods to find an expression for x, the displacement of the particle, in terms of t.

4 marks

SECTION 2 – continued

f. Find, correct to two decimal places, the position of the particle after $\frac{5\pi}{4}$ seconds.

1 mark

g. Show that the position, x in terms of v is $x = \log_e\left(\frac{2}{v^2+1}\right)$.

3 marks

Total 17 marks

END OF QUESTION AND ANSWER BOOK

Multiple choice answer sheet

Instructions: **<u>Circle</u> the correct response**

1.	**A**	**B**	**C**	**D**	**E**
2.	**A**	**B**	**C**	**D**	**E**
3.	**A**	**B**	**C**	**D**	**E**
4.	**A**	**B**	**C**	**D**	**E**
5.	**A**	**B**	**C**	**D**	**E**
6.	**A**	**B**	**C**	**D**	**E**
7.	**A**	**B**	**C**	**D**	**E**
8.	**A**	**B**	**C**	**D**	**E**
9.	**A**	**B**	**C**	**D**	**E**
10.	**A**	**B**	**C**	**D**	**E**
11.	**A**	**B**	**C**	**D**	**E**
12.	**A**	**B**	**C**	**D**	**E**
13.	**A**	**B**	**C**	**D**	**E**
14.	**A**	**B**	**C**	**D**	**E**
15.	**A**	**B**	**C**	**D**	**E**
16.	**A**	**B**	**C**	**D**	**E**
17.	**A**	**B**	**C**	**D**	**E**
18.	**A**	**B**	**C**	**D**	**E**
19.	**A**	**B**	**C**	**D**	**E**
20.	**A**	**B**	**C**	**D**	**E**
21.	**A**	**B**	**C**	**D**	**E**
22.	**A**	**B**	**C**	**D**	**E**