

**‘2016 Examination Package’ -
Trial Examination 3 of 5**

STUDENT NUMBER

Figures

Words

Letter

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SPECIALIST MATHEMATICS

Units 3 & 4 – Written examination 1

(TSSM’s 2013 trial exam updated for the current study design)

Reading time: 15 minutes

Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, and rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, a calculator, blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 11 pages.
- Working space is provided throughout the book.

Instructions

- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ m/s}^2$, where $g = 9.8$.

Question 1

Find $\int \frac{4x-1}{x^2+9} dx$.

3 marks

Question 2

If $2x^2 - x \sin y + y = 10$ find $\frac{dy}{dx}$ at the point $\left(1, \frac{\pi}{4}\right)$. Express your answer in the form $a\sqrt{2} + b$, where a and b are real constants.

3 marks

TURN OVER

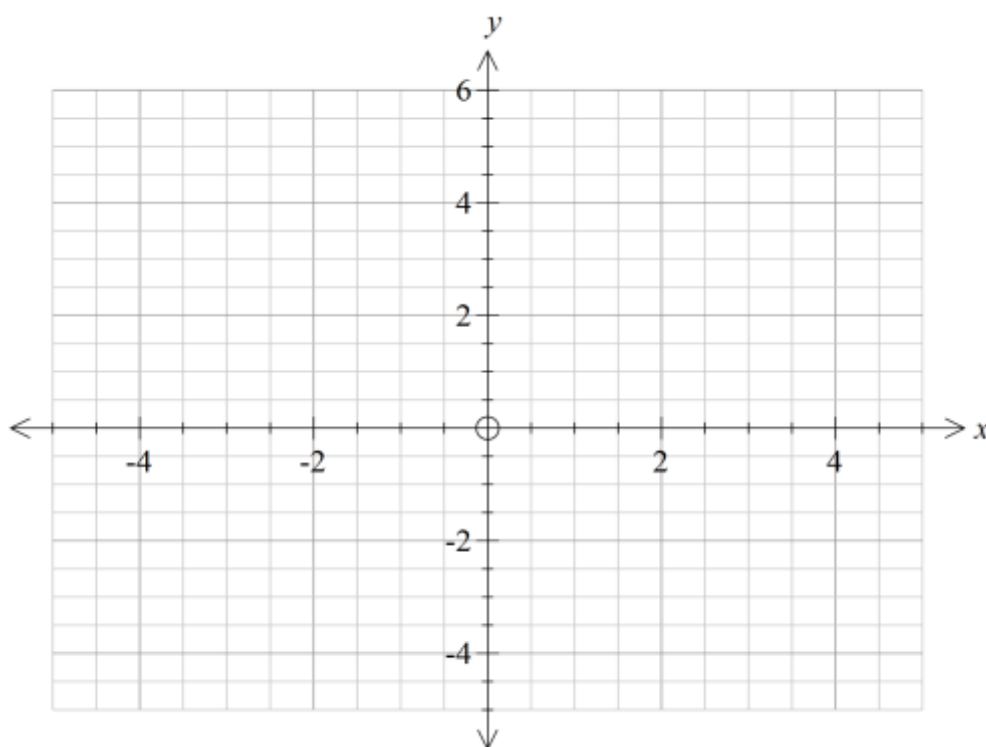
Question 3

- a. Express $f(x) = \frac{2x^2 - 2}{x^2 - 4}$ in the form $f(x) = a + \frac{b}{x^2 - 4}$, where a and b are real constants.

1 mark

- b. Sketch the graph of the relation $y = \frac{2x^2 - 2}{x^2 - 4}$ on the axes below.

Label any asymptotes with their equations and label any intercepts with the axes, writing them as coordinates.



3 marks

- c. Find the area enclosed by the graph of the relation $y = \frac{2x^2 - 2}{x^2 - 4}$ and the x -axis.

[illegible]

3 marks

Total 7 marks

Question 4

Express $z = \left(\frac{-2-2i}{\sqrt{3}-i} \right)^3$, $z \in C$, in Cartesian form.

[illegible]

3 marks

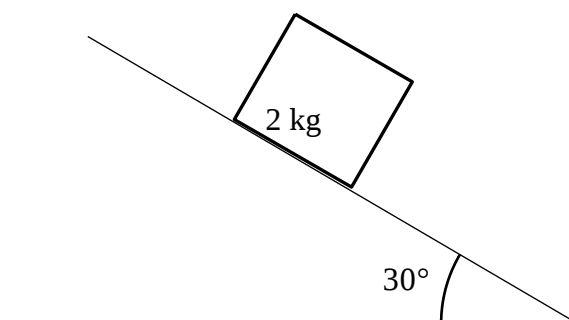
TURN OVER

b. Hence find $\int_0^1 \tan^{-1} x \, dx$.

[illegible]

Total 4 marks

A carton of mass 2 kg is held at rest on a frictionless plane inclined at 30° to the horizontal as shown. After the carton is released from rest, it begins to slide down the plane. Find the speed of the carton after it has travelled a distance of two metres, .

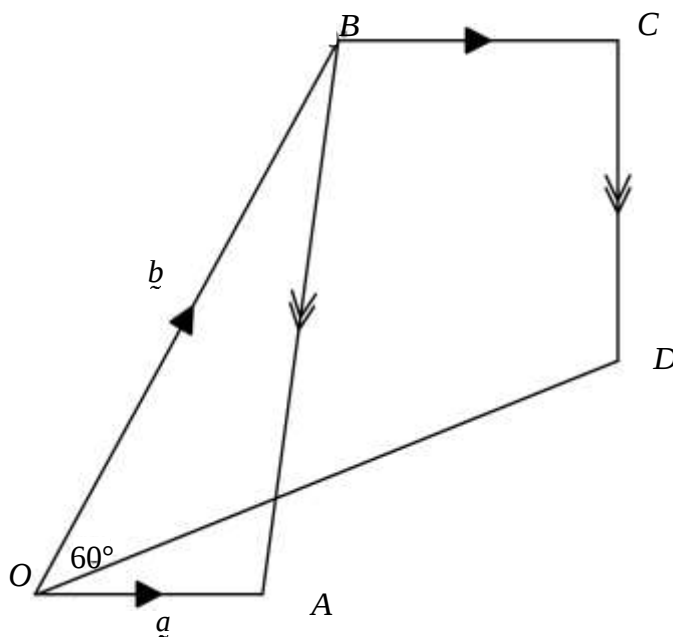


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3 marks

Question 8

In the figure below, $|\overrightarrow{OB}| = 3|\overrightarrow{OA}|$, $\overrightarrow{BC} = \overrightarrow{OA}$, $\overrightarrow{CD}$ is parallel to $\overrightarrow{BA}$ and angle AOB is 60° . The position vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are defined as $\underline{a}$ and $\underline{b}$ respectively, and $\overrightarrow{CD} = m(\overrightarrow{BA})$, where m is a scalar.



- a.** Show that the scalar product, $\underline{a} \cdot \underline{b} = \frac{3}{2} |\underline{a}|^2$.

1 mark

- b.** Express vectors $\overrightarrow{BA}$ and $\overrightarrow{OD}$ in terms of $\underline{a}$, $\underline{b}$ and the scalar m .

2 marks

- c. If $\overrightarrow{OD}$ and $\overrightarrow{BA}$ are **perpendicular** vectors, show that $m = \frac{8}{7}$.

2 marks

Total 5 marks

TURN OVER

Question 9

The region in the first quadrant enclosed by the curve $y = \cos^{-1}(2x)$, the line $y = \frac{\pi}{4}$ and the x - and y -axes is rotated about the y -axis.

Find the volume of the resulting solid of revolution.

[illegible]

3 marks

Question 10

The vector equation of a particle at time, t is defined as

$$\tilde{r}(t) = (2 \sec t + 1) \tilde{i} + (3 \tan t - 2) \tilde{j}, \text{ where } 0 \leq t < \frac{\pi}{2}$$

- a.** Find the Cartesian equation which describes the path of the particle. State the domain and range.

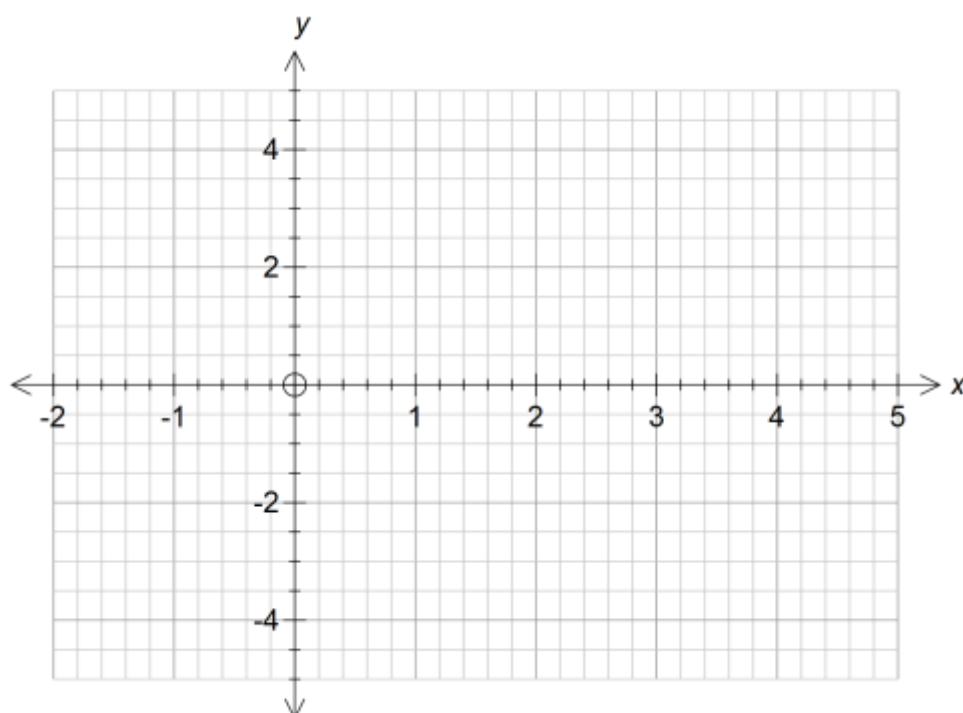
[illegible]

3 marks

b. Find the equations of any asymptotes.

1 mark

c. Hence sketch the path of the particle on the axes below. Indicate with an arrow, the direction of motion.



2 marks
Total 6 marks

END OF QUESTION AND ANSWER BOOK