



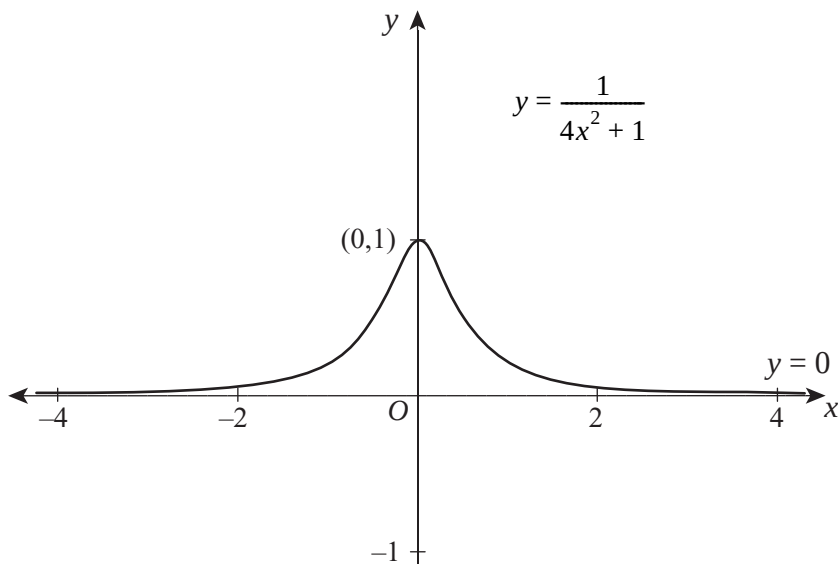
Trial Examination 2013

VCE Specialist Mathematics Units 3 & 4

Written Examination 1

Suggested Solutions

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Question 1 (2 marks)

correct shape

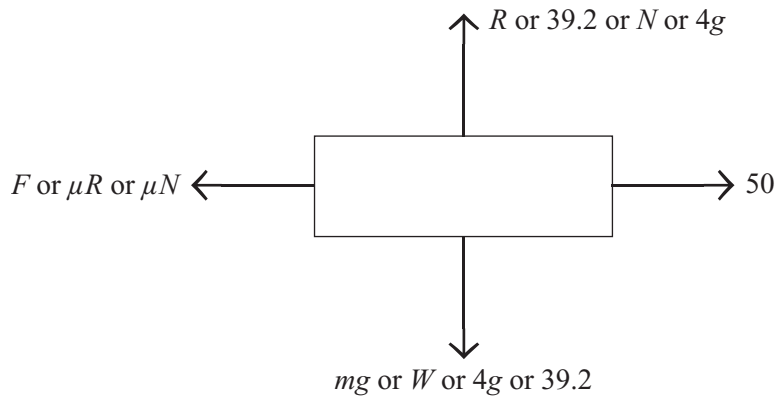
A1

horizontal asymptote $y = 0$; y-axis intercept $(0, 1)$

A1

Question 2 (4 marks)

a.



A1

b. 39.2 (newtons) or $4g$ (newtons)

A1

c. $50 - F = 4 \times 2$

M1

 $F = 42$ (newtons)

A1

Question 3 (4 marks)

$$g'(x) = (2x - x^2)e^{-x}$$

M1

$$g''(x) = (x^2 - 4x + 2)e^{-x}$$

M1 A1

Solving $g''(x) = 0$ for x gives $x = 2 \pm \sqrt{2}$.

A1

Question 4 (4 marks)

Let $u = 1 - \cos(x)$ and so $\frac{du}{dx} = \sin(x)$. M1

$$\left[\log_e(1 - \cos(x)) \right]_{\frac{\pi}{2}}^k = \frac{1}{2} \quad \left(\log_e(1 - \cos(x)) > 0 \text{ for } \frac{\pi}{2} < k < \pi \right) \quad \text{A1}$$

$$1 - \cos(k) = e^{\frac{1}{2}} \quad \text{A1}$$

$$k = \cos^{-1}(1 - \sqrt{e}) \quad \text{A1}$$

Question 5 (4 marks)

Use of $\cot^2(x) = \operatorname{cosec}^2(x) - 1$ to obtain $3\operatorname{cosec}^2(x) + 5\operatorname{cosec}(x) - 2 = 0$. M1

Attempting to factorise or using the quadratic formula. M1

$$\operatorname{cosec}(x) = \frac{1}{3} \text{ or } \operatorname{cosec}(x) = -2 \quad \text{A1}$$

$$\sin(x) = -\frac{1}{2} \Rightarrow x = \frac{7\pi}{6} \quad \text{A1}$$

Note: Award marks as above if a substitution, e.g. $u = \operatorname{cosec}(x)$, is used.

Question 6 (4 marks)

$$(x + yi)^2 = 5 - 12i$$

$$x^2 - y^2 + 2xyi = 5 - 12i \quad \text{M1}$$

Equating real and imaginary parts we obtain:

$$x^2 - y^2 = 5 \quad (1)$$

$$xy = -6 \quad (2) \text{ (or equivalent)}$$

A1

Note: Award A1 for two correct equations.

Solving (1) and (2) for x and y we obtain $x = \pm 3, y = \mp 2$. M1

So $z = 3 - 2i$ and $z = -3 + 2i$. A1

Question 7 (3 marks)

$$\overrightarrow{BA} = \underline{a} - \underline{b} \text{ and } \overrightarrow{CB} = \underline{a} + \underline{b} \quad \text{A1}$$

$$\begin{aligned} \overrightarrow{BA} \cdot \overrightarrow{CB} &= (\underline{a} - \underline{b}) \cdot (\underline{a} + \underline{b}) \\ &= |\underline{a}|^2 - |\underline{b}|^2 \end{aligned} \quad \text{A1}$$

$$\overrightarrow{BA} \cdot \overrightarrow{CB} = 0 \text{ since } |\underline{a}| = |\underline{b}| \text{ and } |\underline{a}|, |\underline{b}| \neq 0. \quad \text{A1}$$

Hence $\angle ABC$ is a right angle.

Question 8 (6 marks)

- a. Substituting $x = k$ and $y = 1$ into $x^3y - \sin\left(\frac{\pi y}{2}\right) = 7$ M1

$$k^3 - \sin\left(\frac{\pi}{2}\right) = 7$$

$$k^3 = 8 \quad \left(\text{as } \sin\left(\frac{\pi}{2}\right) = 1\right)$$

$$k = 2 \quad \text{A1}$$

- b. Using implicit differentiation to differentiate $x^3y - \sin\left(\frac{\pi y}{2}\right) = 7$. M1

$$3x^2y + x^3\frac{dy}{dx} - \frac{\pi}{2}\cos\left(\frac{\pi y}{2}\right)\frac{dy}{dx} = 0 \quad \text{A1}$$

$$\text{At } (2,1), 12 + 8\frac{dy}{dx} = 0 \quad \text{M1}$$

$$\frac{dy}{dx} = -\frac{3}{2} \quad \text{A1}$$

Question 9 (4 marks)

$$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = \frac{2x}{x^2+1} \text{ and so } \int \frac{d}{dx}\left(\frac{1}{2}v^2\right) = \int \frac{2x}{x^2+1} dx$$

$$\frac{1}{2}v^2 = \int \frac{2x}{x^2+1} dx \quad \text{A1}$$

$$v^2 = 2\log_e(x^2+1) + c \text{ where } c \text{ is a constant of integration.} \quad \text{M1}$$

$$\text{When } x = 0, v = 1. \text{ Hence } c = 1. \quad \text{M1}$$

$$v^2 = 2\log_e(x^2+1) + 1$$

$$v = \sqrt{2\log_e(x^2+1) + 1} \quad (\text{as } v = 1 \text{ when } x = 0) \quad \text{A1}$$

Question 10 (5 marks)

a. Given $\frac{(z+i)^n}{z^n} = 1$.

$$\left(\frac{z+i}{z}\right)^n = 1 \Rightarrow \left(1 + \frac{i}{z}\right)^n = 1$$

$$1 + \frac{i}{z} = \text{cis}\left(\frac{0+2k\pi}{n}\right), k = 1, 2, \dots, n-1$$

M1

$$\frac{i}{z} = \text{cis}\left(\frac{2k\pi}{n}\right) - 1$$

$$z = \frac{i}{\text{cis}\left(\frac{2k\pi}{n}\right) - 1}$$

A1

b. $z = \frac{i}{\text{cis}\left(\frac{2k\pi}{n}\right) - 1}$

$$= \frac{i \text{cis}\left(-\frac{k\pi}{n}\right)}{\text{cis}\left(\frac{k\pi}{n}\right) - \text{cis}\left(-\frac{k\pi}{n}\right)}$$

$$\text{(multiplying by } \frac{\text{cis}\left(-\frac{k\pi}{n}\right)}{\text{cis}\left(-\frac{k\pi}{n}\right)} \text{)}$$

M1

$$= \frac{i\left(\cos\left(\frac{k\pi}{n}\right) - i \sin\left(\frac{k\pi}{n}\right)\right)}{2i \sin\left(\frac{k\pi}{n}\right)}$$

A1

$$= \frac{1}{2} \left(\frac{\cos\left(\frac{k\pi}{n}\right)}{\sin\left(\frac{k\pi}{n}\right)} - \frac{i \sin\left(\frac{k\pi}{n}\right)}{\sin\left(\frac{k\pi}{n}\right)} \right)$$

$$\text{So } z = \frac{1}{2} \left(\cot\left(\frac{k\pi}{n}\right) - i \right).$$

A1