

**The Mathematical Association of Victoria**

**Trial Exam 2013**

**SPECIALIST MATHEMATICS**

**Written Examination 1**

**STUDENT NAME** \_\_\_\_\_

**Reading time: 15 minutes**  
**Writing time: 1 hour**

**QUESTION AND ANSWER BOOK**

**Structure of Book**

| <i>Number of questions</i> | <i>Number of questions to be answered</i> | <i>Number of marks</i> |
|----------------------------|-------------------------------------------|------------------------|
| 10                         | 10                                        | 40                     |

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers,
- Students are NOT permitted to bring into the examination room: notes of any kind, a calculator of any type, blank sheets of paper and/or white out liquid/tape.

**Materials supplied**

- Question and answer book of 13 pages with a detachable sheet of miscellaneous formulas at the back.

**Instructions**

- Detach the formula sheet from the back of this book during reading time.
- Write your name in the space provided above on this page.
- All written responses must be in English.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**

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## Instructions

Answer **all** questions in the spaces provided.

Unless otherwise specified an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude  $g \text{ m/s}^2$ , where  $g = 9.8$

### Question 1 (2 marks)

A particular rope will break if its tension exceeds 200g newtons.

Find in terms of  $g$  the greatest acceleration an 80 kg mass can be given when pulled vertically upwards by this rope.

[illegible]

The angle between the two vectors

is  $\frac{2\pi}{3}$ . Find the value of  $a$ .

[illegible]

**Question 3 (5 marks)**

Consider the hyperbola  $\frac{(y-1)^2}{2} - \frac{(x+2)^2}{k} = 1$  where  $k$  is a positive real number.

- a.** Find in terms of  $x, y$  and  $k$  an expression for the gradient at any point on the hyperbola.

2 marks

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- b.** The line  $7y - 3x = -11$  is normal to the hyperbola at a point where  $x = -1$ .  
Find the value of  $k$  in simplest form.

3 marks

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Relative to an origin  $O$ , an object has an acceleration vector given by

At  $t = 0$  the velocity of the object is  $\underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k}$ .

[illegible]

Consider the graph with rule  $\left| \frac{z+1-2i}{z+2-i} \right| = 1$  where  $z \in \mathbb{C}$ .

Write this rule in simplest cartesian form.

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

**a.** Express  $\frac{1}{x^3 - 2x^2 + x}$  in partial fraction form.

2 marks

[illegible]

3 marks

[illegible]

Consider the function  $f:[0,\pi)\rightarrow R$ ,  $f(x)=\tan\left(\frac{x}{2}\right)$ .

The region enclosed by the graph of  $y = f(x)$ , the  $x$ -axis and the vertical line through the point with  $y$ -coordinate  $y = \frac{1}{\sqrt{3}}$  is rotated about the  $x$ -axis to form a solid of revolution.

Find the volume of this solid in the form  $\frac{a\sqrt{b}\pi + c\pi^2}{b}$ , where  $a$ ,  $b$  and  $c$  are integers.

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**Question 8 (4 marks)**

The displacement,  $x$  m, of a body from a fixed point  $O$  after  $t$  seconds is given by

$$x = v - v^2$$

where  $v$  m/s is the velocity of the body. At  $t = 0$ ,  $v = 1$  and  $x = 0$ .

- a.** Find an expression for the acceleration of the body in terms of  $v$ . 2 marks

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- b.** Find the time at which  $v = \frac{1}{2}$ . 2 marks

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Find all solutions to  $1 + \cos(2\theta) = \sqrt{3} \sin(2\theta)$  over the domain  $-\pi \leq \theta \leq \pi$ .

[illegible]

**a. i.** Write  $1 - i\sqrt{3}$  in polar form.

1 mark

[illegible]

3 marks

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- [illegible]

The Mathematical Association of Victoria, 2013