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Specialist Mathematics

2013

Trial Examination 2

SECTION 1 Multiple-choice questions

Instructions for Section 1

Answer **all** questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ ms}^{-2}$, where $g = 9.8$.

Question 1 Given $|z| = 5$, $|z - 4 + 4i| = 1$ and $\text{Arg}(z) = \theta$, $\tan \theta =$

A. -1

B. $-\frac{4}{3}$

C. $\frac{3}{4}$

D. 1

E. $\frac{4}{3}$

Question 2 Given $i^7 z = \frac{\pi}{2} \text{cis}\left(-\frac{2\pi}{3}\right)$, $\text{Arg}(z) =$

A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. $\frac{5\pi}{6}$

D. $-\frac{\pi}{3}$

E. $-\frac{\pi}{6}$

Question 3 Given $a, b, c \in \mathbb{R}^+$, and $z \in \mathbb{C}$ such that $2\text{Re}(z) = \text{Im}(z)$, the maximum number of z satisfying the equation $a|z|^2 + b|z| - c = 0$ is

A. 0

B. 1

C. 2

D. 3

E. 4

Question 4 Given $z = r\text{cis}\theta$ and $w = z - r$, where $r \in R^+$, then $|w| =$

- A. $r \cos \theta$
- B. $r \sin \theta$
- C. $2r \cos \frac{\theta}{2}$
- D. $2r \sin \frac{\theta}{2}$
- E. $2r$

Question 5 The asymptotes of a hyperbola make a 60° angle. A possible equation of the hyperbola is

- A. $\frac{(x+2)^2}{4} - \frac{(y+6)^2}{2} = 1$
- B. $\frac{x^2}{3} - (y-2)^2 = 1$
- C. $\frac{(x-1)^2}{6} - \frac{(y-2)^2}{2} = 1$
- D. $\frac{(x+2)^2}{4} - \frac{(y-2)^2}{9} = 1$
- E. $(x-2)^2 - \frac{(y+2)^2}{9} = 1$

Question 6 The graph of $y = \frac{1}{x^2 - px + q}$, where $p, q \in R \setminus \{0\}$, has a turning point when

- A. $p^2 > 4q$ only
- B. $p^2 < 4q$ only
- C. $p^2 = 4q$
- D. $p^2 \neq 4q$
- E. $p^2 \neq 2q$

Question 7 A sequence of transformations changing the ellipse $\frac{(x-2)^2}{4} + 4y^2 = 1$ to the circle of radius 1 unit centred at the origin O is

- A. Dilate from the y -axis by a factor of 2 and from the x -axis by a factor of 0.5, and then translate in the negative x direction by 1 unit
- B. Dilate from the x -axis by a factor of 2 and from the y -axis by a factor of 0.5, and then translate in the positive x direction by 1 unit
- C. Translate in the negative x direction by 2 units, and then dilate from the y -axis by a factor of 0.5 and from the x -axis by a factor of 2
- D. Translate in the positive x direction by 2 units, and then dilate from the y -axis by a factor of 2 and from the x -axis by a factor of 0.5
- E. Translate in the negative x direction by 2 units, and then dilate from the y -axis by a factor of 2 and from the x -axis by a factor of 0.5

Question 8 Given $\sec(a+b) + \operatorname{cosec}(a-b) = 0$, a possible value for b is

- A. $\frac{a}{2}$
- B. $-\frac{\pi}{3}$
- C. $\frac{\pi}{2}$
- D. $-\frac{\pi}{6}$
- E. $\frac{\pi}{4}$

Question 9 Given $f : \left(0, \frac{1}{a}\right] \rightarrow \mathbb{R}$, $f(x) = \frac{2}{\pi} \cos^{-1}\left(ax - \frac{1}{2}\right)$ and $a \in \mathbb{R}^+$, the domain of f^{-1} is

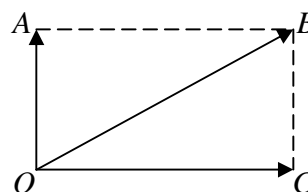
- A. $\left(-\frac{2}{3}, \frac{2}{3}\right]$
- B. $\left[\frac{2}{3}, \frac{4}{3}\right)$
- C. $\left(-\frac{4}{3}, \frac{4}{3}\right]$
- D. $\left[a - \frac{2}{3}, a + \frac{2}{3}\right)$
- E. $\left(a + \frac{2}{3}, a + \frac{4}{3}\right]$

Question 10 The equation $b \tan^{-1}\left(\frac{x-c}{a}\right) - a \tan\left(\frac{x-c}{b}\right) = 0$, where a, b and $c \in \mathbb{R}^+$, and $-\frac{\pi}{2} < \frac{x-c}{b} < \frac{\pi}{2}$, has **more than one** solution when

- A. $a = b$
- B. $ab = 1$
- C. $a > b$
- D. $a < b$
- E. $a + b > 2$

Question 11 $\vec{OA}$, $\vec{OB}$ and $\vec{OC}$ are **position vectors** of points A, B and C respectively, and $OABC$ is a **rectangle**. Which one of the following statements is **undefined** in kinematics?

- A. $\vec{OA} + \vec{OC} = \vec{OB}$
- B. $\vec{OA} + \vec{AB} = \vec{OB}$
- C. $\vec{AB} = \vec{OB} - \vec{OA}$
- D. $\vec{OA} \bullet \vec{OC} = 0$
- E. $|\vec{OA}|^2 + |\vec{OC}|^2 = |\vec{OB}|^2$



Question 12 Vectors $3\tilde{k} - a\tilde{i}$, $\tilde{i} - b\tilde{j}$ and $2\tilde{j} - c\tilde{k}$ are linearly **independent** if the product $abc \in$

- A. $\mathbb{R} \setminus \{3\}$
- B. $\mathbb{R} \setminus \{-4, 2\}$
- C. $\mathbb{R} \setminus \{6\}$
- D. $\mathbb{R} \setminus \{-3\}$
- E. $\mathbb{R} \setminus \{2\}$

Question 13 $\tilde{a}$ and $\tilde{b}$ are any two non-parallel vectors of the same magnitude. The scalar resolute of $\tilde{a}$ in the direction of $\tilde{a} + \tilde{b}$ is

- A. $|\tilde{a}| + \frac{\tilde{a} \cdot \tilde{b}}{|\tilde{a}|}$
- B. $\frac{1}{2}$
- C. $|\tilde{b}| + \frac{\tilde{a} \cdot \tilde{b}}{|\tilde{b}|}$
- D. $\frac{1}{\sqrt{2}}$
- E. $\frac{1}{2}|\tilde{a} + \tilde{b}|$

Question 14 In terms of a the exact value of the definite integral $\int_{-\frac{a}{2}}^{\frac{a}{2}} \sqrt{a^2 - t^2} dt$ is

- A. $\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right)a$
- B. $\left(\frac{\pi}{6} + \frac{\sqrt{3}}{4}\right)a$
- C. $\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right)a^2$
- D. $\left(\frac{\pi}{6} + \frac{\sqrt{3}}{4}\right)a^2$
- E. $\frac{24a^2}{25}$

Question 15 Consider $y = (\tan^{-1} x)^2$, $x \in R$. The area of the region enclosed by the line $y = \frac{\pi^2}{16}$ and the curve $y = (\tan^{-1} x)^2$ is closest to

- A. 0.25
- B. 0.35
- C. 0.5
- D. 0.75
- E. 1

Question 16 The differential equation $(\tan^{-1} x)^2 \frac{dy}{dx} - \frac{1}{1+x^2} = 0$ is equivalent to

- A. $t^2 \frac{dy}{dt} - 1 = 0$ where $t = \tan^{-1} x$
- B. $t^2 \frac{dy}{dt} - \frac{1}{1+t^2} = 0$ where $t = \tan^{-1} x$
- C. $\frac{dy}{dt} - \frac{1}{1+t^2} = 0$ where $t = \tan^{-1} x$
- D. $\frac{dy}{dt} - t^2 = 0$ where $t = \tan^{-1} x$
- E. $\frac{dy}{dt} - \frac{t}{1+t^2} = 0$ where $t = \tan^{-1} x$

Question 17 The graph of $y = (x^2 - 2.5x + 3.1)(x^2 + 2.5x + 3.1)$ has

- A. 3 stationary points and 2 inflection points
- B. 2 stationary points and 2 inflection points
- C. 1 stationary point and 2 inflection points
- D. 1 stationary point and 1 inflection point
- E. 1 stationary point and 0 inflection point

Question 18 The velocity of a particle at position $x \geq 0$ is given by $v = 2e^{-x} - 1$, and $x = 0$ initially. The velocity of the particle at time t is given by

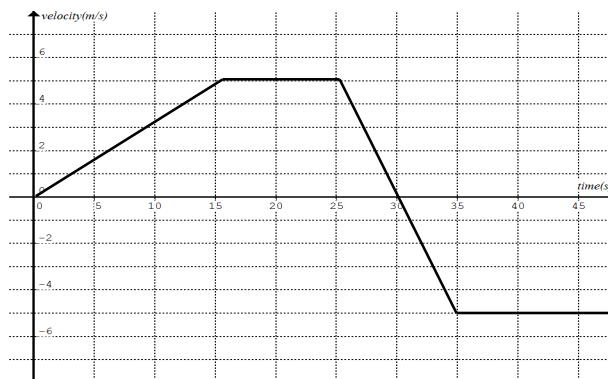
- A. $v = \frac{e^t}{2e^{-t} - 1}$
- B. $v = \frac{e^t}{2e^t - 1}$
- C. $v = \frac{1}{2e^t - 1}$
- D. $v = \frac{e^t}{2 - e^t}$
- E. $v = \frac{e^t}{2 - e^{-t}}$

Question 19 The momentum (in kg m s^{-1}) of a particle changes uniformly from $6\tilde{i} - 6\tilde{j} + 3.0\tilde{k}$ to $3\tilde{i} - 3\tilde{j} + 1.5\tilde{k}$ in 5.0 seconds. The magnitude of the resultant force on the particle is closest to

- A. 0.5 N
- B. 1.0 N
- C. 1.5 N
- D. 3.7 N
- E. 3.8 N

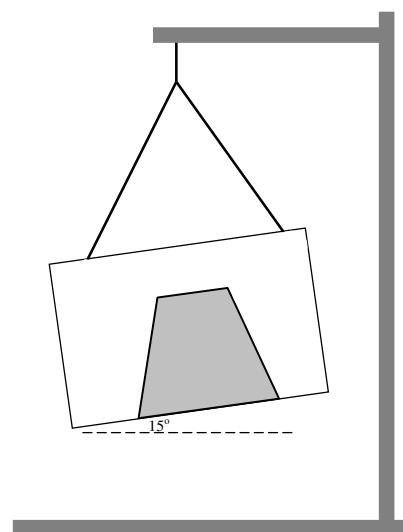
Question 20 The velocity-time graph of a particle in rectilinear motion is shown below. The average *speed* (m s^{-1}) of the particle in the first 40 seconds is closest to

- A. 3.9
- B. 3.85
- C. 3.8
- D. 3.4
- E. 2.5



Question 21 A 1500 kg machine (shaded) is placed inside a 100 kg crate. It is *lowered* by a crane with a speed increasing at a rate of 0.8 m s^{-2} . The machine does not slide when the crate is tilted at an angle of 15° while it is lowered. The reaction force of the crate on the machine is

- A. 12500 N upward and perpendicular to the floor of the crate
- B. 13500 N upward and perpendicular to the floor of the crate
- C. 13500 N vertically upward
- D. 14400 N upward and perpendicular to the floor of the crate
- E. 14400 N vertically upward



Question 22 A M kg crate is pushed along a *rough* horizontal floor by a horizontal force of 99 N. The acceleration of the crate is 1.00 m s^{-2} . Another M kg crate is now stacked securely on top of the first crate. The acceleration is only 0.010 m s^{-2} when the same force of 99 N is used to push the two crates.



The value of M is closest to

- A. 10
- B. 25
- C. 50
- D. 99
- E. 9900

SECTION 2 Extended-answer questions

Instructions for Section 2

Answer **all** questions.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ m s}^{-2}$, where $g = 9.8$.

Question 1 Consider the equation $\frac{z}{2} = \sqrt{a - \sqrt{a + \frac{z}{2}}}$ where $a \in R$ and $z \in C$.

a. $\frac{z}{2} = \sqrt{a - \sqrt{a + \frac{z}{2}}}$ can be expressed in the form $z^4 + lz^2 + mz + n = 0$.

Find the values of l , m and n in terms of a if necessary.

2 marks

b i. $z^4 + lz^2 + mz + n$ can be expressed in factorised form, $(z^2 + 2z + p)(z^2 + rz + q)$.

Find the values of p , q and r in terms of a if necessary.

3 marks

b ii. Hence solve $z^4 + lz^2 + mz + n = 0$ for z in terms of a .

2 marks

c i. Find the values of a such that all the solutions of $z^4 + lz^2 + mz + n = 0$ are real.

1 mark

c ii. Find the values of a such that all the solutions of $z^4 + lz^2 + mz + n = 0$ have imaginary part.

1 mark

c iii. Find the values of a such that $z^4 + lz^2 + mz + n = 0$ has real solutions and solutions with imaginary part.

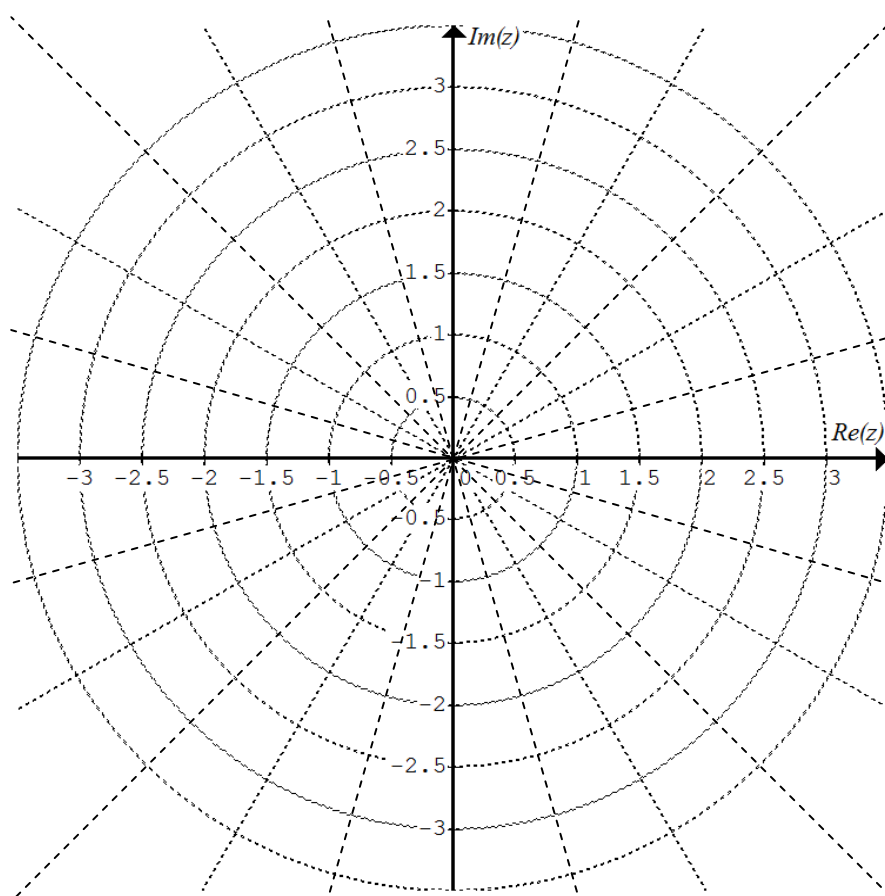
1 mark

d i. Express in polar form the solutions to $\frac{z}{2} = \sqrt{a - \sqrt{a + \frac{z}{2}}}$ for $a = \frac{1}{2}$.

2 marks

d ii. Hence plot accurately the solutions on the grid below. Label each one in polar form.

2 marks



Question 2 The position of a particle moving in the x - y plane is given by $\tilde{r}(t) = \log_e(t+1)[\cos(t)\tilde{i} + \sin(t)\tilde{j}]$ for $t \geq 0$, where $\tilde{i}$ and $\tilde{j}$ are unit vectors pointing to the east (x -direction) and north (y -direction) respectively. Distance is measured in metres, time in seconds and speed in m s^{-1} .

a i. Find the initial position of the particle.

1 mark

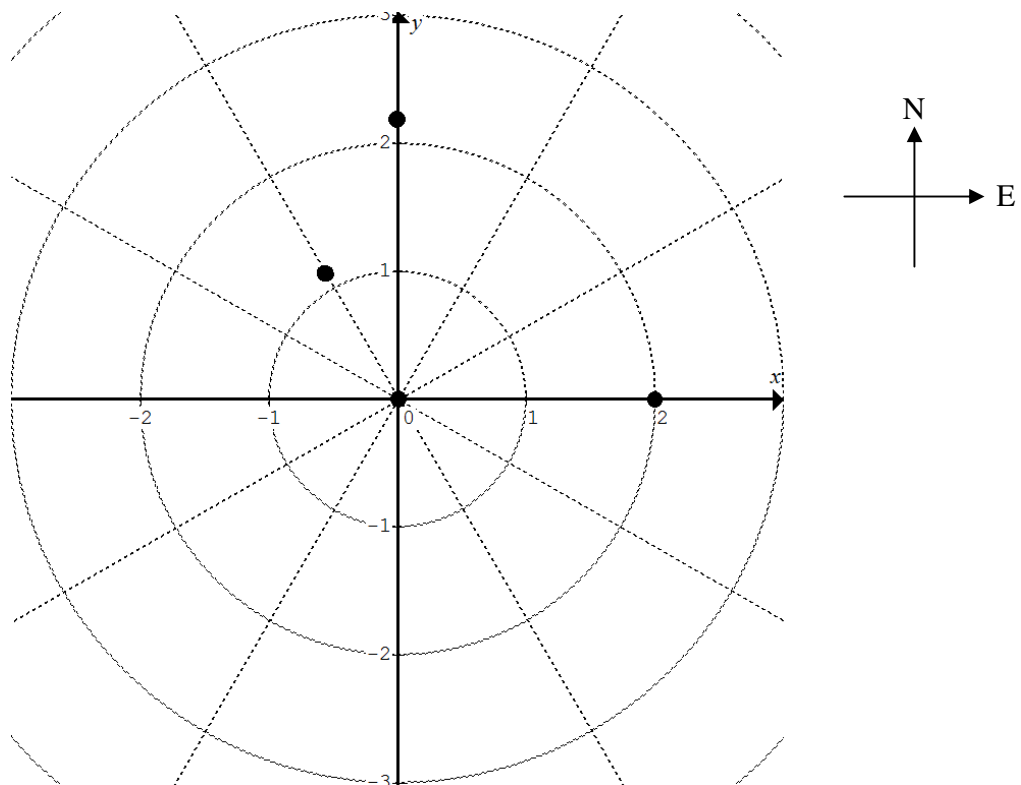
a ii. Complete the following table (correct to two decimal places).

1 mark

t	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{4\pi}{3}$	2π	$\frac{5\pi}{2}$
$ \tilde{r} $	0.00		0.72	0.94	1.13		1.99	2.18

a iii. The positions of the particle at $t = 0$, $\frac{2\pi}{3}$, 2π and $\frac{5\pi}{2}$ are plotted on the following diagram. Label each point with $t = 0$, $\frac{2\pi}{3}$, 2π or $\frac{5\pi}{2}$.

1 mark



a iv. Plot on the above diagram the positions of the particle at $t = \frac{\pi}{6}$, $\frac{\pi}{3}$, $\frac{\pi}{2}$ and $\frac{4\pi}{3}$.

2 marks

b i. Find the velocity $\tilde{v}(t)$ of the particle at time t .

2 marks

b ii. Show that the initial velocity of the particle is $\tilde{i}$.

1 mark

b iii. On the diagram in part **a iii** draw the initial velocity vector of the particle at its initial position.

1 mark

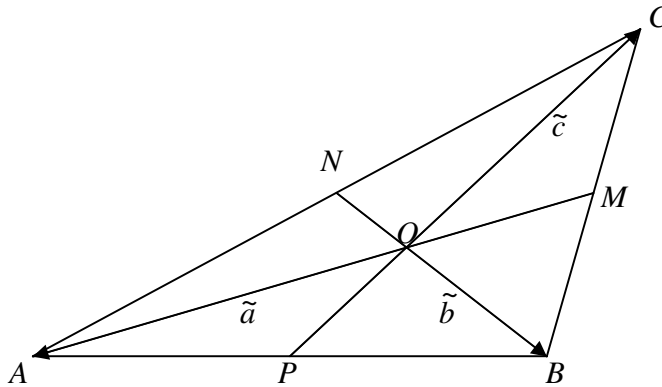
b iv. Find t (correct to two decimal places) when the particle is first heading *south*.

2 marks

b v. What is the speed (correct to two decimal places) of the particle when it is first heading south?

1 mark

Question 3 ABC is any triangle. $\overline{AM}$ and $\overline{BN}$ are *medians* of the triangle, that is M and N are midpoints of line segments $\overline{BC}$ and $\overline{CA}$ respectively. $\overline{CP}$ is a line segment passing through the intersection O of $\overline{AM}$ and $\overline{BN}$. Let $\overrightarrow{OA} = \tilde{a}$, $\overrightarrow{OB} = \tilde{b}$ and $\overrightarrow{OC} = \tilde{c}$.



- a.** Express $\overrightarrow{OM}$ and $\overrightarrow{ON}$ in terms of $\tilde{a}$, $\tilde{b}$ and $\tilde{c}$. 2 marks

Let m , n and p be some positive real numbers such that $\overrightarrow{OM} = -m\tilde{a}$, $\overrightarrow{ON} = -n\tilde{b}$ and $\overrightarrow{OP} = -p\tilde{c}$.

- b i.** Show that $2m\tilde{a} + \tilde{b} + \tilde{c} = \vec{0}$ and $\tilde{a} + 2n\tilde{b} + \tilde{c} = \vec{0}$. 2 marks

Hence show/explain parts **b ii** to **b v**.

- b ii.** $(1 - 2m)\tilde{a} - (1 - 2n)\tilde{b} = \vec{0}$ 1 mark

b iii. $m = n = \frac{1}{2}$. 1 mark

b iv. $\tilde{b} = -\tilde{a} - \tilde{c}$. 1 mark

b v. $\overrightarrow{AB} = -2\tilde{a} - \tilde{c}$. 1 mark

c i. Given $\overrightarrow{AP} = -\tilde{a} - p\tilde{c}$ and $\overrightarrow{AP} = k\overrightarrow{AB}$ where $k \in R$, show that $(2k - 1)\tilde{a} + (k - p)\tilde{c} = \tilde{0}$. 1 mark

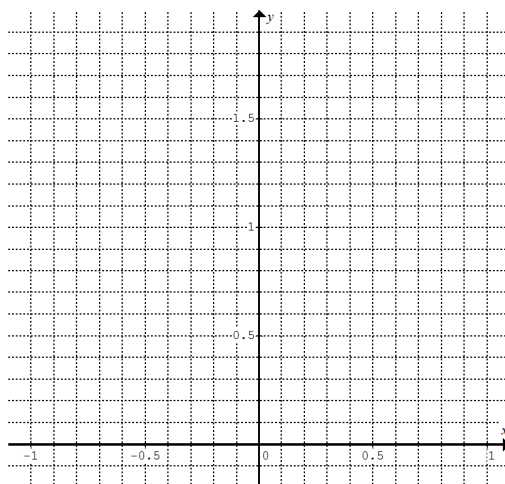
c ii. Hence show that $p = k = \frac{1}{2}$. 1 mark

c iii. Explain why line segment $\overline{CP}$ is a median of triangle ABC . 1 mark

Question 4 A top is formed by revolving $y = \sin^{-1} 2x$, $0 \leq x \leq \frac{1}{2}$, about the y-axis.

- a. Draw an accurate graph of $y = \sin^{-1} 2x$, $0 \leq x \leq \frac{1}{2}$, showing the exact coordinates of the end points.

2 marks



- b. Find the exact coordinates of the point where the gradient of the curve $y = \sin^{-1} 2x$ is 4.

2 marks

- c. Find the exact area under the curve $y = \sin^{-1} 2x$, $0 \leq x \leq \frac{1}{2}$.

3 marks

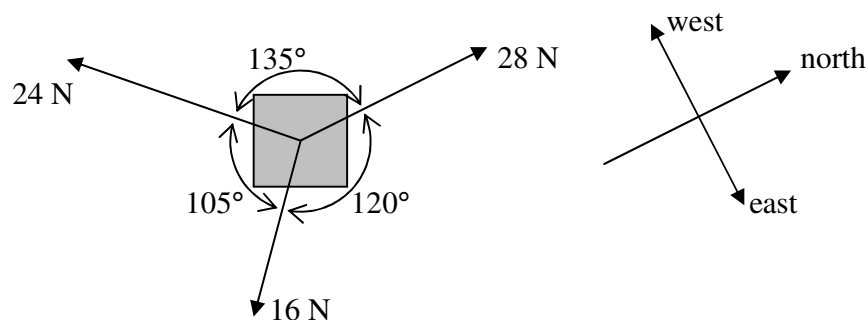
d. The top is cut from a rectangular block of wood of dimensions 1 unit $\times$ 1 unit $\times$ 2 units. Find the exact volume of wood to be removed from the block to make the top.

3 marks

e. The shape of a dowel is formed by revolving $y = \frac{1}{\sqrt{(\frac{1}{2})^2 - x^2}}$, $x \geq 0$ and $y \leq 10$, about the y -axis. Find the exact volume of the dowel.

3 marks

Question 5 A box on a rough horizontal floor is pulled by three forces as shown in the following diagram (not drawn to scale). The box is in *limiting* equilibrium. The coefficient of friction is 0.25 between the box and the floor. The 28-N force is directed towards the north.



- a.** Determine the magnitude (N) and direction (degrees) of the resultant of the three pulling forces, correct to one decimal place. 3 marks

- b.** Write down the magnitude and direction (correct to one decimal place) of the force of friction on the *floor*. 1 mark

- c.** Calculate the mass (kg) of the box, correct to one decimal place. 1 mark

- d.** Determine the magnitude (in m s^{-2} , correct to one decimal place) and direction of the acceleration of the box when the 24 N force is increased to $20\sqrt{2}$ N. 3 marks

End of Exam 2