

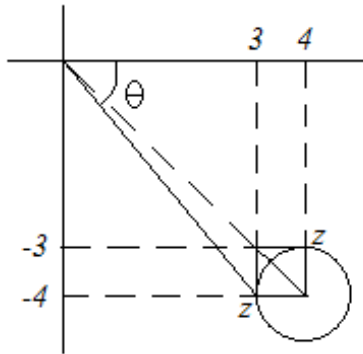
2013 Specialist Maths Trial Exam 2 Solutions
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Section 1

1	2	3	4	5	6	7	8	9	10	11
B	E	C	D	B	D	C	E	B	D	A

12	13	14	15	16	17	18	19	20	21	22
C	E	D	D	A	A	C	B	D	C	C

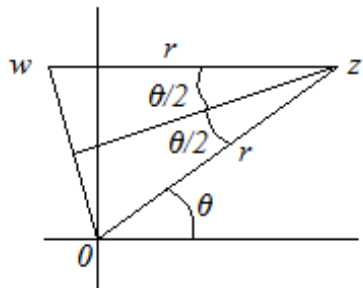
Q1 $\tan \theta = -\frac{4}{3}$, (another possible answer is $\tan \theta = -\frac{3}{4}$) B



Q2 $i^7 z = \frac{\pi}{2} \text{cis}\left(-\frac{2\pi}{3}\right)$, $i^8 z = i \frac{\pi}{2} \text{cis}\left(-\frac{2\pi}{3}\right)$,
 $z = i \frac{\pi}{2} \text{cis}\left(-\frac{2\pi}{3}\right)$, $z = \frac{\pi}{2} \text{cis}\left(-\frac{2\pi}{3} + \frac{\pi}{2}\right) = \frac{\pi}{2} \text{cis}\left(-\frac{\pi}{6}\right)$ E

Q3 $2\text{Re}(z) = \text{Im}(z)$ forms a straight line through the origin.
 $a|z|^2 + b|z| - c = 0$, $\therefore |z| = \frac{-b + \sqrt{b^2 + 4ac}}{2a}$ forms a circle centred at the origin. The two sets have two intersections. C

Q4 Refer to the diagram below. $|w| = 2r \sin \frac{\theta}{2}$ D



Q5 $\frac{b}{a} = \tan 60^\circ$, $\frac{b}{a} = \frac{\sqrt{3}}{1}$, $\frac{b^2}{a^2} = \frac{3}{1}$ B

Q6 $y = \frac{1}{x^2 - px + q}$ has a turning point when $x^2 - px + q$ is not a perfect square, i.e. $\Delta \neq 0$, $p^2 - 4q \neq 0$, $p^2 \neq 4q$ D

Q7 $\frac{(x-2)^2}{4} + 4y^2 = 1 \rightarrow \frac{x^2}{4} + 4y^2 = 1$, $\left(\frac{x}{2}\right)^2 + (2y)^2 = 1$
 $\rightarrow x^2 + y^2 = 1$ C

Q8 $\sec(a+b) = -\text{cosec}(a-b)$, $\frac{1}{\cos(a+b)} = -\frac{1}{\sin(a-b)}$,
 $\cos(a+b) = -\sin(a-b)$,
 $\therefore \sin\left(\frac{\pi}{2} - (a+b)\right) = \sin(-(a-b))$ or
 $\sin\left(\frac{\pi}{2} - (a+b)\right) = \sin(\pi - (a-b))$
 $\therefore \frac{\pi}{2} - (a+b) = -(a-b)$ or $\frac{\pi}{2} - (a+b) = \pi - (a-b)$
 $\therefore b = \frac{\pi}{4}$ or $-\frac{\pi}{4}$ E

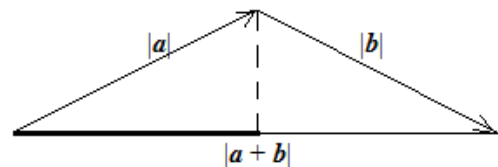
Q9 $f(x)$ is a decreasing function. $f\left(\frac{1}{a}\right) = \frac{2}{3}$, $f(0) = \frac{4}{3}$.
 The range of f is $\left[\frac{2}{3}, \frac{4}{3}\right)$ and it is the domain of f^{-1} . B

Q10 Let $\frac{x-c}{b} = \theta$ where $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$.
 $\therefore \frac{x-c}{a} = \frac{b}{a}\theta$ and the equation is $\frac{b}{a}\tan^{-1}\left(\frac{b}{a}\theta\right) - \tan \theta = 0$.
 More than one solution when $\frac{b}{a} > 1$, $\therefore a < b$ D

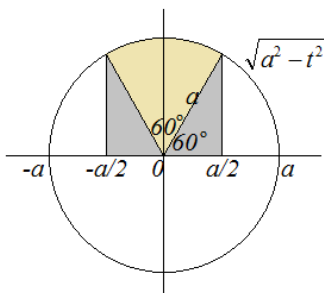
Q11 The addition of position vectors is undefined in kinematics. A

Q12 The vectors $3\tilde{k} - a\tilde{i}$, $\tilde{i} - b\tilde{j}$ and $2\tilde{j} - c\tilde{k}$ are linearly dependent if
 $3\tilde{k} - a\tilde{i} = m(\tilde{i} - b\tilde{j}) + n(2\tilde{j} - c\tilde{k}) = m\tilde{i} + (2n - bm)\tilde{j} - cn\tilde{k}$
 $\therefore m = -a$, $n = -\frac{3}{c}$ and $2n - bm = 0$,
 $\therefore -\frac{6}{c} + ab = 0$, $\therefore abc = 6$
 $\therefore$ the vectors $3\tilde{k} - a\tilde{i}$, $\tilde{i} - b\tilde{j}$ and $2\tilde{j} - c\tilde{k}$ are linearly independent if $abc \neq 6$. C

Q13 Refer to the following diagram. The scalar resolute of $\tilde{a}$ in the direction of $\tilde{a} + \tilde{b}$ is the darker line segment which is half as long as vector $\tilde{a} + \tilde{b}$, i.e. $\frac{1}{2}|\tilde{a} + \tilde{b}|$. E



Q14 $\int_{-\frac{a}{2}}^{\frac{a}{2}} \sqrt{a^2 - t^2} dt = \text{area of the shaded regions}$
 $= \text{area of the sector} + \text{total area of the 2 triangles}$
 $= \left(\frac{\pi}{6} + \frac{\sqrt{3}}{4} \right) a^2$

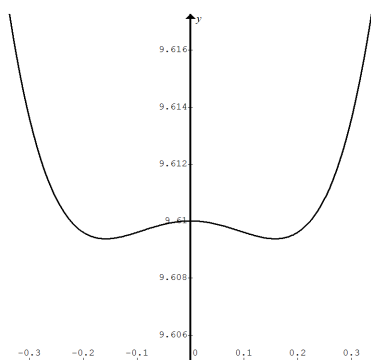


Q15 Find the intersections: $(\tan^{-1} x)^2 = \frac{\pi^2}{16}$, $x = \pm 1$

$\text{Area} = \int_{-1}^1 \left(\frac{\pi^2}{16} - (\tan^{-1} x)^2 \right) dx \approx 0.7431$ by CAS

Q16 $t = \tan^{-1} x$, $\frac{dt}{dx} = \frac{1}{1+x^2}$
 $\frac{dy}{dx} = \frac{dt}{dx} \times \frac{dy}{dt}$, $\therefore \frac{dy}{dx} = \frac{1}{1+x^2} \times \frac{dy}{dt}$
 $(\tan^{-1} x)^2 \frac{dy}{dx} - \frac{1}{1+x^2} = 0$, $t^2 \times \frac{1}{1+x^2} \times \frac{dy}{dt} - \frac{1}{1+x^2} = 0$
 $\therefore \frac{1}{1+x^2} \left(t^2 \frac{dy}{dt} - 1 \right) = 0$
 Since $\frac{1}{1+x^2} \neq 0$, $\therefore t^2 \frac{dy}{dt} - 1 = 0$

Q17



Q18 $v = \frac{dx}{dt} = 2e^{-x} - 1$, $\frac{dx}{dt} = \frac{2-e^x}{e^x}$, $\frac{dt}{dx} = \frac{e^x}{2-e^x}$
 $\therefore t = \int \frac{e^x}{2-e^x} dx = -\log_e(2-e^x)$ satisfying $x=0$ initially.
 $\therefore e^{-t} = 2-e^x$, $\therefore e^{-x} = \frac{1}{2-e^{-t}} = \frac{e^t}{2e^t-1}$
 $\therefore v = 2e^{-x} - 1 = \frac{2e^t}{2e^t-1} - 1 = \frac{1}{2e^t-1}$

Q19 $\Delta \tilde{p} = m \Delta \tilde{v}$, $\Delta \tilde{v} = \frac{\Delta \tilde{p}}{m} = \frac{-3\tilde{i} + 3\tilde{j} - 1.5\tilde{k}}{m}$
 $\tilde{a} = \tilde{a}_{\text{average}} = \frac{\Delta \tilde{v}}{\Delta t} = \frac{-3\tilde{i} + 3\tilde{j} - 1.5\tilde{k}}{5.0m} = \frac{-0.6\tilde{i} + 0.6\tilde{j} - 0.3\tilde{k}}{m}$

D $\tilde{R} = m\tilde{a} = -0.6\tilde{i} + 0.6\tilde{j} - 0.3\tilde{k}$
 $\therefore |\tilde{R}| = \sqrt{(-0.6)^2 + 0.6^2 + (-0.3)^2} = 0.9 \text{ N}$ B

Q20 Total distance travelled in the first 40 seconds

$= \frac{1}{2} \times (10+30) \times 5 + \frac{1}{2} \times (5+10) \times 5 = 137.5 \text{ m}$

Average speed $= \frac{137.5}{40} \approx 3.4 \text{ m s}^{-1}$ D

Q21 Let P be the reaction force of the crate on the machine.

$R = ma$, $1500 \times 9.8 - P = 1500 \times 0.8$
 $\therefore P = 13500 \text{ N}$ C

Q22 For one crate: $99 - \mu M \times 9.8 = M \times 1.00$ (1)

D For two crates: $99 - \mu(2M) \times 9.8 = (2M) \times 0.010$ (2)

(1) - (2): $\mu M \times 9.8 = M \times 0.98$, $\therefore \mu = 0.10$ (3)

Substitute (3) in (1): $M = 50$ C

Section 2

Q1a $\frac{z}{2} = \sqrt{a - \sqrt{a + \frac{z}{2}}}$, $\frac{z^2}{4} = a - \sqrt{a + \frac{z}{2}}$, $a - \frac{z^2}{4} = \sqrt{a + \frac{z}{2}}$
 $\left(a - \frac{z^2}{4} \right)^2 = a + \frac{z}{2}$, $a^2 - \frac{az^2}{2} + \frac{z^4}{16} = a + \frac{z}{2}$

A $\therefore \frac{z^4}{16} - \frac{az^2}{2} - \frac{z}{2} + a^2 - a = 0$

A $\therefore z^4 - 8az^2 - 8z + 16(a^2 - a) = 0$

$\therefore l = -8a$, $m = -8$ and $n = 16(a^2 - a)$

Q1bi $z^4 - 8az^2 - 8z + 16(a^2 - a) = (z^2 + 2z + p)(z^2 + rz + q)$
 $= z^4 + (r+2)z^3 + (p+q+2r)z^2 + (pr+2q)z + pq$
 $\therefore r+2=0$, $p+q+2r=-8a$, $pr+2q=-8$ and $pq=16(a^2-a)$
 $\therefore r=-2$, $p+q=4-8a$, $q-p=-4$
 $\therefore 2q=-8a$, $q=-4a$ and $p=4+q=4-4a$

Q1bii $z^4 - 8az^2 - 8z + 16(a^2 - a) = 0$

$(z^2 + 2z + (4-4a))(z^2 - 2z - 4a) = 0$

$z^2 + 2z + (4-4a) = 0$ or $z^2 - 2z - 4a = 0$

By quadratic formula: $z = -1 \pm \sqrt{4a-3}$ or $z = 1 \pm \sqrt{4a+1}$

Q1ci All real solutions: $4a-3 \geq 0$ AND $4a+1 \geq 0$

i.e. $a \geq \frac{3}{4}$ AND $a \geq -\frac{1}{4}$, $\therefore a \geq \frac{3}{4}$

C Q1cii All solutions has imaginary part:

$4a-3 < 0$ AND $4a+1 < 0$,

i.e. $a < \frac{3}{4}$ AND $a < -\frac{1}{4}$, $\therefore a < -\frac{1}{4}$

Q1cii To have both real solutions and solutions with imaginary

part: $a < \frac{3}{4}$ AND $a \geq -\frac{1}{4}$, $\therefore -\frac{1}{4} \leq a < \frac{3}{4}$

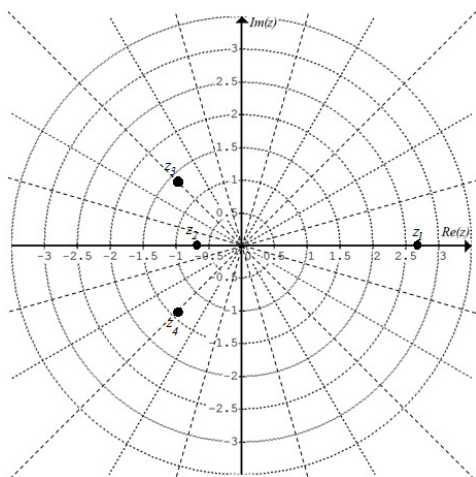
Q1di $z = -1 \pm \sqrt{4a-3}$ OR $z = 1 \pm \sqrt{4a+1}$

When $a = \frac{1}{2}$, $z = -1 \pm i = \sqrt{2} \operatorname{cis}\left(\pm \frac{3\pi}{4}\right)$

OR $z = 1 \pm \sqrt{3} = (1 + \sqrt{3})\operatorname{cis}0$ or $(\sqrt{3} - 1)\operatorname{cis}\pi$

Q1dii $z_1 = (1 + \sqrt{3})\operatorname{cis}0$, $z_2 = (\sqrt{3} - 1)\operatorname{cis}\pi$, $z_3 = \sqrt{2}\operatorname{cis}\left(\frac{3\pi}{4}\right)$,

$z_4 = \sqrt{2}\operatorname{cis}\left(-\frac{3\pi}{4}\right)$

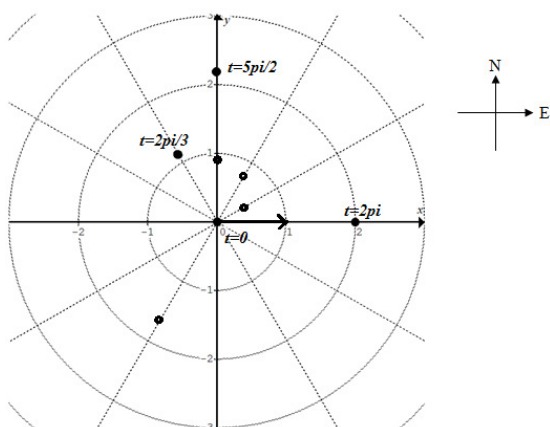


Q2ai $\tilde{r}(0) = \log_e(1)[\cos(0)\tilde{i} + \sin(0)\tilde{j}] = \tilde{0}$

Q2aii

t	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{4\pi}{3}$	2π	$\frac{5\pi}{2}$
$ \tilde{r} $	0.00	0.42	0.72	0.94	1.13	1.65	1.99	2.18

Q2aiii, Q2aiv and Q2biii



Q2bi $\tilde{r}(t) = \log_e(t+1)[\cos(t)\tilde{i} + \sin(t)\tilde{j}]$

$\tilde{v}(t) = \frac{1}{t+1}[\cos(t)\tilde{i} + \sin(t)\tilde{j}] + \log_e(t+1)[- \sin(t)\tilde{i} + \cos(t)\tilde{j}]$
 $= \left(\frac{\cos(t)}{t+1} - \log_e(t+1)\sin(t)\right)\tilde{i} + \left(\frac{\sin(t)}{t+1} + \log_e(t+1)\cos(t)\right)\tilde{j}$

Q2bii $\tilde{v}(0) = \tilde{i}$

Q2biv Let $\frac{\cos(t)}{t+1} - \log_e(t+1)\sin(t) = 0$, by CAS $t \approx 0.78$ s,
 heading north; $t = 3.30$ s, heading south.

Q2bv $\tilde{v}(3.3) \approx 0\tilde{i} - 1.48\tilde{j}$, speed $\approx 1.48 \text{ m s}^{-1}$

Q3a $\overrightarrow{OM} = \frac{1}{2}(\tilde{b} + \tilde{c})$, $\overrightarrow{ON} = \frac{1}{2}(\tilde{c} + \tilde{a})$

Q3bi

$2\overrightarrow{OM} = \tilde{b} + \tilde{c}$, $\therefore -2\overrightarrow{OM} + \tilde{b} + \tilde{c} = \tilde{0}$, $2m\tilde{a} + \tilde{b} + \tilde{c} = \tilde{0} \dots (1)$

$2\overrightarrow{ON} = \tilde{c} + \tilde{a}$, $\therefore -2\overrightarrow{ON} + \tilde{c} + \tilde{a} = \tilde{0}$, $\tilde{a} + 2n\tilde{b} + \tilde{c} = \tilde{0} \dots (2)$

Q3bii (2) - (1): $\tilde{a} - 2m\tilde{a} - \tilde{b} + 2n\tilde{b} = \tilde{0}$

$(1 - 2m)\tilde{a} - (1 - 2n)\tilde{b} = \tilde{0}$

Q3biii Since $\tilde{a}$ and $\tilde{b}$ are independent (non-parallel),

$\therefore 1 - 2m = 0$ and $1 - 2n = 0$

$\therefore m = \frac{1}{2}$ and $n = \frac{1}{2}$

Q3biv Since $2m\tilde{a} + \tilde{b} + \tilde{c} = \tilde{0}$, $\therefore \tilde{a} + \tilde{b} + \tilde{c} = \tilde{0}$, $\therefore \tilde{b} = -\tilde{a} - \tilde{c}$

Q3bv Since $\overrightarrow{AB} = \tilde{b} - \tilde{a}$, $\therefore \overrightarrow{AB} = (-\tilde{a} - \tilde{c}) - \tilde{a} = -2\tilde{a} - \tilde{c}$

Q3ci $\overrightarrow{AP} = k\overrightarrow{AB}$, $-\tilde{a} - p\tilde{c} = k(-2\tilde{a} - \tilde{c})$,

$2k\tilde{a} - \tilde{a} + k\tilde{c} - p\tilde{c} = \tilde{0}$, $\therefore (2k - 1)\tilde{a} + (k - p)\tilde{c} = \tilde{0}$

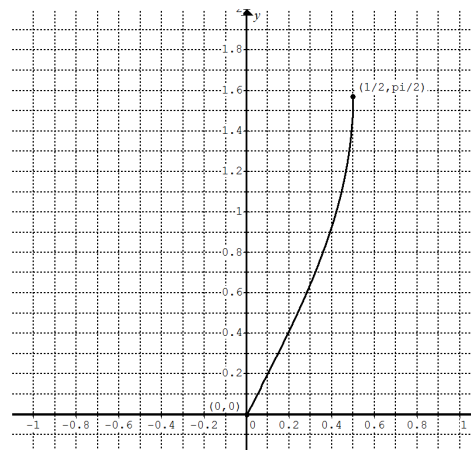
Q3cii Since $\tilde{a}$ and $\tilde{c}$ are independent, $\therefore 2k - 1 = 0$ and $k = p$,

$\therefore k = \frac{1}{2}$ and $p = \frac{1}{2}$

Q3ciii $\overrightarrow{AP} = k\overrightarrow{AB}$, $\therefore \overrightarrow{AP} = \frac{1}{2}\overrightarrow{AB}$, $\therefore P$ is the midpoint of $\overrightarrow{AB}$,

$\therefore \overrightarrow{CP}$ is a median of $\triangle ABC$.

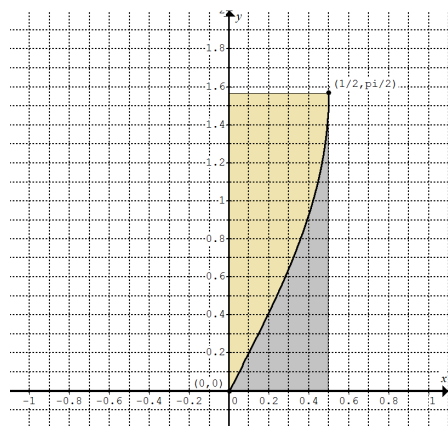
Q4a $x = 0$, $y = 0$; $x = \frac{1}{2}$, $y = \frac{\pi}{2}$



$$\text{Q4b } \frac{dy}{dx} = \frac{2}{\sqrt{1-4x^2}} = 4, \quad 1-4x^2 = \frac{1}{4}, \quad x = \frac{\sqrt{3}}{4},$$

$$\therefore y = \sin^{-1} \frac{\sqrt{3}}{2} = \frac{\pi}{3}, \therefore \text{the point is } \left(\frac{\sqrt{3}}{4}, \frac{\pi}{3} \right).$$

Q4c



$$y = \sin^{-1} 2x, \therefore x = \frac{1}{2} \sin y$$

$$\text{Area} = \int_0^{\frac{\pi}{2}} \sin^{-1}(2x) dx = \frac{1}{2} \times \frac{\pi}{2} - \int_0^{\frac{\pi}{2}} \frac{1}{2} \sin y dy$$

$$= \frac{\pi}{4} + \frac{1}{2} \int_0^{\frac{\pi}{2}} (-\sin y) dy = \frac{\pi}{4} + \frac{1}{2} [\cos y]_0^{\frac{\pi}{2}} = \frac{\pi}{4} - \frac{1}{2}$$

$$\text{Q4d } V_{\text{top}} = \int_0^{\frac{\pi}{2}} \pi x^2 dy = \int_0^{\frac{\pi}{2}} \frac{\pi}{4} \sin^2 y dy = \int_0^{\frac{\pi}{2}} \frac{\pi}{8} (1 - \cos 2y) dy$$

$$= \frac{\pi}{8} \left[y - \frac{\sin 2y}{2} \right]_0^{\frac{\pi}{2}} = \frac{\pi^2}{16} \text{ unit}^3$$

Volume of wood cut out from the block

$$= 1 \times 1 \times 2 - \frac{\pi^2}{16} = 2 - \frac{\pi^2}{16} \text{ unit}^3$$

$$\text{Q4e } y = \frac{1}{\sqrt{\left(\frac{1}{2}\right)^2 - x^2}}, \quad y^2 = \frac{1}{\frac{1}{4} - x^2}, \quad x^2 = \frac{1}{4} - \frac{1}{y^2}$$

When $x = 0$, $y = 10$

$$V_{\text{dowel}} = \int_2^{10} \pi x^2 dy = \pi \int_2^{10} \left(\frac{1}{4} - \frac{1}{y^2} \right) dy = \pi \left[\frac{y}{4} + \frac{1}{y} \right]_2^{10} = \frac{8\pi}{5} \text{ unit}^3$$

Q5a Let $\tilde{i}$ and $\tilde{j}$ be unit vectors pointing to the east and to the north respectively.

Resultant $\tilde{R}$ of the 3 pulling forces

$$= (16 \sin 120^\circ + 24 \sin(-135^\circ))\tilde{i} + (28 + 16 \cos 120^\circ + 24 \cos 135^\circ)\tilde{j}$$

$$= -3.1142\tilde{i} + 3.0294\tilde{j}$$

$$|\tilde{R}| = 4.3446 \approx 4.3 \text{ N}$$

$$\tan \theta = \frac{3.114}{3.029}, \quad \theta \approx 45.8^\circ, \text{ i.e. } N45.8^\circ W$$

Q5b Approximately 4.3 N $N45.8^\circ W$

$$\text{Q5c } \text{Limiting friction} = \mu N, \quad 4.3446 \approx 0.25 \times m \times 9.8, \\ m \approx 1.8 \text{ kg}$$

Q5d Resultant $\tilde{R}$ of the 3 pulling forces

$$= (16 \sin 120^\circ + 20\sqrt{2} \sin(-135^\circ))\tilde{i} + (28 + 16 \cos 120^\circ + 20\sqrt{2} \cos 135^\circ)\tilde{j}$$

$$= -6.1436\tilde{i}$$

Resultant force on the box = ma

$$\therefore -6.1436\tilde{i} + 4.3446\tilde{i} \approx 1.8\tilde{a}$$

$$\therefore \tilde{a} \approx -1.0\tilde{i} \text{ m s}^{-2}, \text{ i.e. } 1.0 \text{ m s}^{-2} \text{ west}$$

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