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Specialist Mathematics

2013

Trial Examination 1

Instructions

Answer **all** questions. Do **not** use calculators.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working or explanation **must** be shown.

Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ ms}^{-2}$, where $g = 9.8$.

Question 1

Consider the relation $\left\{ (x, y) : 2x + \sin \frac{1-y}{2} = 1 \right\}$, where $\sin$ is defined over $\left[-\frac{\pi}{2}, \frac{\pi}{2} \right]$.

a. Find the exact range of the relation.

1 mark

b. Find the maximal domain of the relation.

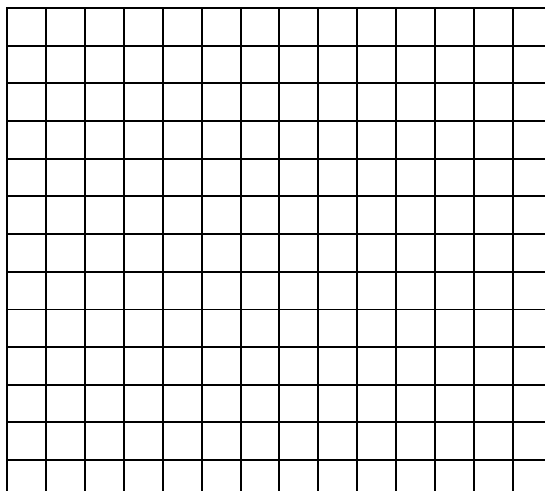
1 mark

c. Find the exact value of y when $x = \frac{1}{4}$.

1 mark

d. Sketch the graph of the relation. Show and label the end points.

2 marks



Question 2

Solve $2z^3 - iz^2 + 4z - 2i = 0$ for z over C .

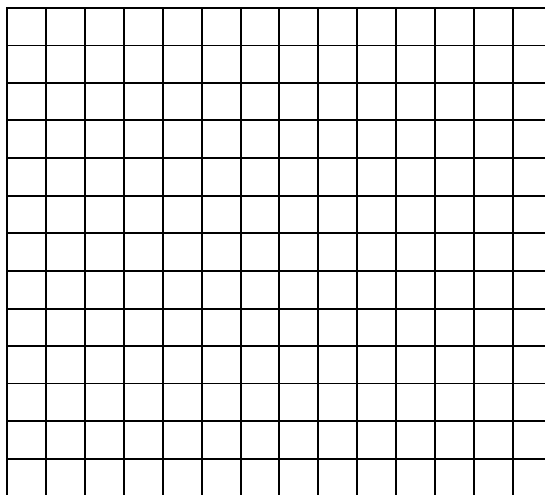
2 marks

Question 3

Consider the set of complex numbers $S = \{z : 8 \geq |2z + 6i - 8|\}$.

a. Sketch the set of complex numbers S .

2 marks



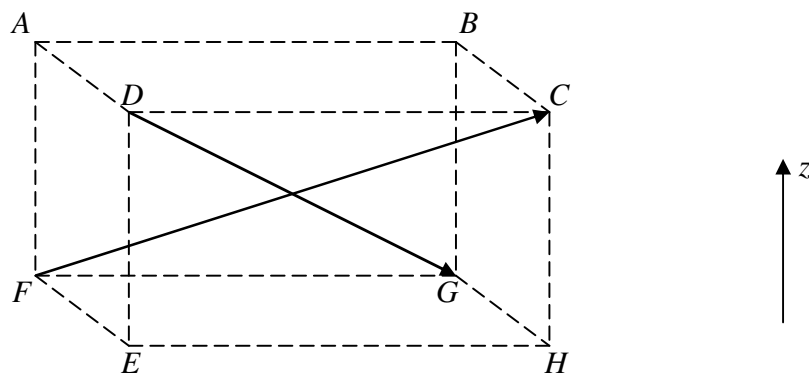
b. Find $z \in S$ in $x + iy$ form such that $|z|$ is a maximum.

2 marks

c. Show that, for $z \in S$, the maximum value of $\text{Arg}(z)$ is $\tan^{-1}\left(\frac{7}{24}\right)$.

3 marks

Question 4



$ABCDEFGH$ shown above (dotted) is a cuboid. Vector $\overrightarrow{FC} = \tilde{i} + 3\tilde{j} + 2\tilde{k}$. $\tilde{i}$, $\tilde{j}$ and $\tilde{k}$ are unit vectors in the directions of orthogonal x , y and z axes respectively.

a. Express vector $\overrightarrow{DG}$ in terms of $\tilde{i}$, $\tilde{j}$ and $\tilde{k}$. 1 mark

b i. Find the exact value of the angle between vectors $\overrightarrow{DG}$ and $\overrightarrow{FC}$. 2 marks

b ii. **Hence** find the exact scalar resolute of $\overrightarrow{DG}$ in the direction of $\overrightarrow{FC}$. 1 mark

Question 5

Let $\tilde{p} = m\tilde{i} - \tilde{j}$, $\tilde{q} = m\tilde{j} + \tilde{k}$ and $\tilde{r} = \tilde{i} - 8m\tilde{k}$.

Find the values of m such that $\tilde{p}$, $\tilde{q}$ and $\tilde{r}$ are linearly **independent**. 4 marks

Question 6

The position of a body at time $t \geq 0$ (in seconds) is given by $\tilde{r} = t\tilde{i} + \sqrt{3}t\tilde{j} - 4.9(1-t)^2\tilde{k}$, where $\tilde{i}$ and $\tilde{j}$ are horizontal orthogonal unit vectors, and $\tilde{k}$ is a vertically upward unit vector. Distance is measured in metres.

- a. Find the horizontal speed of the body. 1 mark

- b. Find the minimum speed of the body. 1 mark

- c. Find the acceleration of the body when $t = 1$. 1 mark

Question 7

Consider $f'(x) = \frac{16 \tan^{-1} x}{1+x^2}$.

- a. Find $f(x)$. 2 marks

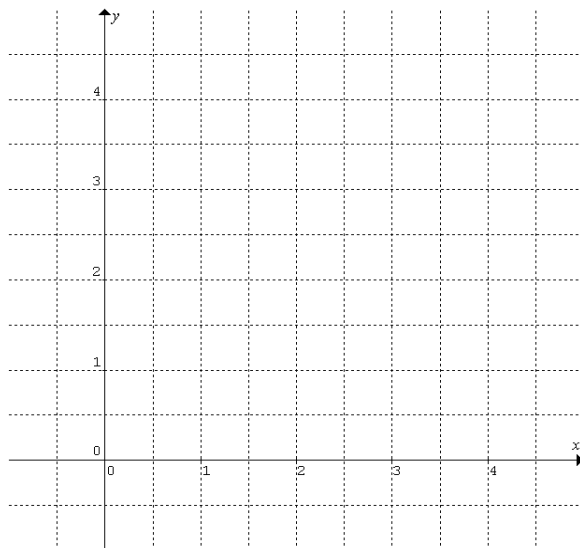
- b. Hence find the exact area of the region bounded by the x -axis, $y = f'(x)$, $x = -1$ and $x = 1$. 2 marks

Question 8

Consider the differential equation $\frac{dy}{dx} + \frac{y}{x} = 0$.

- a. Construct a slope field for the differential equation using 1 unit interval for both x and y within the intervals $1 \leq x \leq 4$ and $1 \leq y \leq 4$. Draw each tangent line about 0.5 unit long.

3 marks

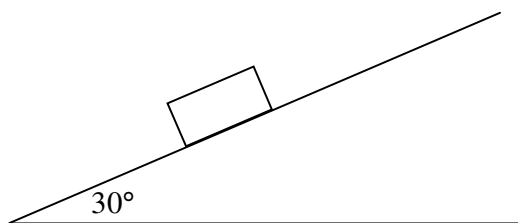


- b. Sketch the solution curve to the differential equation $\frac{dy}{dx} + \frac{y}{x} = 0$ through $(1,2)$.

1 mark

Question 9

A 5 kg body slides down a slope at **constant velocity**. The slope makes a 30° angle with the horizontal.



- a. Determine the magnitude of the reaction force of the slope on the body.

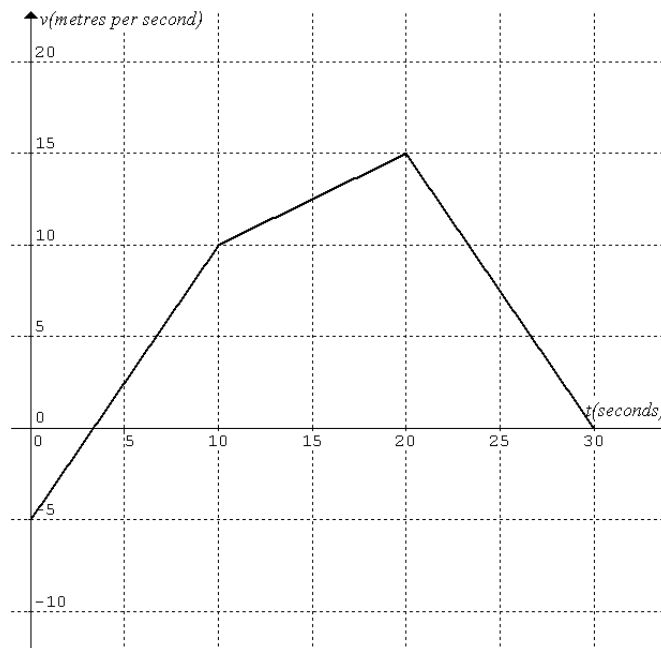
1 mark

- b. Determine the magnitude of the force on the body due to friction.

2 marks

Question 10

A particle moves in a straight line, the x -axis. At time $t = 0$ s, it is at $x = -15$ m. The velocity-time graph of the particle for the first 30 seconds is shown below.



a. Determine the position of the particle at $t = 10$ s.

2 marks

b. Determine the average velocity of the particle from $t = 0$ to $t = 30$.

2 marks

End of Exam 1