



INSIGHT

YEAR 12 Trial Exam Paper

2013

SPECIALIST MATHEMATICS

Written examination 2

STUDENT NAME:

Reading time: 15 minutes

Writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

<i>Section</i>	<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
1	22	22	22
2	5	5	58
			Total 80

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT have to be cleared.

- Students are NOT permitted to bring sheets of paper or white-out liquid/tape into the examination.

Materials provided

- The question and answer book of 31 pages, a formula sheet, and an answer sheet for the multiple-choice questions.

Instructions

- Write your **name** in the box provided and on the multiple-choice answer sheet.
- Remove the formula sheet during reading time.
- You must answer the questions in English.

At the end of the examination

- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones or any other electronic devices into the examination.

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SECTION 1**Instructions for Section 1**

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Take the **acceleration due to gravity** to have magnitude $g \text{ m/s}^2$, where $g = 9.8$.

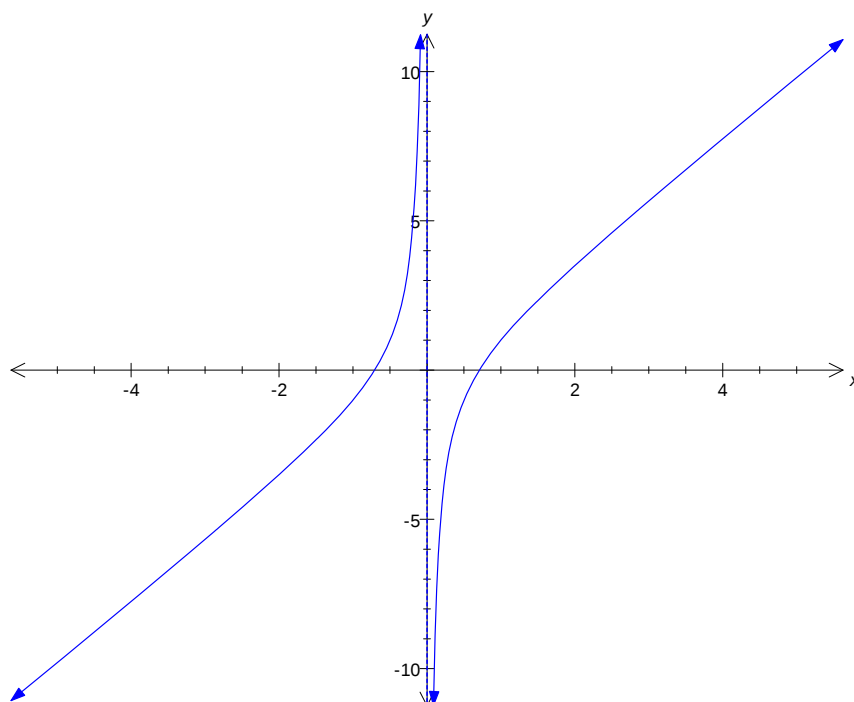
Question 1

For the hyperbola $\frac{x^2}{9} - \frac{(y+1)^2}{16} = 1$, the equations of the asymptotes are

- A. $y = \pm \frac{4x}{3} - 1$
- B. $y = \pm \frac{4x}{3} + 1$
- C. $y = \pm \frac{3x}{4} - 1$
- D. $y = \pm \frac{3}{4}(x-1)$
- E. $y = \pm \frac{4}{3}(x-1)$

Question 2

Which expression could define the function shown in the graph?



- A. $f(x) = \frac{ax^2 + b}{x}, a < 0 \text{ and } b > 0$
- B. $f(x) = \frac{ax^2 + bx}{x}, a > 0 \text{ and } b < 0$
- C. $f(x) = \frac{ax^2 + b}{x}, a > 0 \text{ and } b < 0$
- D. $f(x) = \frac{ax^2 + bx}{x^2}, a > 0 \text{ and } b < 0$
- E. $f(x) = \frac{ax^2 + b}{x^2}, a < 0 \text{ and } b > 0$

Question 3

If $\tan(2x) = -\frac{3}{4}$ and $\frac{\pi}{2} < x < \pi$, then $\operatorname{cosec}(x)$ is

- A. $\frac{3}{\sqrt{10}}$
- B. $\frac{\sqrt{10}}{3}$
- C. $\frac{1}{3}$
- D. $\sqrt{10}$
- E. $\frac{1}{\sqrt{10}}$

Question 4

For $y = a - \pi \sin^{-1}(2x - k)$, $a > 0, k > 0$, the maximal domain and range are

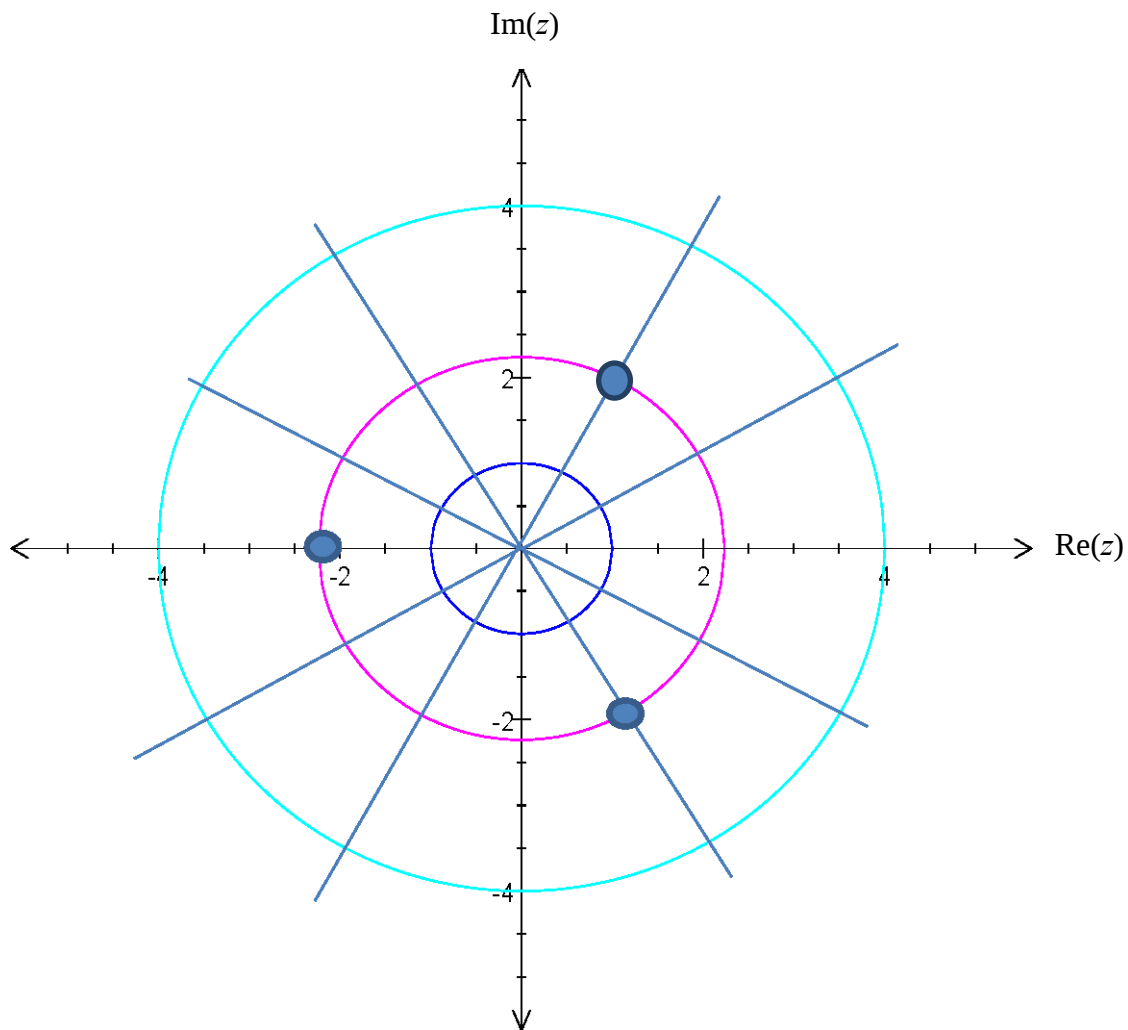
- A. domain $\left[\frac{-k}{2}, \frac{k}{2}\right]$, range $\left[\frac{\pi - a\pi}{2}, \frac{\pi + a\pi}{2}\right]$
- B. domain $\left[\frac{k-1}{2}, \frac{k+1}{2}\right]$, range $\left[\frac{\pi - a\pi}{2}, \frac{\pi + a\pi}{2}\right]$
- C. domain $\left[\frac{k-1}{2}, \frac{k+1}{2}\right]$, range $\left[\frac{2a - \pi^2}{2}, \frac{2a + \pi^2}{2}\right]$
- D. domain $\left[\frac{-k}{2}, \frac{k}{2}\right]$, range $\left[\frac{a - 2\pi}{2}, \frac{a + 2\pi}{2}\right]$
- E. domain $\left[\frac{k}{2} - 1, \frac{k}{2} + 1\right]$, range $\left[a - \frac{\pi^2}{2}, a + \frac{\pi^2}{2}\right]$

Question 5

If $z = 1 - i$, then $\text{Arg}\left(\frac{i^3}{z}\right)$ is

- A. $\frac{\pi}{4}$
- B. $\frac{7\pi}{4}$
- C. $\frac{-3\pi}{4}$
- D. $\frac{3\pi}{4}$
- E. $\frac{-\pi}{4}$

Question 6



The three points marked on the Argand diagram above represent solutions to the cubic equation $z^3 = a + bi$, where

- A. $a > 0$ and $b = 0$
- B. $a = 0$ and $b < 0$
- C. $a = 0$ and $b > 0$
- D. $a > 0$ and $b < 0$
- E. $a < 0$ and $b = 0$

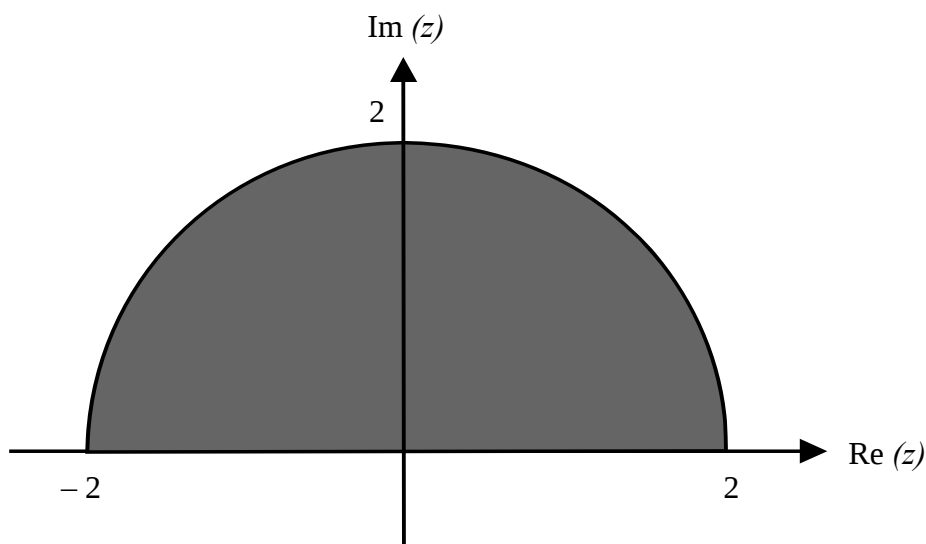
SECTION 1 – continued
TURN OVER

Question 7

If $P(z) = az^3 + bz^2 + cz + d$ and $P(-ki) = 0$ and $P\left(\frac{m}{n}\right) = 0$, where $a, b, c, d, k, m, n \in \mathbb{R} \setminus \{0\}$,

then the possible values of a, b, c and d could be

- A. $a = 1, b = -m, c = -nk^2, d = -mk^2$
- B. $a = 1, b = \frac{-m}{n}, c = -k^2, d = \frac{-mk^2}{n}$
- C. $a = n, b = -m, c = -nk^2, d = \frac{-mk^2}{n}$
- D. $a = 1, b = -m, c = -k^2, d = \frac{-mk^2}{n}$
- E. $a = n, b = -m, c = nk^2, d = -mk^2$

Question 8

The shaded region, including the boundaries, shown above on the Argand diagram could be described by

- A. $\frac{|z|}{4} \leq 2$
- B. $|z| \leq 2 \cup \text{Im}(z) \geq 0$
- C. $|z| \leq 4 \cap \text{Im}(z) \geq 0$
- D. $|z| \leq 2 \cap \text{Re}(z) \geq 0$
- E. $|z| \leq 2 \cap \text{Im}(z) \geq 0$

Question 9

If $\frac{dx}{dy} = \sqrt{9 - 4x^2}$ then

A. $y = \int \sqrt{9 - 4x^2} \, dx$

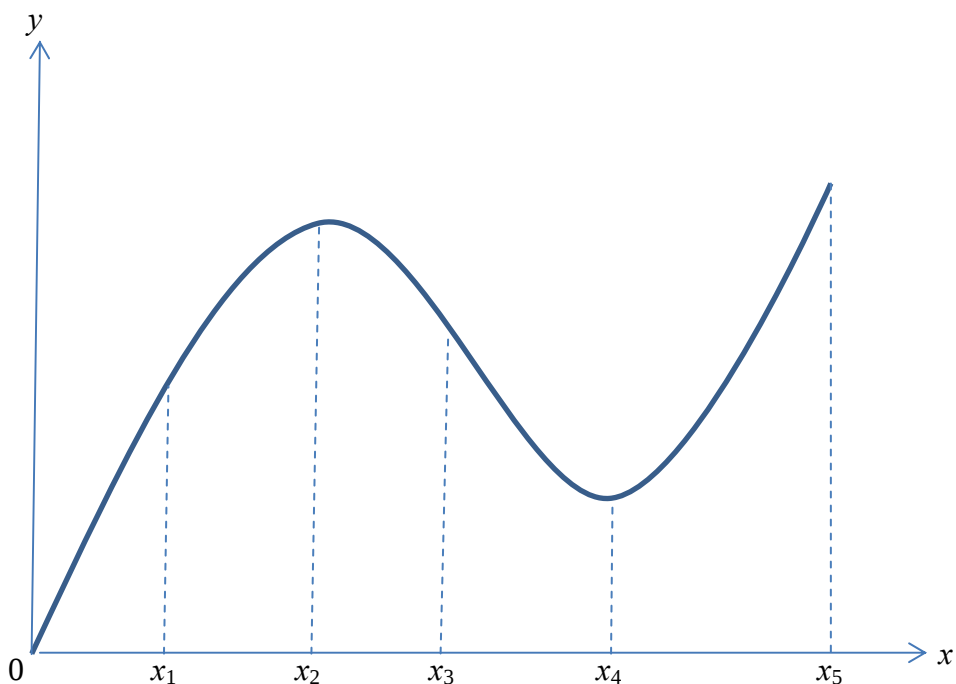
B. $y = \frac{1}{2} \int \frac{1}{\sqrt{\frac{9}{4} - x^2}} \, dx$

C. $y = 2 \int \frac{1}{\sqrt{\frac{9}{4} - x^2}} \, dx$

D. $y = \frac{1}{2} \int \frac{1}{\sqrt{\frac{4}{9} - x^2}} \, dx$

E. $y = \frac{2}{3} \int \frac{1}{\sqrt{1 - x^2}} \, dx$

Question 10



Part of the graph of function f is shown above for $0 \leq x \leq x_5$.

For this function, $f'(x_2) = 0$, $f'(x_4) = 0$ and there are points of inflexion at $x = x_1$, $x = x_3$, $x = x_5$.

Then $f''(x) > 0$ for

- A. $0 < x < x_1$
- B. $0 < x < x_1$ and $x_3 < x < x_5$
- C. $x_3 < x < x_5$
- D. $x_1 < x < x_3$
- E. $x_1 < x < x_2$ and $x_4 < x < x_5$

Question 11

For the curve $x \sin(y) - x^2 = 5$

- A. $\frac{dy}{dx} = \frac{2x - \sin(y)}{x \cos(y)}$
- B. $\frac{dy}{dx} = x \cos(y) - 2x$
- C. $\frac{dy}{dx} = x \cos(y) + \sin(y) - 2x$
- D. $\frac{dy}{dx} = \frac{2x - x \sin(y)}{x \cos(y)}$
- E. $\frac{dy}{dx} = \frac{2x - \sin(y)}{\cos(y)}$

Question 12

$\int \frac{3x-2}{(x-1)^2} dx$ can be written as

- A. $\int \frac{3}{(x-1)^2} - \frac{2}{(x-1)} dx$
- B. $\int \frac{3}{(x-1)} - \frac{2}{(x-1)^2} dx$
- C. $\int \frac{3}{(x-1)} + \frac{1}{(x-1)^2} dx$
- D. $\int \frac{3}{(x-1)} - \frac{1}{(x-1)^2} dx$
- E. $\int \frac{3}{(x-1)^2} - \frac{1}{(x-1)} dx$

Question 13

Initially, a salt solution has a volume of 100 litres. Pure water is poured into the solution at 10 litres per minute and the mixture, after stirring well, is removed at the rate of 7 litres per minute. A differential equation for the mass, x kg, of salt present at time t minutes is given by

- A. $\frac{dx}{dt} = 10 - \frac{7x}{100}$
- B. $\frac{dx}{dt} = 10 - \frac{7x}{100+t}$
- C. $\frac{dx}{dt} = \frac{-7x}{100+t}$
- D. $\frac{dx}{dt} = \frac{-7x}{100+3t}$
- E. $\frac{dx}{dt} = 10 - \frac{7x}{100+3t}$

Question 14

The position of an object from a fixed point, O , is x metres at time t seconds, where

$$x = \frac{e^{2t} + 1}{e^t}, \quad t \geq 0.$$

The acceleration of the object, in m/s^2 , when $t = 2$ seconds is

- A. $e^2 - e^{-2}$
- B. $e - e^{-1}$
- C. $\frac{e^4 + 1}{e^2}$
- D. $\frac{e^3 - 1}{e^2}$
- E. $\frac{e^4 - e^2}{e^2}$

Question 15

The velocity of an object v metres/second is $v = \sin(x) \cos(x)$, where x metres is the distance of the object from a fixed point O .

The acceleration of the object is

- A. $a = \cos(2x)$
- B. $a = -\cos(x) \sin(x)$
- C. $a = \frac{\sin(2x)}{2}$
- D. $a = \frac{\sin(4x)}{4}$
- E. $a = \cos(2x) \sin 2(x)$

Question 16

If $\underline{a} = 5\underline{i} - \underline{j} + 2\underline{k}$ and $\underline{b} = \underline{i} - 2\underline{j} + \underline{k}$ then the vector resolute of $\underline{b}$ in the direction of $\underline{a}$ is

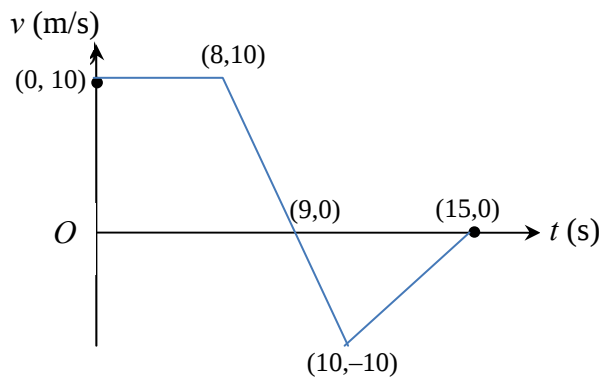
- A. $\frac{3}{25}\underline{i} - \frac{3}{10}\underline{j} + \frac{3}{5}\underline{k}$
- B. $\frac{3}{2}\underline{i} - \frac{3}{10}\underline{j} + \frac{3}{5}\underline{k}$
- C. $3\underline{i} - 3\underline{j} + \underline{k}$
- D. $\frac{3}{2}\underline{i} + \frac{3}{10}\underline{j} + \frac{3}{5}\underline{k}$
- E. $\frac{3}{2}\underline{i} - \frac{3}{10}\underline{j} - \frac{3}{5}\underline{k}$

Question 17

The position vector of a particle at time t is given by $\underline{r}(t) = \cos(2t)\underline{i} + \sin^2(t)\underline{j}$, $t \geq 0$.

The equation of the particle's path is

- A. $y = \frac{1}{2}(1-x)$, $-1 \leq x \leq 1$
- B. $y^2 = \frac{1}{2}(1-x)$, $x \geq 0$
- C. $y = \frac{1}{2}(1-x)^2$, $x \geq 0$
- D. $y^2 = \frac{1}{2}(1-x)^2$, $-1 \leq x \leq 1$
- E. $y^2 = \frac{1}{2}(1-x)$, $-1 \leq x \leq 1$

Question 18

The velocity–time graph of a particle moving in a straight line starting from a fixed position, O , is shown above. The particle moves with a constant velocity for 8 seconds in a westerly direction. Where is the particle located 7 seconds later?

- A. 55 m west of O
- B. 115 m west of O
- C. 120 m east of O
- D. 55 m east of O
- E. 100 m east of O

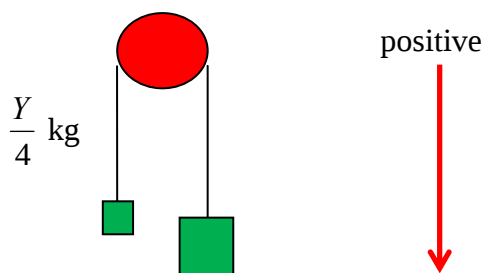
Question 19

An approximate solution to the differential equation $\frac{dy}{dx} = \log_e(2x + 1)$ is found using Euler's method, with a step size of 0.1 and with $y_0 = 0$ and $x_0 = 0$. The value obtained for y_2 would be

- A. $\frac{\log_e 0.2}{10}$
- B. $\frac{\log_e 1.2}{10}$
- C. $\frac{1 + \log_3 1.2}{10}$
- D. $\log_e 0.1$
- E. $\log_e 1.2$

Question 20

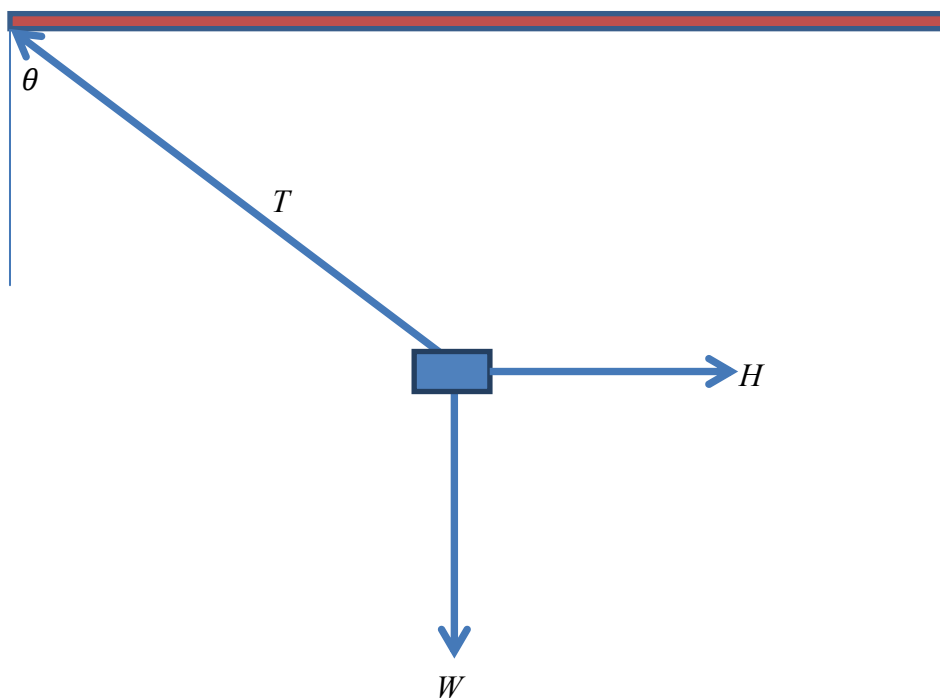
The diagram shows a smooth pulley with two objects attached to either end of an inextensible string. The mass of the smaller object is 40% less than the mass of the larger object.



The acceleration of the larger object is

- A. $\frac{7g}{13} \text{ m/s}^2$
- B. $\frac{g}{3} \text{ m/s}^2$
- C. $\frac{g}{4} \text{ m/s}^2$
- D. $\frac{1}{4} \text{ m/s}^2$
- E. $4g \text{ m/s}^2$

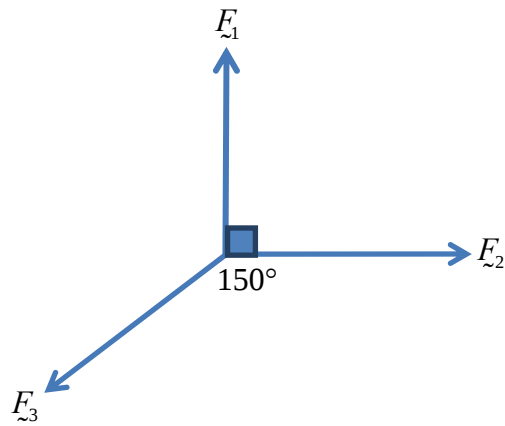
Question 21



An object of mass 10 kg is attached to a beam by a light inelastic string, making an angle of θ with the vertical. The tension force acting on the object is T newtons, and W newtons is the weight force. A horizontal force, H , of 60 newtons keeps the object in equilibrium. The value of θ (in degrees, to 2 decimal places) is

- A. 0.55°
- B. 31.47°
- C. 31.48°
- D. 58.52°
- E. 30.96°

Question 22



Three co-planar forces, F_1 , F_2 and F_3 , act on a particle in equilibrium, then

- A. $2\sqrt{3}F_1 = 2F_2 = \sqrt{3}F_3$
- B. $F_1 = F_2 = F_3$
- C. $2F_1 = 2\sqrt{3}F_2 = F_3$
- D. $\sqrt{3}F_1 = 2F_2 = F_3$
- E. $2\sqrt{3}F_1 = F_2 = 2F_3$

END OF SECTION 1

**END OF SECTION 1
TURN OVER**

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SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

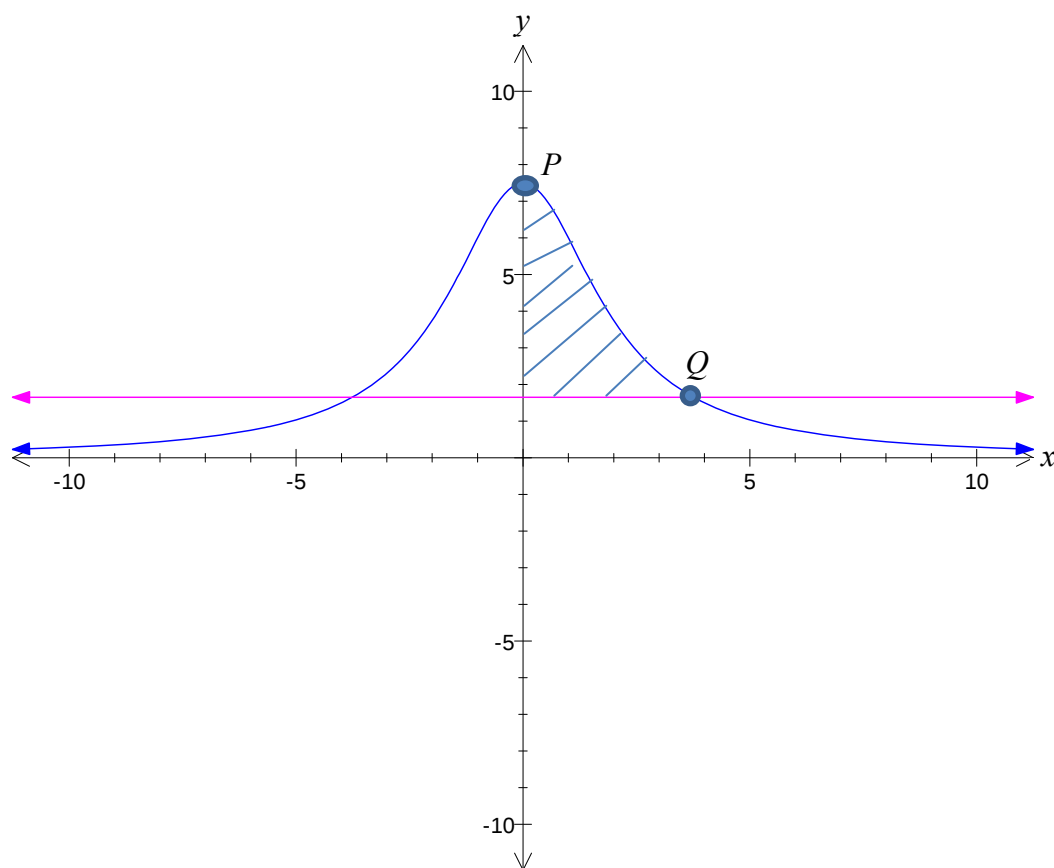
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ m/s}^2$, where $g = 9.8$.

Question 1 (8 marks)

The graphs of the functions $y = \frac{a}{x^2 + b}$, $a > 0$, $b > 0$, and $y = c$, $c > 0$ are shown on the axes below.

P is the y -intercept of $y = \frac{a}{x^2 + b}$ and Q is one of the points of intersection of $y = \frac{a}{x^2 + b}$ and $y = c$.



- a.** Find the coordinates of P and Q in terms of the constants a , b and c .

3 marks

- b. i.** Write a definite integral for A , the shaded area enclosed by the graphs and the y -axis, using the constants a , b , and c .

1 mark

- ii.** Write a definite integral for V , the volume of the shaded area rotated around the y -axis, using the constants a , b and c .

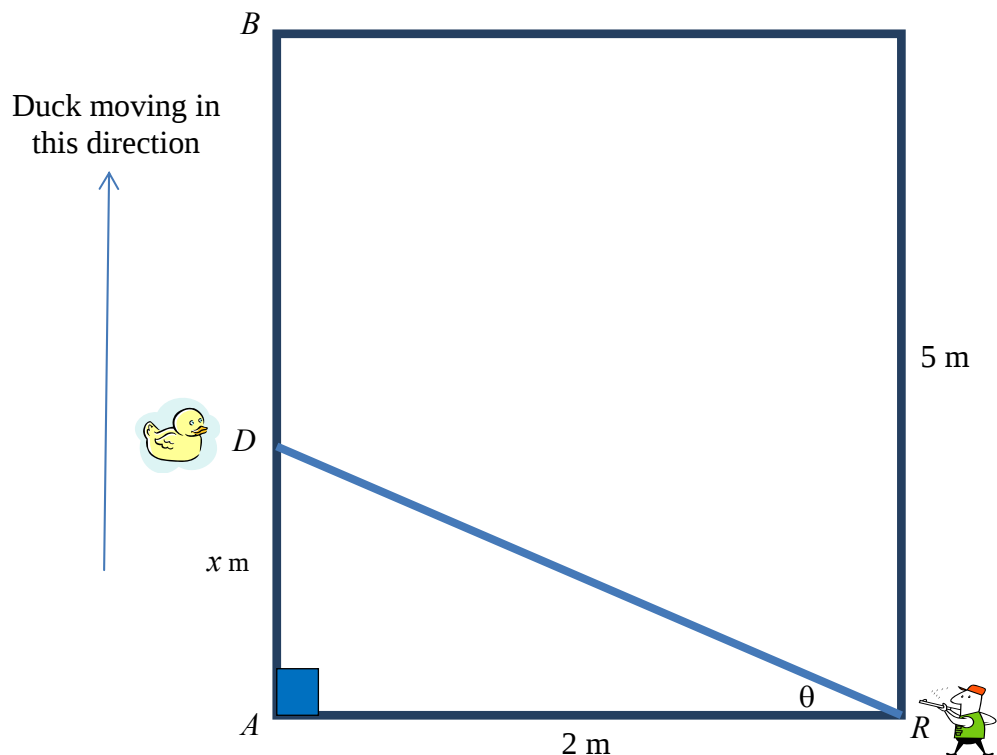
2 marks

- c.** If $a = 30$, $b = 4$ and $c = 1.6$, evaluate A and V , to 1 decimal place.

2 marks

Question 2 (14 marks)

The rectangle in the diagram represents an aerial view of a shooting gallery at an amusement park. A rifle is fixed at one side of the gallery at point R and a duck, D , moves along the line AB (5 m in length). The duck's velocity, in m/s, is given by $\frac{dx}{dt} = \frac{3x}{2}$, where x m is the distance from A to the duck. Initially, the duck is at the point where $x = 1$ and is moving towards B .



- a. Find θ in terms of x .

1 mark

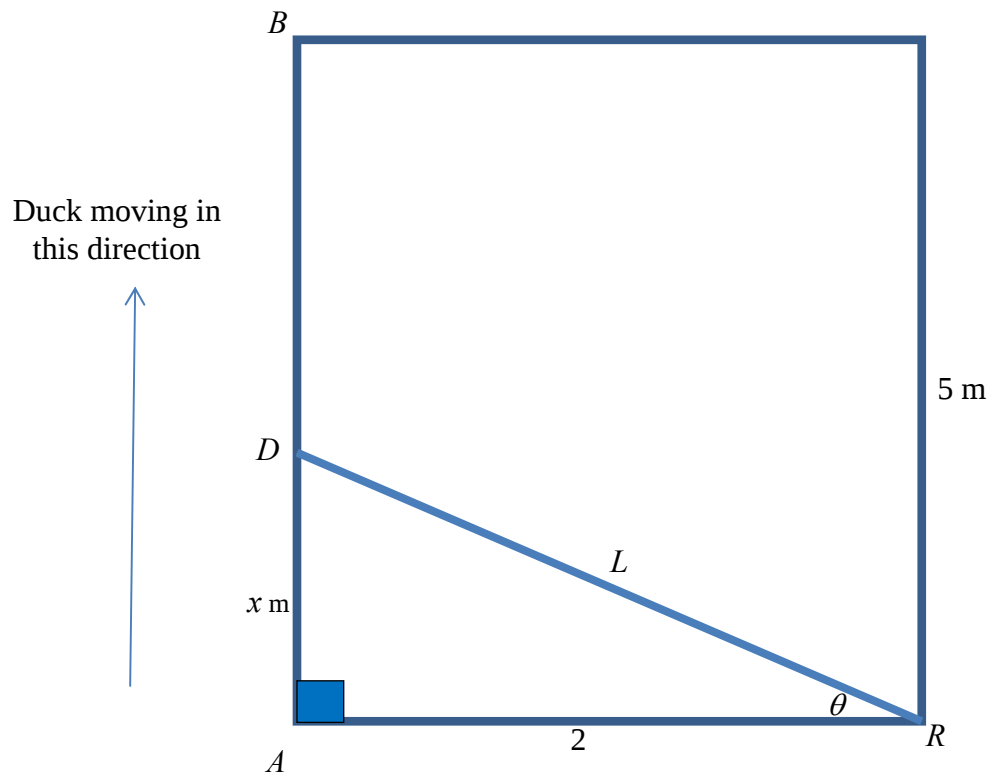
- b.** Find the value of $\frac{d\theta}{dt}$ when $x = 1$.

2 marks

- c.** Find a , b and c such that $\frac{d\theta}{dt} = \frac{a \sin(b\theta)}{c}$.

3 marks

- d. Let L be the direct distance from the rifle to the duck.



- i. Find L in terms of θ only.

1 mark

- ii. Find $\frac{dL}{dt}$ in terms of θ .

2 marks

- iii. Find $\frac{dL}{dt}$ in terms of L and, hence, find L in terms of t .

5 marks

[illegible]

Question 3 (12 marks)

In a chemical reaction, a compound C (kg) is formed by combining one part of chemical L and two parts of chemical M. The rate at which C is formed is proportional to the product of the amounts of chemicals L and M that are still present in the mixture at any given time t minutes. Initially, there are 5 kg of L, 8 kg of M and no amount of C present, but after some time, x kg of compound C has been formed.

- a. How much of this x kg of C formed is chemical M and how much is chemical L?

1 mark

- b. How many kilograms of chemical M and chemical L remain in the mixture?

1 mark

- c. Show that $\frac{dx}{dt} = k(12 - x)(15 - x)$, where k is a constant.

2 marks

After 1 minute, 2 kg of chemical C has been formed.

- d.** Using calculus, solve the differential equation, writing t in terms of x .

7 marks

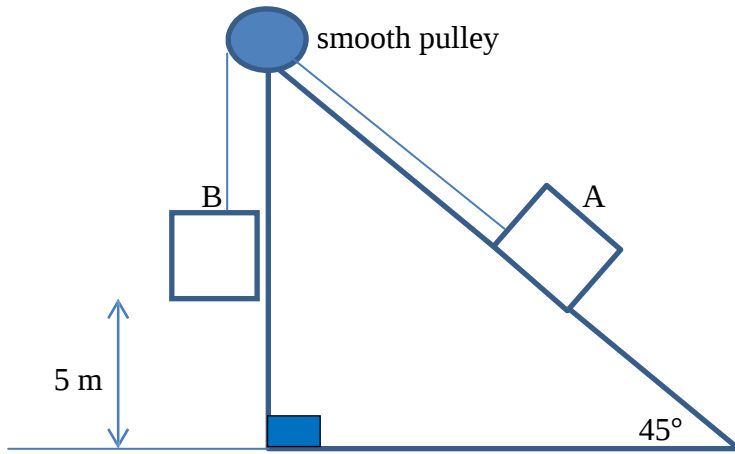
[illegible]

- e. How long, to the nearest minute, would it take to form 10 kg of compound C?

1 mark

Question 4 (10 marks)

The diagram shows a load, A, of mass M kg, being held stationary on a rough inclined slope by another load, B, also of mass M kg, which is hanging vertically 5 m above the ground. The two loads are connected by a light inelastic rope passing over a smooth pulley.



- a. Initially, the load A is on the verge of moving up the slope. Show that μ , the coefficient of friction, is $\sqrt{2}-1$.

3 marks

- b.** Load A is now reduced by 20% and both loads begin to move from rest. Show that the acceleration of B is $\frac{g}{9} \text{ m/s}^2$.

4 marks

[illegible]

- c. The rope breaks after load B has been moving for 2 seconds. Find the values of a , b and c , if the speed with which B hits the ground after the rope breaks is $\frac{\sqrt{ag + bg^2}}{c}$.

3 marks

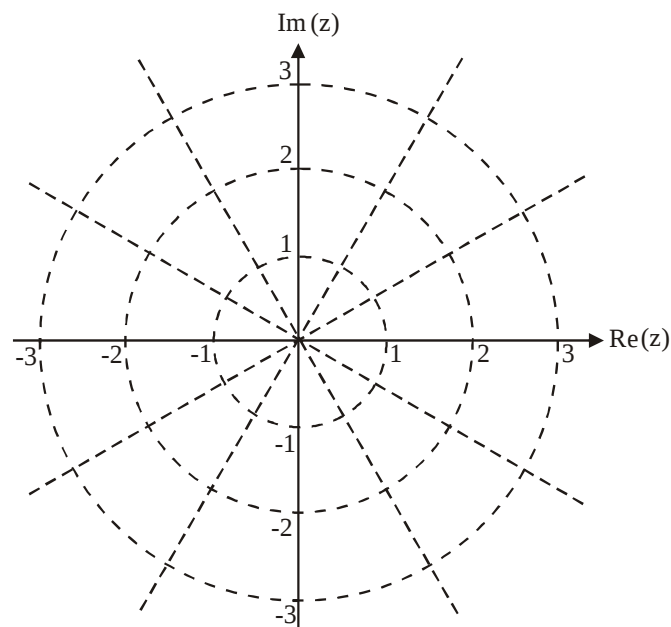
[illegible]

Question 5 (14 marks)

a. Let $u = -1 + \sqrt{3}i$.

- i. Find and plot z_1 and z_2 , where z_1 and z_2 are solutions to $z^2 = \bar{u}$ where $\text{Arg } z_1 < 0$ and $\text{Arg } z_2 > 0$.

3 marks



- ii.** The complex relationship given by $|z - z_1| = 2|z - z_2|$ represents a circle. Find the Cartesian equation of the circle and state the exact coordinates of its centre and the value of its radius.

4 marks

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

b. Let $z = \cos\theta + i\sin\theta$.

i. Show that $z^3 + \frac{1}{z^3} = 2\cos(3\theta)$.

2 marks

ii. Show $z + \frac{1}{z} = 2\cos(\theta)$.

1 mark

- iii.** Hence, using parts **i** and **ii** or otherwise, show that $4\cos^3(\theta) = \cos(3\theta) + 3\cos(\theta)$.

4 marks

END OF QUESTION AND ANSWER BOOK