



INSIGHT

Year 12 Trial Exam Paper

2013

SPECIALIST MATHEMATICS

Written examination 1

STUDENT NAME:

Reading time: 15 minutes

Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
9	9	40

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring sheets of paper, notes of any kind or white out liquid/tape into the examination.
- Calculators are not permitted in this examination.

Materials provided

- The question and answer book of 15 pages with a separate sheet of miscellaneous formulas.
- Working space is provided throughout this book.

Instructions

- Write your **name** in the box provided.
- Remove the formula sheet during reading time.
- You must answer the questions in English.

Students are NOT permitted to bring mobile phones and/or any other electronic devices into the examination.

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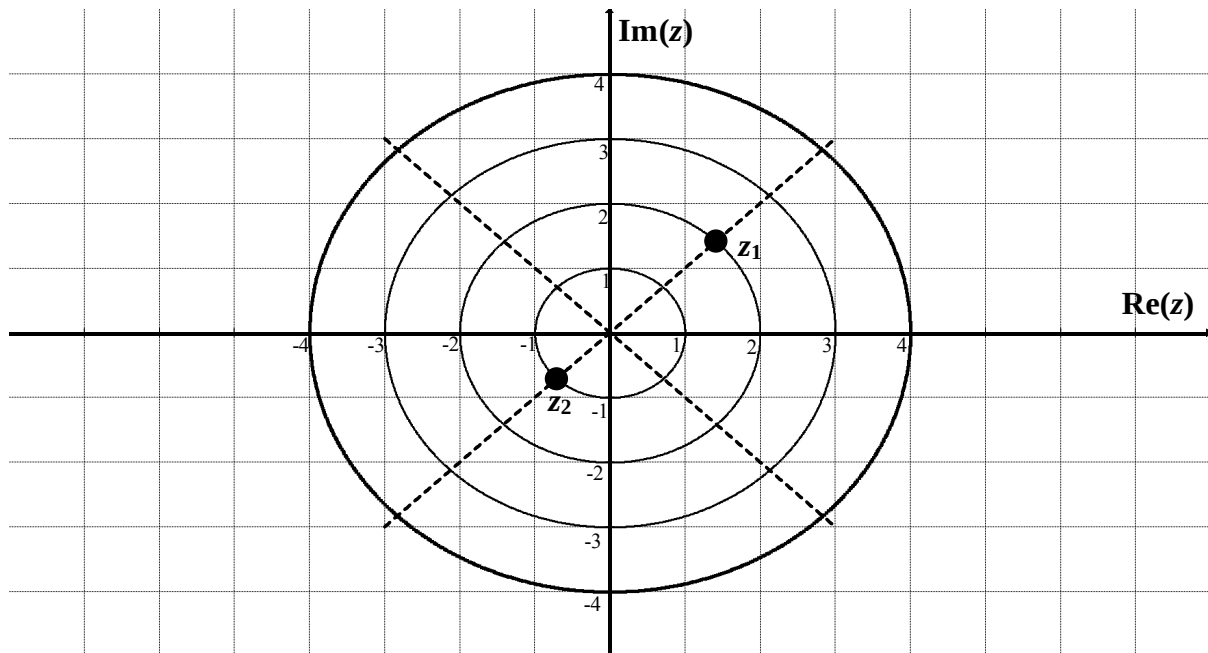
Question 1 (4 marks)

- a. Find all solutions to the equation $z^4 + 2z^2 = 8$, $z \in \mathbb{C}$.

2 marks

- b. Consider z_1 and z_2 as shown on the Argand diagram below. On the same diagram plot the following points:

$$u = \frac{z_1}{z_2} \text{ and } w = i^2(\bar{z}_1)^2$$



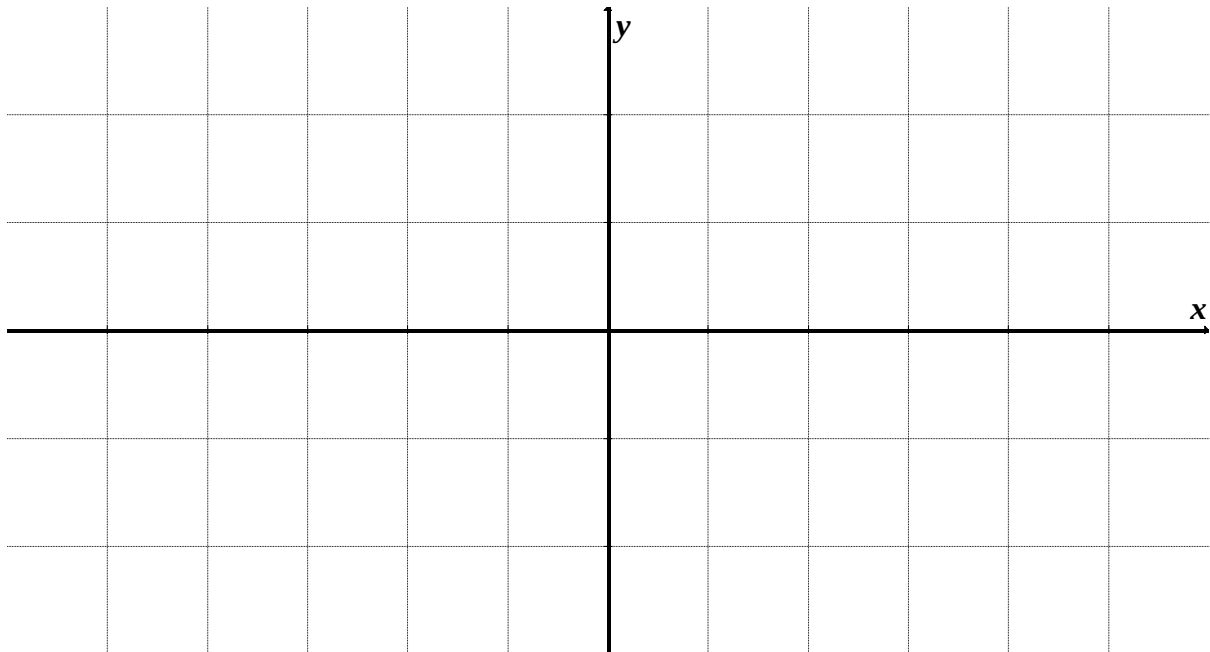
2 marks

**End of Question 1
TURN OVER**

Question 2 (6 marks)

- a. Given that $f(x) = \frac{x^2 + 2}{x^2 + 4} = 1 - \frac{2}{x^2 + 4}$, sketch the graph of $y = f(x)$ on the axes below.
Label any asymptotes with their equations and any axes intercepts as coordinates.

3 marks



- b.** Find the exact area enclosed by the graph of $f(x) = \frac{x^2 + 2}{x^2 + 4}$, the x -axis, the y -axis and the line $x = 2$.

3 marks

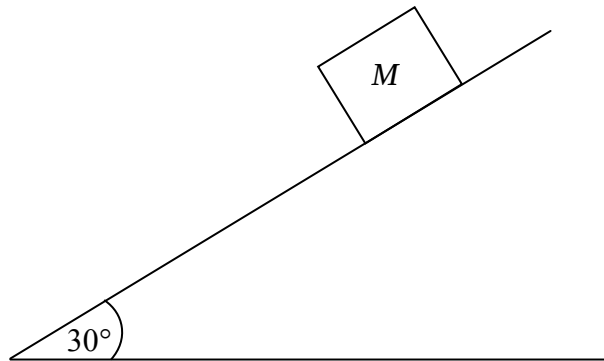
End of Question 2
TURN OVER

Question 3 (3 marks)

Evaluate $\int_0^{e-1} \frac{\sin(\pi \log_e(x+1))}{x+1} dx$.

Question 4 (5 marks)

A mass of M kg slides from rest down a rough plane, which makes an angle of 30° with the horizontal ground. The coefficient of friction is μ .



- a.** On the diagram above, mark all the forces acting on the mass, M .

1 mark

- b.** Find the acceleration of the mass in terms of g and μ .

2 marks

- c. If the mass reaches a speed of 3 m/s after sliding for 3 metres, find μ in the form $\mu = \frac{g-c}{\sqrt{cg}}$ where c is an integer.

2 marks

Question 5 (5 marks)

A particle moves in a straight line with an acceleration of $a \text{ m/s}^2$, which is given by $a = x + 1$.

At time t seconds, its displacement is x metres from a fixed point and its velocity is $v \text{ m/s}$.

When $t = 0$, $x = 0$ and $v = 1$.

- a. Show that $v = x + 1$.

3 marks

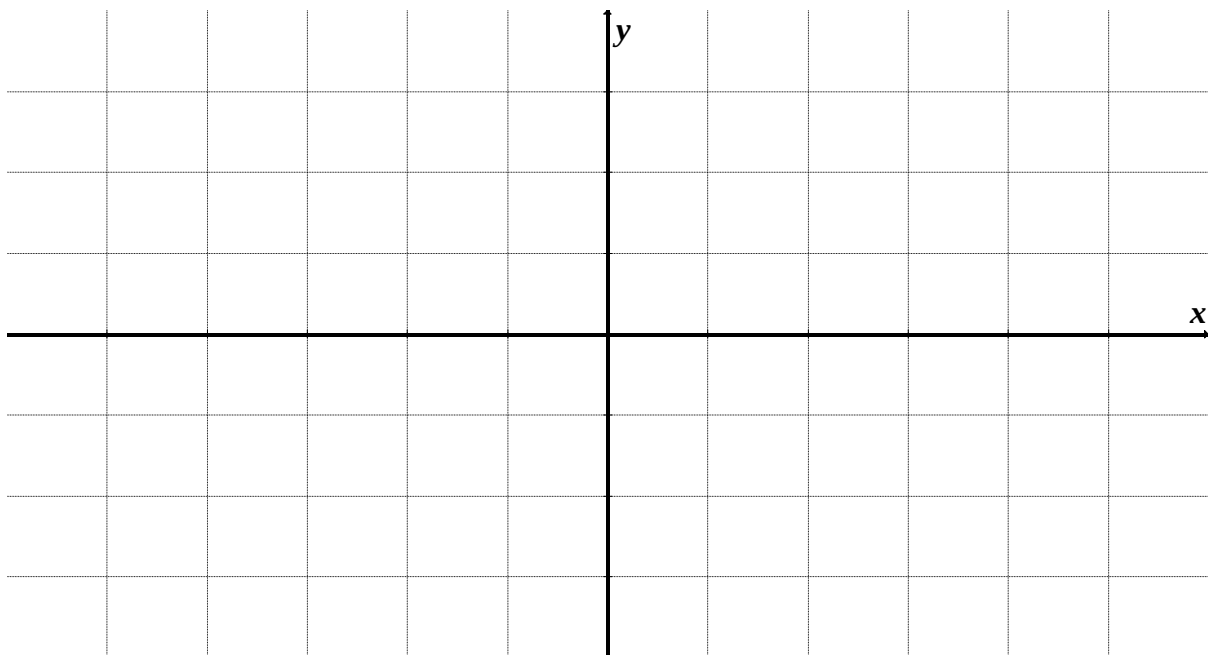
- b.** Hence, express x as a function of t .

2 marks

End of Question 5
TURN OVER

Question 6 (2 marks)

On the axes below, sketch the graph of the relation $y^2 = 4 - 4(x-1)^2$.
Show all axes intercepts.



Question 7 (6 marks)

The motion of a particle is described by the following parametric equations:

$$x - y = 0.5 \sec(t)$$

$$y = 2 + \tan^2(t)$$

- a.** Show that the Cartesian equation is given by the relation

$$y = 4x^2 - 8xy + 4y^2 + 1.$$

2 marks

- b.** Show that when time $t = \frac{\pi}{3}$ the coordinate of the particle is (6, 5).

2 marks

- c. Hence, find the gradient of the curve at the point $(6, 5)$.

2 marks

Question 8 (4 marks)

Three points, A , B and C , are given by $A(2, 2, 1)$, $B(3, 0, 4)$ and $C(5, -4, 10)$.

- a. Show that A , B and C are collinear.

2 marks

- b.** The respective unit vectors in the direction of $\vec{OA}$ and $\vec{OB}$ are $\underline{a} = \frac{1}{3}(2\underline{i} + 2\underline{j} + \underline{k})$ and $\underline{b} = \frac{1}{5}(3\underline{i} + 4\underline{k})$.

Find a vector that bisects the angle between $\vec{OA}$ and $\vec{OB}$.

2 marks

End of Question 8
TURN OVER

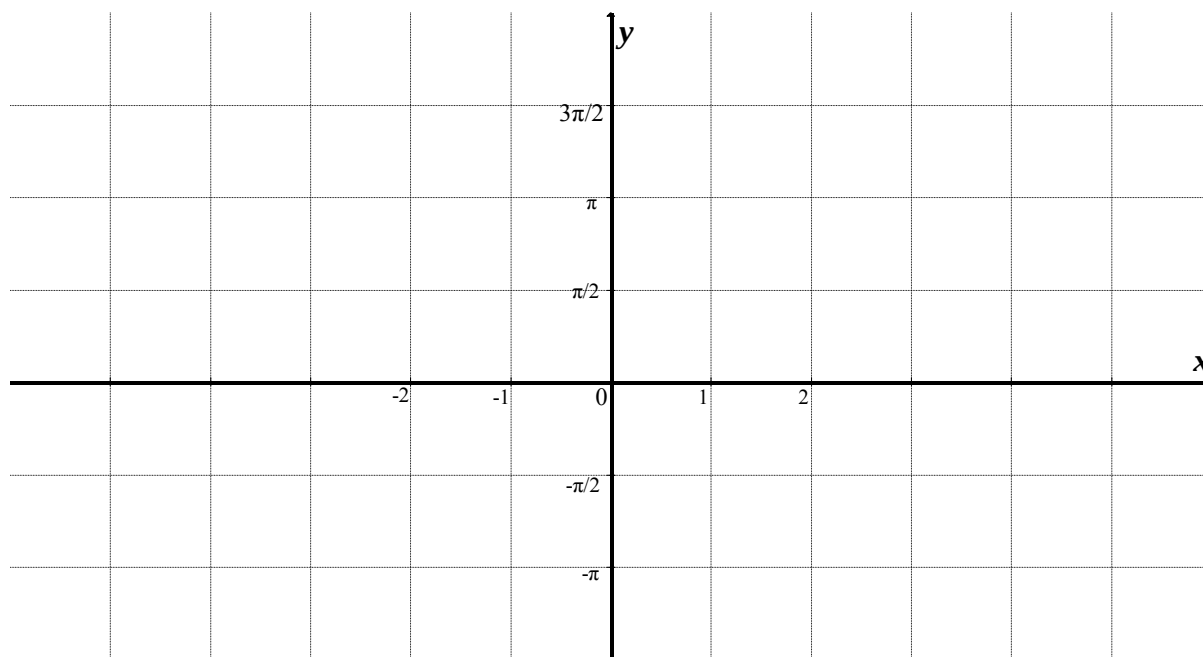
Question 9 (5 marks)

- a. On the set of axes below, sketch the graph of the curve with equation

$$f(x) = \frac{\pi}{2} + \sin^{-1}(x).$$

Clearly show the coordinates of the end points.

2 marks



- b.** Find the volume generated when the area between the curve $y = f(x)$ and the y -axis is rotated about the y -axis to form a solid of revolution.

3 marks

END OF QUESTION AND ANSWER BOOK