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Engage
Education
Foundation

Units 3 and 4 Specialist Maths: Exam 1

Practice Exam Question and Answer Booklet

Duration: 15 minutes reading time, 60 minutes writing time

Structure of book:

Number of questions	Number of questions to be answered	Number of marks
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers and rulers.
- Students are not permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.
- No calculator is allowed in this examination.

Materials supplied:
- This question and answer booklet of 11 pages, including a sheet of miscellaneous formulas.

Instructions:
- You must complete all questions of the examination.
- Write all your answers in the spaces provided in this booklet.

Instructions

Answer all questions in the spaces provided.
In all questions where a numerical answer is required an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working must be shown.
Unless otherwise indicated, the diagrams in this book are not drawn to scale.

Questions

Question 1

Given $f(z) = z^4 - 2z^3 + 2z^2 + 10$, and that $f(2+i) = 0$, factorize $f(z)$ over C

4 marks

Question 2

Suppose a mass has acceleration $a = e^{v^2}$. Find x, the position of the mass, in terms of v, given that $x = e$ when $v = 0$. Hint: use an appropriate substitution to solve the integral.

4 marks

Question 3

a. Find the area enclosed by the function $f(x) = \frac{2}{4+x^2}$, the x-axis and the lines $x = \pm\sqrt{3}$

2 marks

b. Find an antiderivative of the function $f(x) = \frac{x^2}{\sqrt{2x-1}}$

4 marks

Total: 6 marks

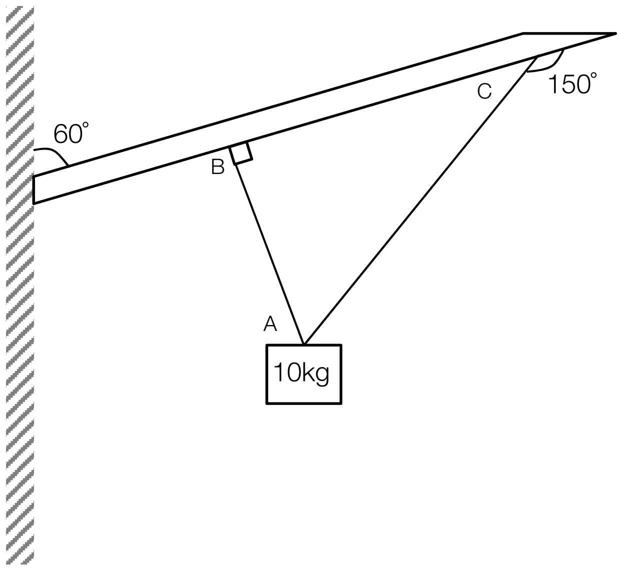
Question 4

Express $\sin(3\theta) - \cos(3\theta)$ as a function of $\sin(\theta)$ and $\cos(\theta)$ only.

3 marks

Question 5

A 10kg weight is held in static equilibrium by two ropes, $\overline{AB}$ and $\overline{AC}$, fixed to a slanted beam, which is attached to a nearby wall, as shown below.



a. Show that the line bisecting the angle made by the two ropes at the weight is parallel to the wall.

3 marks

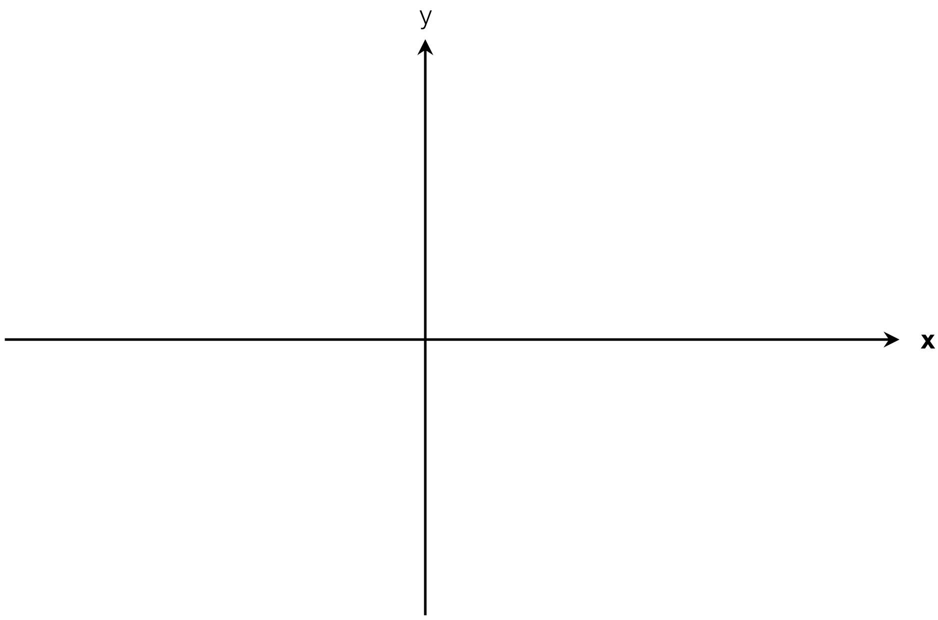
b. Hence, or otherwise, find the magnitude of the force exerted by $\overline{AB}$ and $\overline{AC}$ on the weight.

2 marks

Total: 5 marks

Question 6

Sketch the graph of $y = \frac{1}{x} + \frac{x}{2} + \frac{3}{4}$. Include all asymptotes and axes intercepts (do not include turning points).

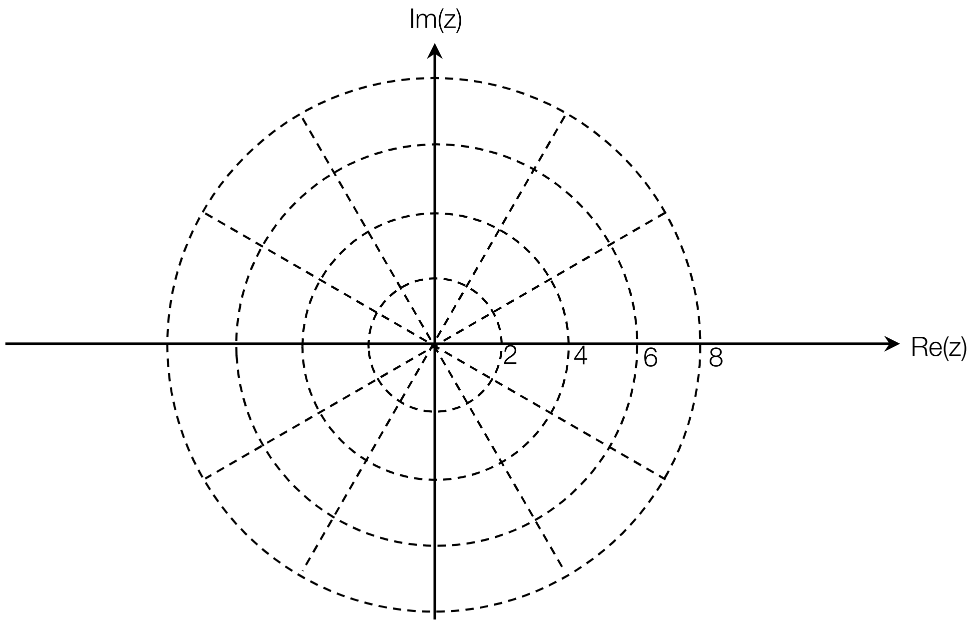


3 marks

Question 7

Given $z = 2cis(\frac{\pi}{6})$, plot:

a. z
b. z^2
c. z^3



3 marks

Question 8

Use implicit differentiation to find $\frac{dy}{dx}$ if:

$$xy\log_e(xy) = 1$$

4 marks

Question 9

A particle follows a helical path given by $\boldsymbol{r}(t) = 3\cos t\,\boldsymbol{i} + 3\sin t\,\boldsymbol{j} + t\boldsymbol{k}$

a. Find $\dot{\boldsymbol{r}}(t)$ and $\ddot{\boldsymbol{r}}(t)$

2 marks

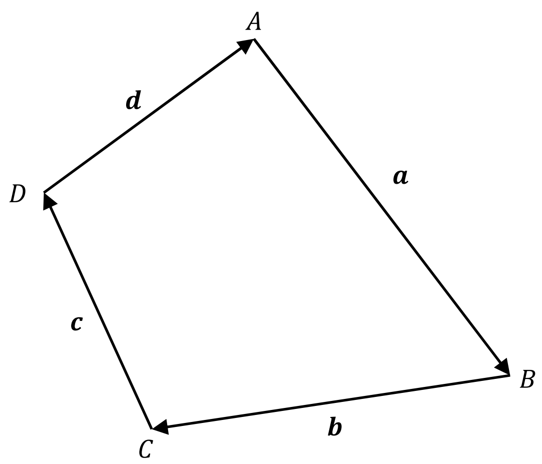
b. Find $\dot{\boldsymbol{r}}(t) \cdot \ddot{\boldsymbol{r}}(t)$. What does this say about the direction of the particle's acceleration relative to its velocity?

2 marks

Total: 4 marks

Question 10

Consider an arbitrary quadrilateral $ABCD$, where vectors $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ and $\boldsymbol{d}$ are the vectors shown below, and where P, Q, R and S are the midpoints of $\overline{AB}, \overline{BC}, \overline{CD}$ and $\overline{DA}$, respectively.



Prove, using vector methods, that the quadrilateral $PQRS$ is a parallelogram.

4 marks

End of Booklet

Looking for solutions? Visit www.engageeducation.org.au/practice-exams

Formula sheet

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder	$2\pi rh$
volume of a cylinder	$\pi r^2 h$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a pyramid	$\frac{1}{3}Ah$
volume of a sphere	$\frac{4}{3}\pi r^3$
area of a triangle	$\frac{1}{2}bc\sin A$
sine rule	$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$
cosine rule	$c^2=a^2+b^2-2ab\cos C$

Coordinate geometry

ellipse $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ hyperbola $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$

Circular (trigonometric) functions

$\cos^2(x)+\sin^2(x)=1$

$1+\tan^2(x)=\sec^2(x)$ $\cot^2(x)+1=\operatorname{cosec}^2(x)$

$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y)$ $\sin(x-y)=\sin(x)\cos(y)-\cos(x)\sin(y)$

$\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$ $\cos(x-y)=\cos(x)\cos(y)+\sin(x)\sin(y)$

$\tan(x+y)=\frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$ $\tan(x-y)=\frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$

$\cos(2x)=\cos^2(x)-\sin^2(x)=2\cos^2(x)-1=1-2\sin^2(x)$

$\sin(2x)=2\sin(x)\cos(x)$ $\tan(2x)=\frac{2\tan(x)}{1-\tan^2(x)}$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1,1]$	$[-1,1]$	$\mathbb{R}$
range	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	$[0,\pi]$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Algebra (complex numbers)

$z = x + yi = r(\cos\theta + i\sin\theta) = r\operatorname{cis}\theta$

$z^n = r^n \operatorname{cis}(n\theta)$ (de Moivre's theorem)

$|z| = \sqrt{x^2 + y^2} = r$

$-\pi < \operatorname{Arg} z \leq \pi$

$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$

$\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}\operatorname{cis}(\theta_1 - \theta_2)$

Calculus

$\dfrac{d}{dx}(x^n) = nx^{n-1}$

$\displaystyle\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$

$\dfrac{d}{dx}(e^{ax}) = ae^{ax}$

$\displaystyle\int e^{ax} dx = \frac{1}{a}e^{ax} + c$

$\dfrac{d}{dx}(\log_e x) = \dfrac{1}{x}$

$\displaystyle\int \frac{1}{x} dx = \log_e|x| + c$

$\dfrac{d}{dx}(\sin(ax)) = a\cos(ax)$

$\displaystyle\int \sin(ax)\, dx = -\frac{1}{a}\cos(ax) + c$

$\dfrac{d}{dx}(\cos(ax)) = -a\sin(ax)$

$\displaystyle\int \cos(ax)\, dx = \frac{1}{a}\sin(ax) + c$

$\dfrac{d}{dx}(\tan(ax)) = \dfrac{a}{\cos^2(ax)} = a\sec^2(ax)$

$\displaystyle\int \sec^2(ax)\, dx = \frac{1}{a}\tan(ax) + c$

$\dfrac{d}{dx}(\sin^{-1}(x)) = \dfrac{1}{\sqrt{1-x^2}}$

$\displaystyle\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c, a > 0$

$\dfrac{d}{dx}(\cos^{-1}(x)) = -\dfrac{1}{\sqrt{1-x^2}}$

$\displaystyle\int \frac{-1}{\sqrt{a^2-x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, a > 0$

$\dfrac{d}{dx}(\tan^{-1}(x)) = \dfrac{1}{1+x^2}$

$\displaystyle\int \frac{a}{a^2+x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$

product rule $\dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

quotient rule $\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{\left(v\dfrac{du}{dx} - u\dfrac{dv}{dx}\right)}{v^2}$

chain rule $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx}$

Euler's method If $\frac{dy}{dx} = f(x), x_0 = a$ and $y_0 = a$, then $y_{n+1} = y_n + hf(x_n)$

acceleration $a = \dfrac{d^2x}{dt^2} = \dfrac{dv}{dt} = v\dfrac{dv}{dx} = \dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right)$

constant (uniform) acceleration $v = u + at$, $s = ut + \frac{1}{2}at^2$, $v^2 = u^2 + 2as$, $s = \frac{1}{2}(u+v)t$

Vectors in two and three dimensions

$\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$

$\dot{\boldsymbol{r}} = \frac{d\boldsymbol{r}}{dt} = \frac{dx}{dt}\boldsymbol{i} + \frac{dy}{dt}\boldsymbol{j} + \frac{dz}{dt}\boldsymbol{k}$

$|\boldsymbol{r}| = \sqrt{x^2 + y^2 + z^2} = r$

$\boldsymbol{r}_1.\boldsymbol{r}_2 = r_1 r_2 \cos\theta = x_1x_2 + y_1y_2 + z_1z_2$

Mechanics

momentum	$\boldsymbol{p} = m\boldsymbol{v}$
equation of motion	$\boldsymbol{R} = m\boldsymbol{a}$
friction	$F \leq \mu N$