



Units 3 and 4 Specialist Maths: Exam 1

Practice Exam Solutions

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Marks allocated are indicated by a number in square brackets, for example, [1] indicates that the line is worth one mark.

Question 1a

$$3xy^2 + 4y = 12 - 2x$$

$$3y^2 + 6xy \frac{dy}{dx} + 4 \frac{dy}{dx} = -2$$

implicit differentiation using the chain rule [1]

$$\frac{dy}{dx}(6xy + 4) = -2 - 3y^2$$

factorise out $\frac{dy}{dx}$

$$\frac{dy}{dx} = -\frac{3y^2+2}{6xy+4}$$

make $\frac{dy}{dx}$ the argument [1]

Question 1b

$$\frac{dy}{dx} = -\frac{3y^2+2}{6y^2+4}$$

substitute in $x = y$ or $y = x$

$$\frac{dy}{dx} = -\frac{3y^2+2}{2(3y^2+2)}$$

simplify the terms

$$\frac{dy}{dx} = -\frac{1}{2}$$

simplify [1]

Question 2

$$y = \int x\sqrt{x^2 + 9} dx + c$$

fundamental theorem of calculus

$$y = \frac{1}{2} \int 2x\sqrt{x^2 + 9} dx + c$$

add a $2x$ factor because of the x^2

Use substitution $u = x^2 + 9$, $\frac{du}{dx} = 2x$ [1]

$$y = \frac{1}{2} \int \frac{du}{dx} \sqrt{u} dx + c$$

substitute in known terms

$$y = \frac{1}{2} \int \sqrt{u} du + c$$

chain rule

$$y = \frac{1}{2} * \frac{2}{3} * u^{\frac{3}{2}} + c$$

integrate [1]

$$y = \frac{1}{3} (x^2 + 9)^{\frac{3}{2}} + c$$

replace u by $x^2 + 9$

Now $y = 8$ when $x = 0$, so:

$$8 = \frac{1}{3} (3^2)^{\frac{3}{2}} + c$$

substitute in known point

$$c = -1$$

[1]

$$\text{Hence, } y = \frac{1}{3} (x^2 + 9)^{\frac{3}{2}} - 1$$

[1]

Question 3a

$$y = \int -x + \frac{1}{x-2} dx$$

$$y = -\frac{x^2}{2} + \log_e(x-2) + c$$

[1] for integration, [1] for including c

Question 3b

Find the x-axis intercept:

$$\text{Let } y = 0, \text{ then } 0 = -x + \frac{1}{x-2}$$

$$0 = x^2 - 2x - 1$$

$$x = \frac{2 \pm \sqrt{4+4}}{2}$$

$$x = 1 \pm \sqrt{2} \quad [1]$$

Let A be the area:

$$A = -\int_{1+\sqrt{2}}^3 \left(-x + \frac{1}{x-2}\right) dx \quad [1]$$

$$A = -\left[-\frac{x^2}{2} + \log_e|x-2|\right]_{1+\sqrt{2}}^3 \quad \text{using answer from 3a}$$

$$A = \frac{9}{2} - \log_e 1 + \left(-\frac{(1+\sqrt{2})^2}{2} + \log_e|\sqrt{2}-1|\right)$$

$$A = \frac{9-1-2\sqrt{2}-2}{2} + \log_e(\sqrt{2}-1)$$

$$A = 3 - \sqrt{2} + \log_e(\sqrt{2}-1) \quad [2]$$

Question 4a

i is the motion in the x plane, j is the motion in the y plane [1 for this fact, or its use, doesn't need to be stated] Hence:

$$y = \sin^2 t$$

$$x = \cos 2t = 1 - 2\sin^2 t \quad [1]$$

$$\therefore x = 1 - 2y$$

$$y = \frac{1-x}{2} \quad [1]$$

Question 4b

A straight line between (1, 0) and (-1, 1). [1 mark for each endpoints (total 2), 1 mark for shape]

Question 5

$$z^2 + 2z + 5 = 0 \quad [1]$$

$$z = \frac{-2 \pm \sqrt{4-20}}{2}$$

$$z = \frac{-2 \pm \sqrt{16}i}{2} \quad [1]$$

$$z = -1 \pm 2i \quad [1]$$

Question 6

$$y = \sin(mx)$$

$$\frac{dy}{dx} = m \cos(mx)$$

$$\frac{d^2y}{dx^2} = -m^2 \sin(mx) \quad [1]$$

$$\sin(mx) = 3m^2 \sin(mx) \quad [1]$$

$$1 = 3m^2, \text{ and also } m = 0 \quad [1]$$

$$\frac{1}{3} = m^2$$

$$m = 0, \pm \frac{1}{\sqrt{3}} \quad [1]$$

Question 7a

$$-1 \leq \frac{x}{2} + 1 \leq 1 \quad [1]$$

$$-2 \leq \frac{x}{2} \leq 0$$

$$-4 \leq x \leq 0$$

Therefore the domain is $[-4, 0]$. [1]

Alternatively, you could build up the expression (rather than reducing it to x), as is used to find the range:

$$0 \leq \cos^{-1} a \leq \pi, \text{ where } a = \frac{x}{2} + 1 \quad [1]$$

$$0 \leq \frac{1}{\pi} \cos^{-1} a \leq 1$$

$$-2 \leq \frac{1}{\pi} \cos^{-1} a - 2 \leq -1$$

Therefore the range is $[-2, -1]$. [1]

Question 7b

$$\frac{d}{dx} \left(\frac{1}{\pi} \arccos \left(\frac{x}{2} + 1 \right) - 2 \right)$$

$$= \frac{1}{\pi} \frac{d}{dx} \left(\arccos \left(\frac{x}{2} + 1 \right) \right) \quad \text{take out constant, } \frac{d}{dx} \text{ of } 2 \text{ is } 0$$

Use chain rule where $u = \left(\frac{x}{2} + 1 \right)$: [1]

$$= \frac{-\frac{d}{dx} \left(\frac{x}{2} + 1 \right)}{\pi \sqrt{1 - \left(\frac{x}{2} + 1 \right)^2}} \quad [1]$$

$$= \frac{-1}{2\pi \sqrt{1 - \left(\frac{x}{2} + 1 \right)^2}} \quad [1]$$

$$= \frac{-1}{\pi \sqrt{-x(x+4)}} \quad [2]$$

Hence, $a = -1$, $b = -1$, and $c = 4$

Question 8a

$$v(t) = 0$$

$$\frac{5(1-2t)}{1-2t} = 0$$

$$t = \frac{1}{2} \quad [1]$$

Question 8b

$$v(t) = \frac{5(1-2t)}{1+2t} = \frac{10}{1+2t} - 5 \quad [1]$$

From part a, the particle is moving forwards for $0 \leq t \leq \frac{1}{2}$ and backwards for $\frac{1}{2} \leq t \leq 1$.

Therefore,

$$d = \int_0^{\frac{1}{2}} \left(\frac{10}{1+2t} - 5 \right) dt - \int_{\frac{1}{2}}^1 \left(\frac{10}{1+2t} - 5 \right) dt \quad [1]$$

$$= [5 \log_e(1+2t) - 5t]_0^{\frac{1}{2}} - [5 \log_e(1+2t) - 5t]_{\frac{1}{2}}^1$$

$$= 5 \log_e 2 - \frac{5}{2} - (5 \log_e 3 - 5) + 5 \log_e 2 - \frac{5}{2} \quad [1]$$

$$= 10 \log_e 2 - 5 \log_e 3$$

$$= 5 \log_e 4 - 5 \log_e 3$$

$$= 5 \log_e \frac{4}{3} \quad [1]$$