

THIS BOX IS FOR ILLUSTRATIVE PURPOSES ONLY

**‘2016 Examination Package’ -
Trial Examination 2 of 5**

STUDENT NUMBER

Figures

Words

Letter

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SPECIALIST MATHEMATICS

Units 3 & 4 – Written examination 2

(TSSM’s 2012 trial exam updated for the current study design)

Reading time: 15 minutes

Writing time: 2 hours

QUESTION & ANSWER BOOK

Structure of book

<i>Section</i>	<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
1	22	22	22
2	5	5	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved graphics calculator or approved CAS calculator and a scientific calculator.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question book of 33 pages, including a multiple choice answer sheet.

Instructions

- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other electronic devices into the examination room.

SECTION 1

Instructions for Section 1

Answer **all** questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks are **not** deducted for incorrect answers.

If more than 1 answer is completed for any question, no mark will be given.

Take the **acceleration due to gravity**, to have magnitude $g \text{ m/s}^2$, where $g = 9.8$

Question 1

The asymptotes $y = \frac{2}{3}x - \frac{11}{3}$ and $y = -\frac{2}{3}x - \frac{7}{3}$ are characteristic of the hyperbola

A. $9(y+3)^2 - 4(x-1)^2 = 36$

B. $4(y+3)^2 - 9(x-1)^2 = 36$

C. $\frac{(x+1)^2}{9} - \frac{(y-3)^2}{4} = 1$

D. $9(x+1)^2 - 4(y-3)^2 = 36$

E. $\frac{(x-1)^2}{4} - \frac{(y+3)^2}{9} = 1$

Question 2

The graph of $y = \frac{-2}{2x^2 + kx + 5}$ has two vertical asymptotes for

A. $k \in (-\infty, -2\sqrt{10}] \cup [2\sqrt{10}, \infty)$

B. $k \in [-2\sqrt{10}, 2\sqrt{10}]$

C. $k \in (2\sqrt{10}, \infty)$

D. $k \in (-\infty, -2\sqrt{10}) \cup (2\sqrt{10}, \infty)$

E. $k \in (-2\sqrt{10}, 2\sqrt{10})$

SECTION 1 - continued

Question 3

The implied domain and range of the function with rule $f(x) = 2 \cos^{-1}(3x - 6) - 1$ respectively are

- A. $[-1, 1]$ and $[0, \pi]$
- B. $[-9, -3]$ and $[-1, 2\pi - 1]$
- C. $\left[\frac{5}{3}, \frac{7}{3}\right]$ and $[-1, 2\pi - 1]$
- D. $\left[-\frac{7}{3}, -\frac{5}{3}\right]$ and $[-1, 2\pi - 1]$
- E. $\left[\frac{5}{3}, \frac{7}{3}\right]$ and $[1, 2\pi + 1]$

Question 4

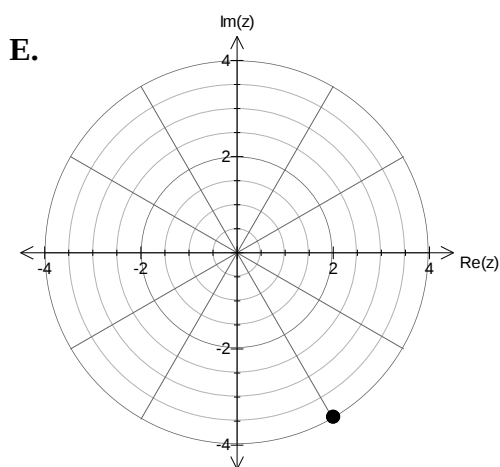
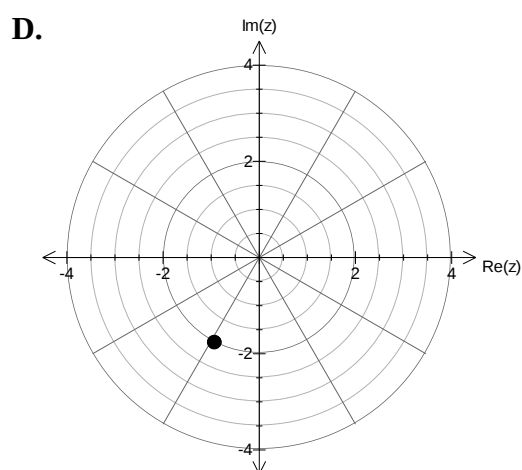
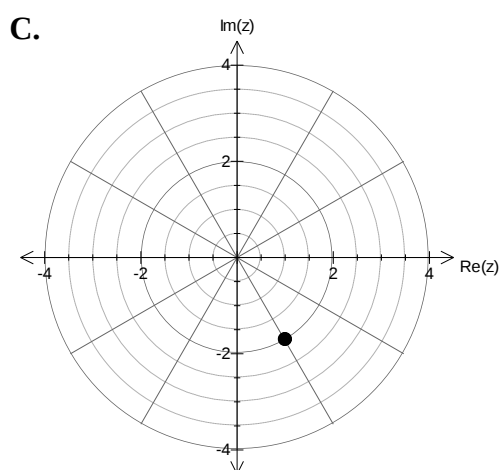
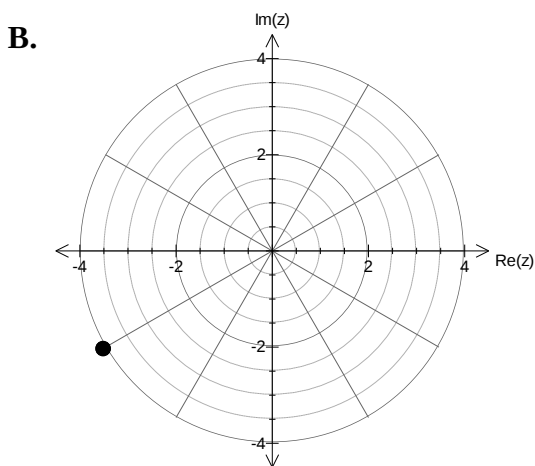
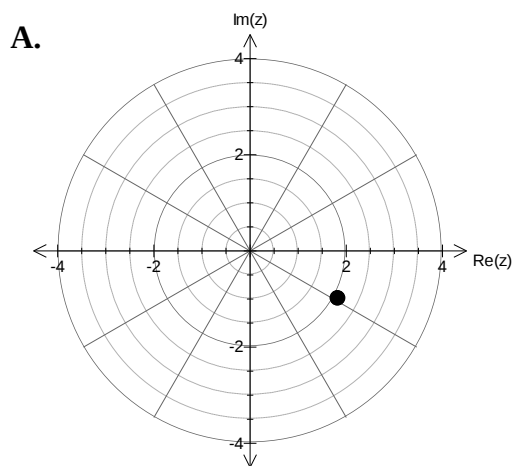
The fourth roots of $-8i$ are

- A. $2 \operatorname{cis}\left(-\frac{5\pi}{8}\right)$, $2 \operatorname{cis}\left(-\frac{\pi}{8}\right)$, $2 \operatorname{cis}\left(\frac{3\pi}{8}\right)$ and $2 \operatorname{cis}\left(\frac{7\pi}{8}\right)$
- B. $2^{\frac{3}{4}} \operatorname{cis}\left(-\frac{5\pi}{8}\right)$, $2^{\frac{3}{4}} \operatorname{cis}\left(-\frac{\pi}{8}\right)$, $2^{\frac{3}{4}} \operatorname{cis}\left(\frac{3\pi}{8}\right)$ and $2^{\frac{3}{4}} \operatorname{cis}\left(\frac{7\pi}{8}\right)$
- C. $2^{\frac{3}{4}} \operatorname{cis}\left(-\frac{3\pi}{8}\right)$, $2^{\frac{3}{4}} \operatorname{cis}\left(\frac{\pi}{8}\right)$, $2^{\frac{3}{4}} \operatorname{cis}\left(\frac{5\pi}{8}\right)$ and $2^{\frac{3}{4}} \operatorname{cis}\left(\frac{9\pi}{8}\right)$
- D. $2^{\frac{3}{4}} \operatorname{cis}\left(-\frac{\pi}{2}\right)$, $2^{\frac{3}{4}} \operatorname{cis}(0)$, $2^{\frac{3}{4}} \operatorname{cis}\left(\frac{\pi}{2}\right)$ and $2^{\frac{3}{4}} \operatorname{cis}(\pi)$
- E. $\pm 2 \operatorname{cis}\left(-\frac{\pi}{4}\right)$ and $\pm 2 \operatorname{cis}\left(\frac{\pi}{4}\right)$

SECTION 1 - continued
TURN OVER

Question 5

A complex number, $z \in \mathbb{C}$ is defined as $z = -2 + 2\sqrt{3}i$. The complex number $\sqrt{z}$ is best represented on the argand diagram by



Question 6

$\operatorname{Im}\left(\frac{(-1-i\sqrt{3})^2}{(\sqrt{3}+i)^3}\right)$ is equal to

- A. $-\frac{1}{4}$
- B. $\frac{1}{4}$
- C. $\frac{i}{4}$
- D. $-\frac{1}{2}\sin\left(\frac{\pi}{12}\right)$
- E. $\frac{1}{2}$

Question 7

If $\sec A = \frac{5}{2}$ and $\operatorname{cosec} B = -3$, where $A \in \left(0, \frac{\pi}{2}\right)$ and $B \in \left(\frac{3\pi}{2}, 2\pi\right)$, then $\sin(A+B)$ is equal to

- A. $\frac{1}{15}(\sqrt{21} - 4\sqrt{2})$
- B. $\frac{2}{15}(\sqrt{42} + 1)$
- C. $\frac{1}{15}(4\sqrt{2} - \sqrt{21})$
- D. $\frac{11}{15}$
- E. $\frac{2}{15}(\sqrt{42} - 1)$

SECTION 1 – continued
TURN OVER

Question 8

Given $\underline{a} = 2\underline{i} - 3\underline{j} + \underline{k}$, $\underline{b} = \underline{i} + 2\underline{j} - 3\underline{k}$ and $\underline{c} = \underline{i} - \underline{j} - \underline{k}$, the vector resolute of $\underline{a}$ in the direction of $\underline{b} - \underline{c}$ is equal to

- A. $\frac{11}{13}(3\underline{j} - 2\underline{k})$
- B. $-\frac{1}{13}(26\underline{i} - 6\underline{j} - 9\underline{k})$
- C. $-\frac{11}{13}(3\underline{j} - 2\underline{k})$
- D. $\frac{1}{13}(26\underline{i} - 6\underline{j} - 9\underline{k})$
- E. $\frac{11\sqrt{13}}{13}(2\underline{k} - 3\underline{j})$

Question 9

If M divides the line segment AB internally in the ratio $5:4$ and $\overrightarrow{OM}$ is the position vector of M relative to the origin, O , then $\overrightarrow{OM}$ is equal to

- A. $\frac{1}{9}(\overrightarrow{OA} + \overrightarrow{OB})$
- B. $\frac{1}{9}(4\overrightarrow{OA} - 5\overrightarrow{OB})$
- C. $\frac{1}{9}(5\overrightarrow{OA} + 4\overrightarrow{OB})$
- D. $-\frac{1}{9}(4\overrightarrow{AO} + 5\overrightarrow{BO})$
- E. $-\frac{5}{4}(\overrightarrow{OA} + \overrightarrow{OB})$

Question 10

On the Cartesian Plane, let the positive x -direction be aligned with $\mathbf{i}$ and the positive y -direction be aligned with $\mathbf{j}$. A vector of length 12 units that is perpendicular to the line $3x - 5y + 10 = 0$ could be equal to

- A. $\frac{6\sqrt{34}}{17}(3\mathbf{i} - 5\mathbf{j})$
- B. $-\frac{6\sqrt{34}}{17}(5\mathbf{i} + 3\mathbf{j})$
- C. $-\frac{\sqrt{34}}{34}(-5\mathbf{i} + 3\mathbf{j})$
- D. $12(3\mathbf{i} + 5\mathbf{j})$
- E. $12(5\mathbf{i} - 3\mathbf{j})$

Question 11

The position vector of a particle is described as $\mathbf{r}(t) = (1 + \cos 2t)\mathbf{i} + \sin t \mathbf{j}$, $t \in \left[0, \frac{\pi}{2}\right]$.

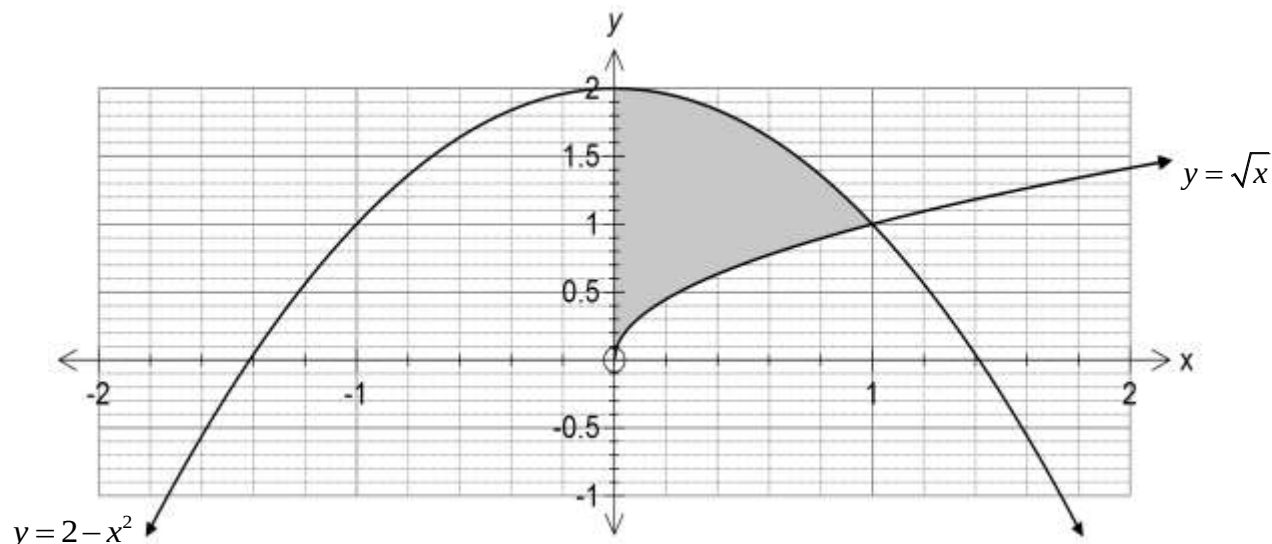
The Cartesian equation of the path of the particle is

- A. $y = -\frac{\sqrt{4-2x}}{2}$
- B. $y = \pm\sqrt{1-\frac{x}{2}}$
- C. $y = \frac{\sqrt{4-2x}}{2}$
- D. $y = -\sqrt{\frac{x}{2}-1}$
- E. $y = \frac{\sqrt{4+2x}}{2}$

SECTION 1 - continued
TURN OVER

Question 12

The area enclosed by $y = 2 - x^2$, $y = \sqrt{x}$ and the y -axis, as shown below, is rotated 2π radians about the x -axis.



The volume of the solid of revolution formed is represented by

- A. $\pi \int_0^1 y^2 dy + \pi \int_1^2 \sqrt{2-y} dy$
- B. $\pi \int_0^1 2 - x^2 - \sqrt{x} dx$
- C. $\pi \int_0^1 y^4 dy + \pi \int_1^2 2 - y dy$
- D. $\pi \int_0^1 (2 - x^2 - \sqrt{x})^2 dx$
- E. $\pi \int_0^1 (2 - x^2)^2 - x dx$

Question 13

Using a suitable substitution, the definite integral $\int_{-1}^2 \frac{x}{\sqrt{1-2x}} dx$ is equivalent to

- A. $\frac{1}{4} \int_{-3}^3 \frac{u-1}{\sqrt{u}} du$
- B. $\frac{1}{4} \int_{-3}^3 \frac{1-u}{\sqrt{u}} du$
- C. $\frac{1}{4} \int_3^{-3} \frac{1-u}{\sqrt{u}} du$
- D. $\frac{1}{4} \int_{-1}^2 \frac{1-u}{\sqrt{u}} du$
- E. $\frac{1}{2} \int_{-3}^3 \frac{1-u}{\sqrt{u}} du$

Question 14

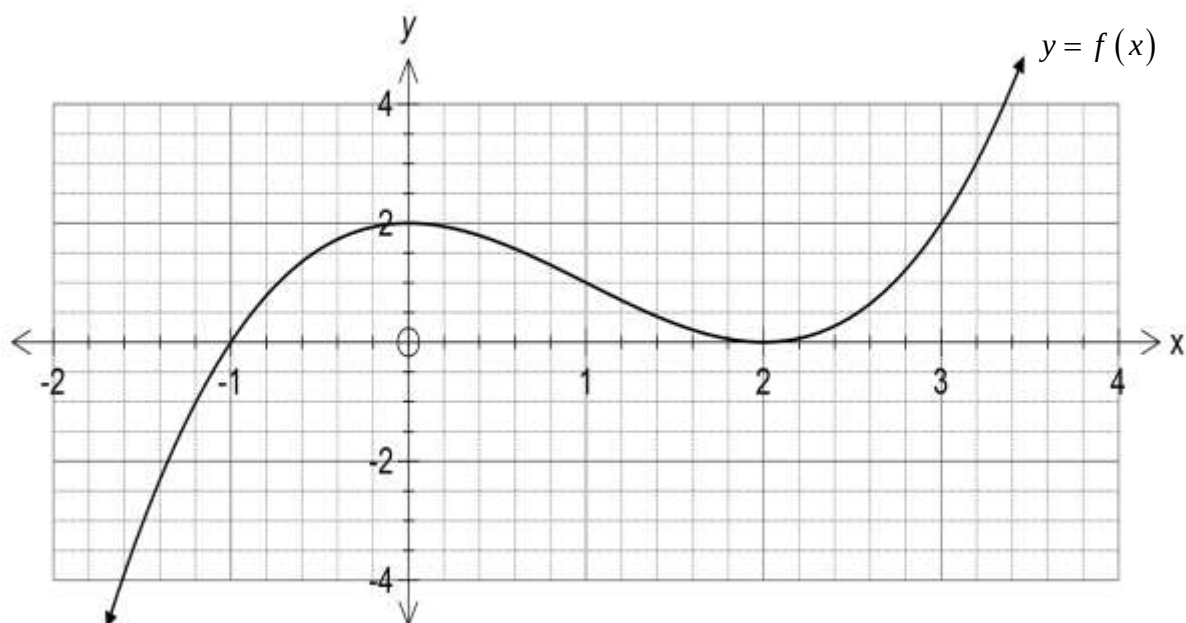
The tangent to the curve $x^2y - y^2 = 10$ is undefined at the point

- A. $(2, -2)$
- B. $(2\sqrt{2}, -4)$
- C. $(\sqrt{7}, -3)$
- D. $(-2\sqrt{2}, 4)$
- E. $(-\sqrt{10}, -5)$

SECTION 1 - continued
TURN OVER

Question 15

The graph of a function, $y = f(x)$ is provided below.



The graph of an anti-derivative function, $y = F(x)$ could have

- A. A local maximum point at $(-1, 0)$, a non-stationary point of inflection at $(1, 1)$ and a stationary point of inflection at $(2, 0)$
- B. A local minimum point at $(-1, 0)$, a non-stationary point of inflection at $(0, 2)$ and a stationary point of inflection at $(2, 0)$
- C. A local maximum point at $(0, 2)$, a non-stationary point of inflection at $(1, 1)$ and a local minimum point at $(2, 0)$
- D. A local maximum point at $(-1, 0)$, a stationary point of inflection at $(0, 2)$ and a non-stationary point of inflection at $(2, 0)$
- E. A local minimum point at $(-1, 0)$, a non-stationary point of inflection at $(1, 1)$ and a stationary point of inflection at $(2, 0)$

SECTION 1 – continued

Question 16

Euler's method with a step size of 0.2 is used to solve the differential equation

$$\frac{dy}{dx} = 2 \cos^{-1}\left(\frac{x}{3}\right) \text{ with } x_0 = 1 \text{ and } y_0 = 2.$$

The value of y_3 correct to four decimal places is equal to

- A. 3.3902
- B. 2.4924
- C. 3.8243
- D. 2.4923
- E. 2.9561

Question 17

A partly filled tank contains 300 litres of water in which 1200 grams of salt has been dissolved. Water is poured into the tank at the rate of 8 litres per minute. The mixture is kept uniform by stirring and it leaves the tank through a hole at the rate of 6 litres per minute.

There are x grams of salt in the tank after t minutes.

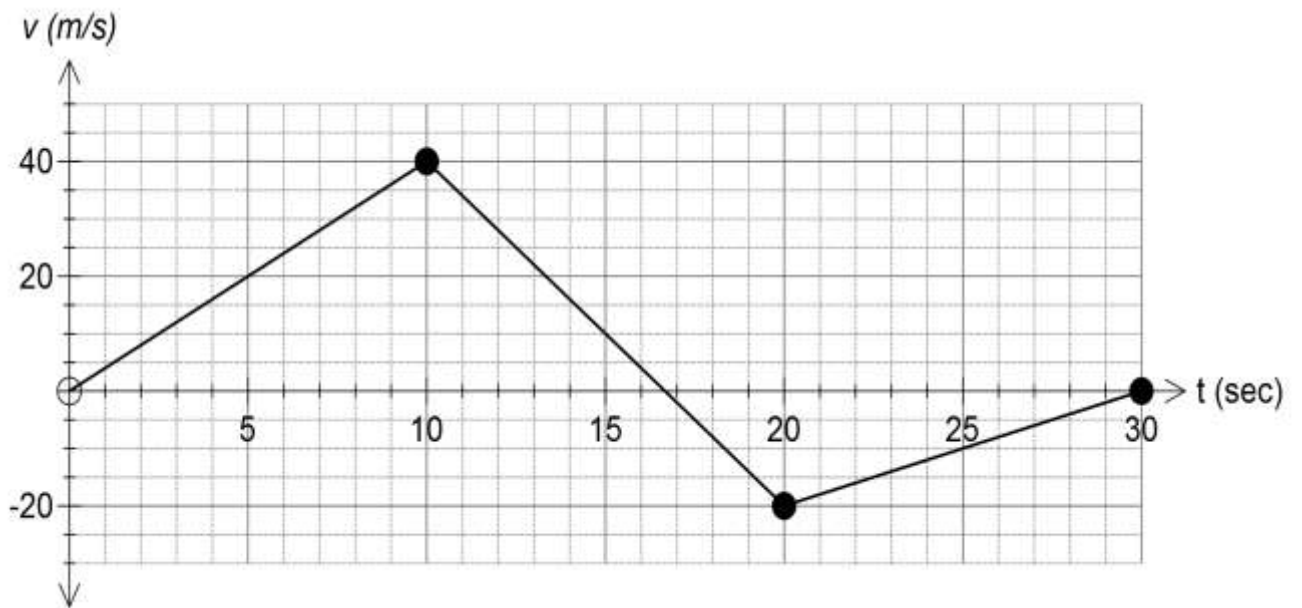
An expression for the differential equation describing this situation is

- A. $\frac{dx}{dt} = 8 - \frac{3x}{150+t}$, where $x = 300$ litres when $t = 0$
- B. $\frac{dx}{dt} = \frac{-x}{300+2t}$, where $x = 1200$ grams when $t = 0$
- C. $\frac{dx}{dt} = \frac{-3x}{150-t}$, where $x = 1200$ grams when $t = 0$
- D. $\frac{dx}{dt} = 8 - \frac{3x}{150+t}$, where $x = 1200$ grams when $t = 0$
- E. $\frac{dx}{dt} = \frac{-3x}{150+t}$, where $x = 1200$ grams when $t = 0$

SECTION 1 - continued
TURN OVER

Question 18

The velocity-time graph for a particle is provided below.



The total distance travelled by the particle, in metres, is equal to

- A. $\frac{1400}{3}$
- B. 200
- C. $\frac{1000}{3}$
- D. 470
- E. 1800

SECTION 1 – continued

Question 19

An object is projected vertically upwards from ground level with an initial velocity of 20ms^{-1} . Two seconds later, a second object is projected vertically upwards from the same location with an initial velocity of 18ms^{-1} .

Ignoring any effects due to air resistance, it can be determined that

- A. The two objects will collide at a height greater than 15 metres above ground level
- B. The two objects will collide approximately 0.76 seconds after the projection of the first object
- C. The two objects will collide approximately 3.16 seconds after the projection of the second object
- D. The two objects will collide approximately 0.76 seconds after the projection of the second object
- E. The two objects will collide approximately 3.16 seconds after the projection of the first object

Question 20

The acceleration of a particle is defined as $a(v) = v^2 + 4$, where $v = 2\text{ms}^{-1}$ when $x = 1$ metre.

An expression for the velocity, $v(x)$ is

- A. $\log_e \sqrt{\frac{x^2 + 4}{5}} + 2$
- B. $2\sqrt{2e^{2(x-1)} - 1}$
- C. $2 \tan\left(\frac{4x - 4 + \pi}{4}\right)$
- D. $\sqrt{4 - e^{2(x-1)}}$
- E. $\frac{1}{2} \tan(4x - 4 + \pi) + 4$

SECTION 1 - continued
TURN OVER

Question 21

Three forces $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$ act on a particle, causing the particle to move in a straight line with an acceleration of 2.5ms^{-2} . If $\vec{F}_1 = 2\vec{i} - 3\vec{j} - \vec{k}$, $\vec{F}_2 = -\vec{i} + 4\vec{j} + 2\vec{k}$ and $\vec{F}_3 = \vec{i} + 4\vec{j} - 2\vec{k}$ newtons, then the mass of the particle is equal to

- A. $\frac{5\sqrt{30}}{2}$ kg
- B. $\frac{2\sqrt{30}}{5}$ kg
- C. $\frac{\sqrt{30}}{12}$ kg
- D. $\frac{5\sqrt{7}}{4}$ kg
- E. $\frac{4\sqrt{7}}{5}$ kg

Question 22

The acceleration of a 25kg block sliding down a frictionless plane making an angle of 46° with the horizontal is:

- A. 3.05 m/s^2
- B. 6.81 m/s^2
- C. 6.95 m/s^2
- D. 7.05 m/s^2
- E. 7.0 m/s^2

**END OF SECTION 1
TURN OVER**

SECTION 2

Instructions for Section 2

Answer **all** questions.

A decimal approximation will not be accepted if the question specifically asks for an **exact** answer.

For questions worth more than one mark, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams are **not** drawn to scale.

Take the **acceleration due to gravity**, to have magnitude $g \text{ m/s}^2$, where $g = 9.8$.

Question 1

a. Show that $\text{cis}\left(\theta - \frac{3\pi}{2}\right) = -\sin \theta + i \cos \theta$.

2 marks

SECTION 2 – Question 1- continued

b. Hence or otherwise, show that $\frac{1}{\operatorname{cis}\left(\theta - \frac{3\pi}{2}\right)} = -\operatorname{cis}\left(\frac{\pi}{2} - \theta\right)$.

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

3 marks

c. Expand $(-\sin \theta + i \cos \theta)^4$ without simplifying terms.

1 mark

SECTION 2 – Question 1- continued
TURN OVER

- d.** Use de Moivre's theorem to show that $\left(\operatorname{cis}\left(\theta - \frac{3\pi}{2}\right)\right)^4 = \operatorname{cis}(4\theta)$.

2 marks

- e.** Hence, find $\cos(4\theta)$ in terms of $\sin \theta$.

2 marks

- f.** Hence, find the exact value of $\cos(4\theta)$ when $\cot \theta = -\frac{5}{2}$, $\theta \in \left(\frac{\pi}{2}, \pi\right)$.

2 marks

Total 12 marks

SECTION 2- continued

b. A differential equation is defined as

$$2f''(x) + \frac{1}{2}f'(x) + m = 0$$

- i.** For $x \in [-\pi, \pi]$, find the values of m , correct to three decimal places, for which there are exactly three solutions to the differential equation.

2 marks

- ii.** Hence, solve the differential equation $2f''(x) + \frac{1}{2}f'(x) - \frac{1}{4} = 0$ for $x \in [-\pi, \pi]$, correct to two decimal places.

1 mark

The equation $f(x) = \frac{1}{e^x \sec x}$ is reflected in the x -axis, dilated by a factor of 2 away from the x -axis and then translated $\frac{\pi}{8}$ units in the positive x -direction.

- c.** State the transformed equation, $g(x)$ in the form $g(x) = af(x-h)$

1 mark

SECTION 2 – Question 2- continued

- d.** For $x \in [-\pi, \pi]$, state the coordinates of the points of intersection of the curves $y = f(x)$ and $y = g(x)$, correct to two decimal places.

2 marks

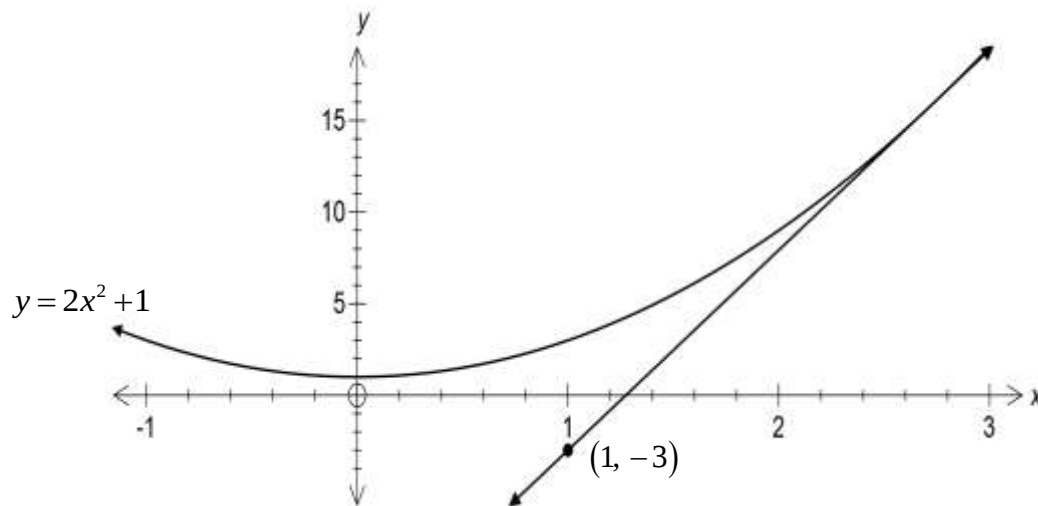
- e.** Hence, for $x \in [-\pi, \pi]$, find the area bounded by the curves $y = f(x)$ and $y = g(x)$, correct to one decimal place.

1 mark
Total 10 marks

SECTION 2- continued
TURN OVER

Question 3

A tangent drawn to the curve $y = 2x^2 + 1$ passes through the point $(1, -3)$ as shown on the graph below.



- a. Show that the tangent touches the curve at the point $(\sqrt{3} + 1, 4\sqrt{3} + 9)$

3 marks

SECTION 2 – Question 3- continued

- d.** The bounded area in **part c.** is rotated 2π radians about the y -axis. Find the exact volume of the solid of revolution formed..

[illegible]

3 marks

Total 10 marks

SECTION 2- continued

Question 4

The position vectors of two remotely controlled model racing cars, A and B are defined as

$$\underline{r}_A = (t+3)\underline{i} + (t^2 - 4t)\underline{j}, \quad t \geq 0$$

$$r_B = \left(\frac{t^2 - 8}{2} \right) \tilde{i} + (t - 3) \tilde{j}, \quad t \geq 0$$

where displacement components are measured in metres.

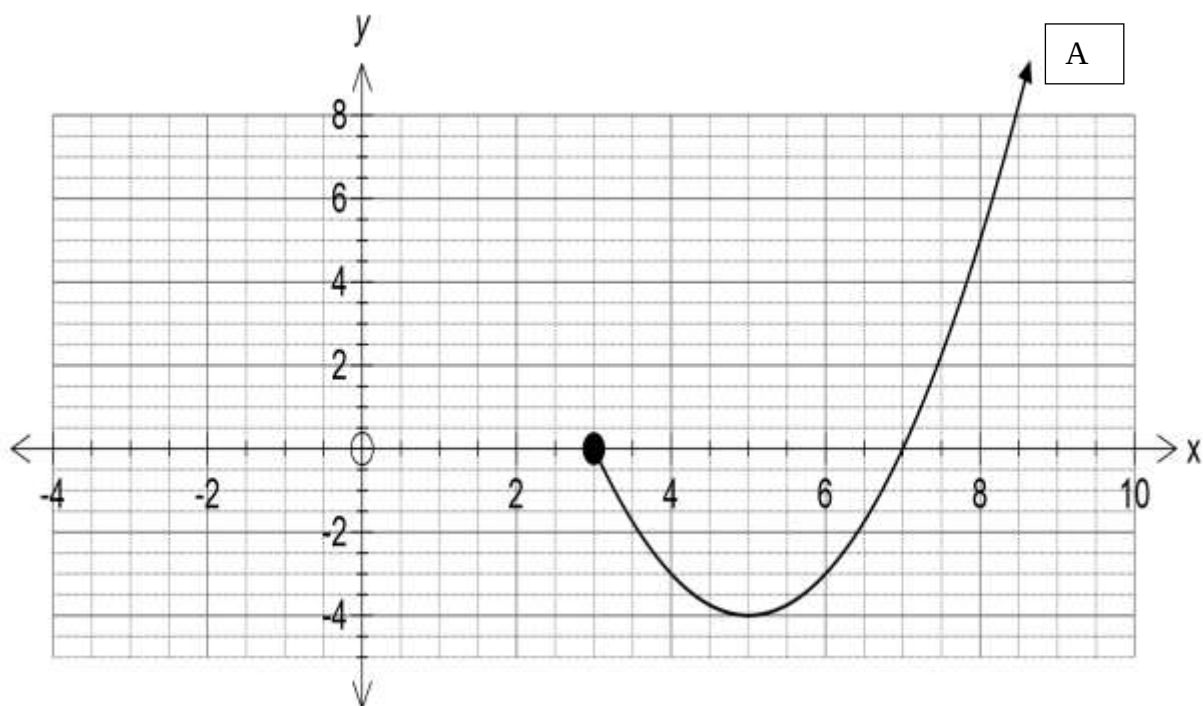
- a.** The Cartesian equation of the path of car A is $y = x^2 - 10x + 21$, where $x \geq 3$. Find the Cartesian equation of the path of car B in the form $y = f(x)$ and state the implied domain.

[illegible]

2 marks

SECTION 2 – Question 4- continued
TURN OVER

- b. The path of car A is provided on the set of axes below.
On the same set of axes sketch the path of car B over its implied domain.



1 mark

- c. Verify that the cars do not collide.

2 marks

SECTION 2 – Question 4- continued

- d.** Find, to the nearest degree, the acute angle between the paths of the two model cars at the point where their paths intersect.

3 marks

- e.** Find an expression for the distance between the two model cars in terms of t .

1 mark

- f.** Hence, find the times, in seconds, when the two model cars are 5 metres apart, correct to two decimal places.

1 mark

SECTION 2 – Question 4- continued
TURN OVER

- g.** Find expressions for the velocities of the two model cars, $v_A(t)$ and $v_B(t)$.

1 mark

- h.** Hence, find the exact time, in seconds, when the two model cars are travelling perpendicular to each other.

2 marks
Total 13 marks

SECTION 2- continued

Question 5

The ‘*Burj Khalifa*’ in downtown Dubai is the tallest building in the world. Its height is 828 metres. Alex, a keen BASE jumper, plans to free-fall from its highest point for a distance and then parachute to the ground below.

In readiness, Alex stands atop the ‘*Burj Khalifa*’ at exactly 828 metres above ground level. Alex then falls for exactly 90 metres before opening his parachute. In free-fall, his initial velocity is zero and any effects due to air resistance are considered to be negligible.

- a. Find Alex’s speed after he has free-fallen for exactly 90 metres.

1 mark

After Alex has fallen 90 metres, his parachute opens instantaneously. He immediately experiences an air resistance force of $5v^2$ newtons, where $v \text{ ms}^{-1}$ is the velocity of Alex, t seconds after his parachute has opened.

- b. If Alex (with gear) weighs 85 kg show that his acceleration, $a \text{ ms}^{-2}$ can be expressed as

$$a = \frac{17g - v^2}{17}.$$

2 marks

SECTION 2 – Question 5- continued
TURN OVER

- c. Hence, show that the speed of Alex's descent is given by $v(x) = \sqrt{1597.4e^{\frac{2x}{17}} + 166.6}$, where $x \in [0, 738]$ metres is the vertical distance travelled after his parachute has opened.

[illegible]

4 marks

SECTION 2 – Question 5- continued

- d.** Find, correct to two decimal places, Alex's 'limiting (or terminal) speed' after his parachute has opened.

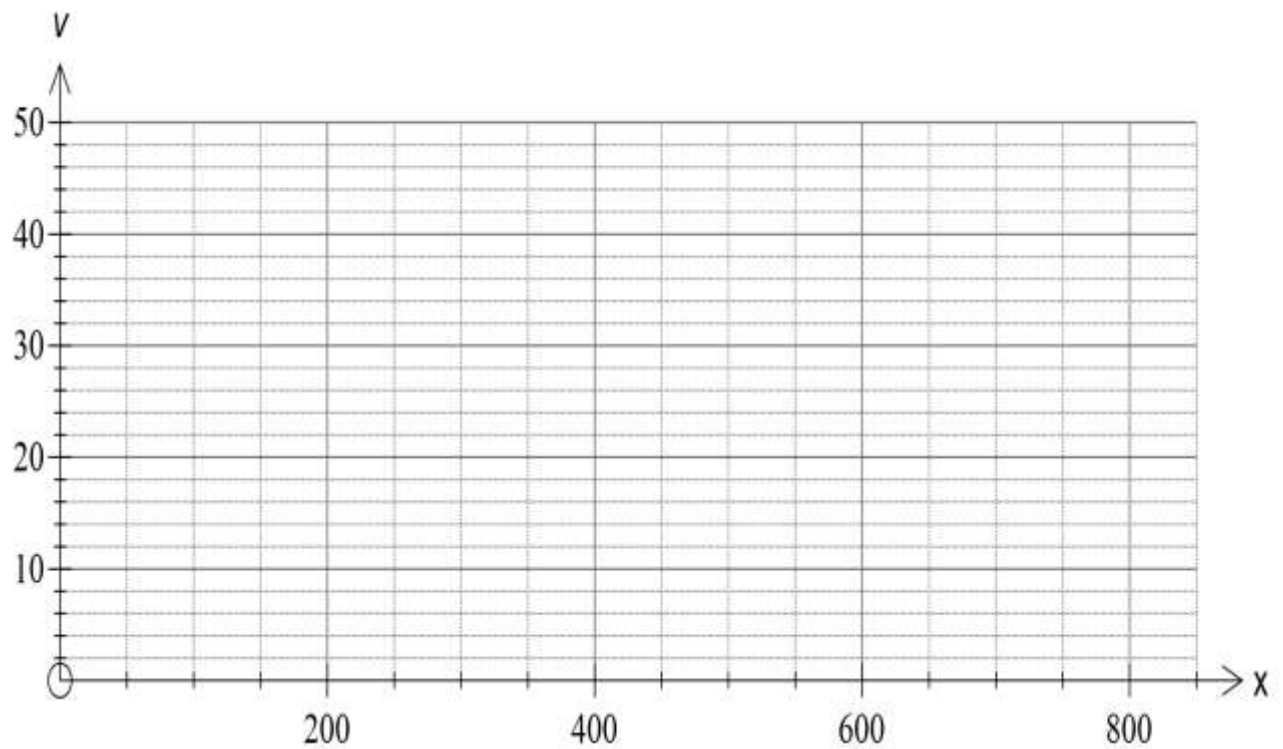
1 mark

- e.** While his parachute is opened, determine the vertical distance that Alex is from ground level, correct to the nearest metre, at the point when he is descending at twice his 'limiting speed'.

2 marks

SECTION 2 – Question 5- continued
TURN OVER

- f. Sketch a velocity-time graph for Alex's motion on the axes provided below for $x \in [0, 828]$, displaying all key features.



3 marks
Total 13 marks

END OF QUESTION AND ANSWER BOOK

Multiple Choice Answer Sheet

Circle or shade in the correct response

1.	A	B	C	D	E
2.	A	B	C	D	E
3.	A	B	C	D	E
4.	A	B	C	D	E
5.	A	B	C	D	E
6.	A	B	C	D	E
7.	A	B	C	D	E
8.	A	B	C	D	E
9.	A	B	C	D	E
10.	A	B	C	D	E
11.	A	B	C	D	E
12.	A	B	C	D	E
13.	A	B	C	D	E
14.	A	B	C	D	E
15.	A	B	C	D	E
16.	A	B	C	D	E
17.	A	B	C	D	E
18.	A	B	C	D	E
19.	A	B	C	D	E
20.	A	B	C	D	E
21.	A	B	C	D	E
22.	A	B	C	D	E