

Trial Examination 2012

VCE Specialist Mathematics Units 3 & 4

Written Examination 2

Suggested Solutions

SECTION 1

1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E

12	A	B	C	D	E
13	A	B	C	D	E
14	A	B	C	D	E
15	A	B	C	D	E
16	A	B	C	D	E
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18	A	B	C	D	E
19	A	B	C	D	E
20	A	B	C	D	E
21	A	B	C	D	E
22	A	B	C	D	E

Question 1 D

Using the Proper Fraction command with a CAS,

$$\frac{2x^2 - x + 3}{4 - x} = -\frac{31}{x - 4} - 2x - 7$$

oblique asymptote occurs as $x \rightarrow \pm\infty$ $\therefore$ oblique asymptote $y = -2x - 7$

Question 2 C

The ellipse has centre $(-5, 3)$, a horizontal semi-axis of 5, and a vertical semi-axis of 3.

$$\text{Therefore } \frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \text{ becomes } \frac{(x-(-5))^2}{5^2} + \frac{(y-3)^2}{3^2} = 1$$

$$\therefore 225\left(\frac{(x+5)^2}{25} + \frac{(y-3)^2}{9}\right) = 1$$

$$\therefore (9(x+5)^2 + 25(y-3)^2) = 225 \text{ gives C.}$$

Question 3 E

A hyperbola of the form $\frac{x^2}{9} - \frac{y^2}{16} = 1$ has x -intercepts of $(-3, 0)$ and $(3, 0)$, and therefore domain $x \in R \setminus (-3, 3)$.

Translating this hyperbola 2 units to the right, $\frac{(x-2)^2}{9} - \frac{y^2}{16} = 1$ will give domain $x \in R \setminus \{-3+2, 3+2\}$, i.e. $x \in R \setminus \{-1, 5\}$.

Question 4 C

The range of $y = \sin^{-1}(x)$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

The range of $y = a\sin^{-1}(x)$ is $\left[-\frac{\pi}{2} \times a, \frac{\pi}{2} \times a\right]$

The range of $y = a\sin^{-1}(x) + c$ is $\left[-\frac{a\pi}{2} + c, \frac{a\pi}{2} + c\right]$, which is equivalent to $\left[\frac{-a\pi - 2c}{2}, \frac{a\pi + 2c}{2}\right]$

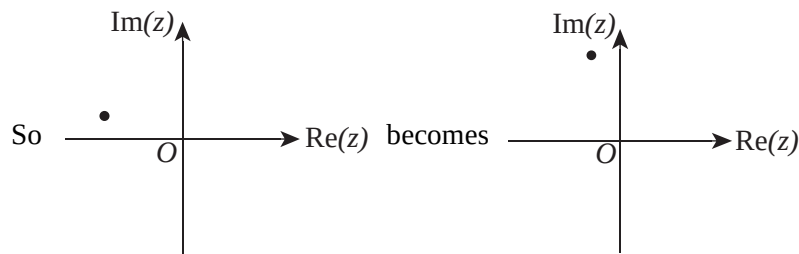
Question 5 D

If $z = r\text{cis}(\theta)$, then $\frac{\bar{z}}{i} = \frac{r\text{cis}(-\theta)}{\text{cis}(\frac{\pi}{2})} = r\text{cis}(-\theta - \frac{\pi}{2}) = r\text{cis} - \left[\theta + \frac{\pi}{2}\right]$.

This represents a rotation of $\frac{\pi}{2}(90^\circ)$ counter-clockwise followed by a reflection of z in the real axis giving alternative **D**.

Or let $z = -a + bi$ where $a > 0, b > 0$

$$\begin{aligned}\therefore \frac{\bar{z}}{i} &= \frac{-a - bi}{i} \\ &= \frac{-a - bi}{i} \times \frac{i}{i} \\ &= ai - b\end{aligned}$$

**Question 6 C**

If $z^5 = ai$

$$z^5 = a\text{cis}\left(\frac{\pi}{2}\right)$$

$$z = \sqrt[5]{a}\text{cis}\left(\frac{\pi}{10} + \frac{2k\pi}{5}\right), k = 0, 1, 2, 3, 4$$

$$= \sqrt[5]{a}\text{cis}\left(\frac{\pi}{10} \text{ or } \frac{5\pi}{10} \text{ or } \frac{9\pi}{10} \text{ or } \frac{13\pi}{10} \text{ or } \frac{17\pi}{10}\right)$$

$$= \sqrt[5]{a}\text{cis}\left(\frac{\pi}{10} \text{ or } \frac{\pi}{2} \text{ or } \frac{9\pi}{10} \text{ or } \frac{-7\pi}{10} \text{ or } \frac{-3\pi}{10}\right)$$

We require $\text{Arg}(z) > 0$. This leaves $\frac{\pi}{10}, \frac{\pi}{2}, \frac{9\pi}{10}$.

There are 3 solutions.

Question 7 D

As all coefficients are real, $z = 1 + i$ is also a solution.

$$(z - 3)(z - 1 + i)(z - 1 - i) = 0$$

$$z^3 - 5z^2 + 8z - 6 = 0$$

$$z^3 - 5z^2 + 8z = 6$$

Question 8 E

$$|z - ai| - |z + a| = 0$$

$$|z - ai| = |z + a|$$

$$|x + (y - a)i| = |(x + a) + yi|$$

$$\sqrt{x^2 + (y - a)^2} = \sqrt{(x + a)^2 + y^2}$$

$$x^2 + y^2 - 2ay + a^2 = x^2 + 2ax + a^2 + y^2$$

$$-2ay = 2ax$$

$$y = -x$$

i.e. a straight line through origin, gradient = -1

Question 9 D

Using the Algebra Solve command with a CAS suggests $x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$ or 2π over $x \in [0, 2\pi]$

However, $\cos(x) \neq 0$

$$\therefore x \neq \frac{\pi}{2} \text{ or } \frac{3\pi}{2}$$

$$\therefore x = 0 \text{ or } \pi \text{ or } 2\pi, \text{ which sum to } 3\pi.$$

Question 12 D

$\mathbf{a} = 4\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$ and $\mathbf{b} = 3\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$, so $\mathbf{a} - \mathbf{b} = \mathbf{i} + \mathbf{k}$.

If the angle between $\mathbf{a}$ and $\mathbf{a} - \mathbf{b}$ is θ , then $\cos(\theta) = \frac{(\mathbf{a} - \mathbf{b}) \cdot \mathbf{a}}{|\mathbf{a} - \mathbf{b}||\mathbf{a}|} = \frac{(\mathbf{i} + \mathbf{k}) \cdot (4\mathbf{i} + 2\mathbf{j} + 4\mathbf{k})}{\sqrt{1^2 + 1^2}\sqrt{4^2 + 2^2 + 4^2}}$

$$\cos(\theta) = \frac{4 + 4}{\sqrt{2} \cdot \sqrt{36}} = \frac{8}{6\sqrt{2}} = \frac{2\sqrt{2}}{3}$$

$$\theta = \arccos\left(\frac{2\sqrt{2}}{3}\right)$$

Question 13 E

$$x = \tan(\theta) \Rightarrow \frac{dx}{d\theta} = \sec^2(\theta) \text{ and } y = \tan(2\theta) \Rightarrow \frac{dy}{d\theta} = 2\sec^2(2\theta)$$

$$\text{Hence } \frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx} = 2\sec^2(2\theta) \times \frac{1}{\sec^2(\theta)}$$

$$\text{Thus } \frac{dy}{dx} = \frac{2\cos^2(\theta)}{\cos^2(2\theta)}$$

$$\text{At } \theta = \frac{\pi}{6}, \frac{dy}{dx} = \frac{2\cos^2\left(\frac{\pi}{6}\right)}{\cos^2\left(\frac{\pi}{3}\right)} = \frac{2\left(\frac{\sqrt{3}}{2}\right)^2}{\left(\frac{1}{2}\right)^2} = \frac{\frac{6}{4}}{\frac{1}{4}} = 6$$

So the gradient of the normal equals $-\frac{1}{6}$

$$\theta = \frac{\pi}{6}, x = \tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}} \text{ and } y = \tan\left(\frac{\pi}{3}\right) = \sqrt{3}$$

$$\text{The equation of the normal is } y - \sqrt{3} = -\frac{1}{6}\left(x - \frac{1}{\sqrt{3}}\right) \Rightarrow y = -\frac{1}{6}x + \frac{1}{6\sqrt{3}} + \sqrt{3}$$

Multiplying both sides by 18 gives

$$18y = -3x + \frac{18}{6\sqrt{3}} + 18\sqrt{3}, \text{ i.e. } 3x + 18y = 19\sqrt{3}$$

Question 14 A

$\int_0^{\frac{\pi}{4}} \frac{\cos(2x)}{\sqrt{1+2\sin(2x)}} dx$ can be simplified in a number of ways, but the denominator in the alternatives suggests

$\sqrt{1+u}$ or $\sqrt{1+2u}$ where $u = \sin(2x)$ or $u = 2\sin(2x)$

Let $u = 2\sin(2x) \Rightarrow du = 4\cos(2x)dx$ when $x = 0, u = 0, x = \frac{\pi}{2}, u = 2$

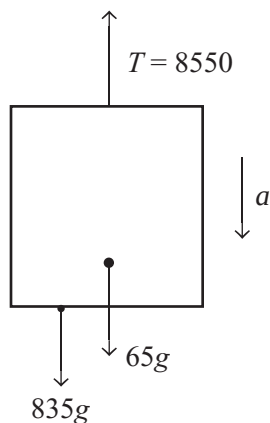
Thus $\int_0^{\frac{\pi}{4}} \frac{\cos(2x)}{\sqrt{1+2\sin(2x)}} dx$ becomes $\frac{1}{4} \int_0^{\frac{\pi}{4}} \frac{4\cos(2x)}{\sqrt{1+2\sin(2x)}} dx = \frac{1}{4} \int_0^2 \frac{du}{\sqrt{1+u}}$

Question 15 A

Motion downwards is treated as the positive direction.

$$900g - 8550 = 900a$$

$$a = 0.3 \text{ m/s}^2$$



Now consider the girl of mass 65 kg having a reaction force R exerted on her by the lift floor:

$$65g - R = 65 \times 0.3$$

$$R = 617.5 \text{ N}$$

Question 16 D

Let $\vec{r}(t) = 2t\vec{i} + t\vec{j} - 2t\vec{k} + \vec{c}_1$

As $\dot{\vec{r}}(0) = 4\vec{i} - \vec{j}$, $\vec{c}_1 = 4\vec{i} - \vec{j}$, so $\dot{\vec{r}}(t) = (2t + 4)\vec{i} + (t - 1)\vec{j} - 2t\vec{k}$

$$\vec{r}(t) = (t^2 + 4t)\vec{i} + \left(\frac{t^2}{2} - t\right)\vec{j} - t^2\vec{k} + \vec{c}_2$$

At $t = 0$, $r = 0$, so $\vec{c}_2 = 0$, giving $\vec{r}(t) = (t^2 + 4t)\vec{i} + \left(\frac{t^2}{2} - t\right)\vec{j} - t^2\vec{k}$

$$\vec{r}(4) = 32\vec{i} + 4\vec{j} - 16\vec{k}$$

The distance the object is from O is $|\vec{r}(4)| = \sqrt{32^2 + 4^2 + 16^2} = 36$

Question 17 A

The formula sheet gives the volume of a pyramid as $V = \frac{1}{3}Ah$.

The cross sectional area, A , is found by using the sine rule, $\text{Area} = \frac{1}{2}bc \sin(A)$, as each face of the tetrahedron is an equilateral triangle of side L .

$$\text{Thus } A = \frac{1}{2}L^2 \sin(60^\circ) = \frac{1}{2}L^2 \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}L^2}{4}$$

$$V = \frac{1}{3}Ah = \frac{1}{3} \times \frac{\sqrt{3}L^2}{4} \times \frac{\sqrt{6}L}{3} = \frac{\sqrt{18}L^3}{36} = \frac{\sqrt{2}L^3}{12}$$

$$\text{We require } \frac{dL}{dt} = \frac{dL}{dV} \times \frac{dV}{dt}$$

$$\text{Now } \frac{dV}{dL} = \frac{\sqrt{2}L^2}{4} \Rightarrow \frac{dL}{dV} = \frac{4}{\sqrt{2}L^2}$$

$$\frac{dL}{dt} = \frac{4}{\sqrt{2}L^2} \times 6 = \frac{24}{\sqrt{2}L^2}$$

$$\text{If the area of a face is } \sqrt{3} \text{ cm}^2 \text{ then } \sqrt{3} = \frac{\sqrt{3}L^2}{4} \Rightarrow L = 2$$

$$\therefore \frac{dL}{dt} = \frac{24}{4\sqrt{2}} = \frac{6}{\sqrt{2}} = 3\sqrt{2}$$

Question 18 A

Using the Euler approximation $y_{n+1} = y_n + h \frac{dy}{dx} \Big|_{x=x_n}$ we have $y_1 = y_0 + h \frac{dy}{dx} \Big|_{x=x_1}$

Now $y_0 = f(4) = 1$, $h = -0.2$, $\frac{dy}{dx} \Big|_{x=x_1} = \frac{dy}{dx} \Big|_{x=4} = f'(4) = -1$ using the graph.

Thus $y_1 = 1 + (-0.2)(-1) = 1.2$

Since $f'(x)$ is decreasing near $x = 4$, the graph of $y = f(x)$ must be concave near $x = 4$, meaning the tangent line drawn from $x = 4$ must lie above the graph of $y = f(x)$, as shown below.



Thus the Euler approximation to $f(3.8)$ overestimates the value of $f(3.8)$.

Question 19 E

The statement $|v(t)| \neq v(t)$ implies that for some of the time, the body has a speed which does not equal its velocity. Thus the particle for part of the time interval $[0, 20]$ is travelling with a negative velocity.

So $\int_0^{20} v(t) dt$ represents the displacement of the particle over the time interval $[0, 20]$.

Note that in alternative **B**, the position of the particle at $t = 20$ represents its displacement from the origin. The particle cannot be assumed to have started from the origin.

Question 20 B

Given $\frac{dv}{dt} = \frac{k}{v}$, then $3 = \frac{k}{1} \Rightarrow k = 3$ so that $\frac{dv}{dt} = \frac{3}{v}$

$$\frac{dt}{dv} = \frac{v}{3} \Rightarrow t = \int \frac{v}{3} dv \Rightarrow \frac{v^2}{6} + c$$

Using initial conditions, $t = 0$, $v = -2$, $c = -2$, thus $t = \frac{v^2}{6} - 2 \Rightarrow v^2 = 6t + 4$

$v = \pm\sqrt{6t + 4}$, as initial velocity is to the left.

$$\therefore v = -\sqrt{6t + 4}$$

Question 21 D

As the acceleration is constant, $s = ut + \frac{1}{2}at^2$ gives $3 = \frac{1}{2}a(2\sqrt{3})^2 \Rightarrow a = \frac{1}{2}$

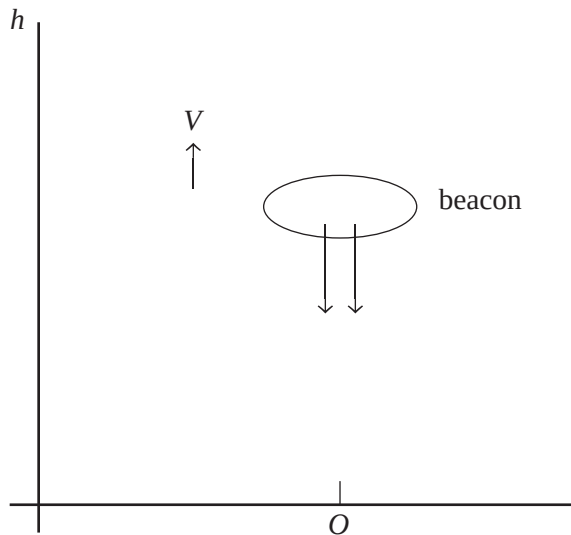
equations of motion:

$$2g - T = 2 \times \frac{1}{2} \Rightarrow T = 2g - 1$$

$$T - xg = x \times \frac{1}{2} \Rightarrow T = xg + \frac{1}{2}x$$

$$\text{Thus } 2g - 1 = xg + \frac{1}{2}x \Rightarrow 4g - 2 = 2xg + x$$

$$\therefore 2(2g - 1) = x(2g + 1) \Rightarrow x = \frac{2(2g - 1)}{2g + 1}$$

Question 22 A

Applying Newton's second law:

The resultant force is in the opposite direction to the motion and

$$F_{\text{resultant}} = -25g - 25kV^2 = -25(g + kV^2)$$

Using $F = ma$ we get $-25(g + kV^2) = 25a$

$$\text{Thus } a = -(g + kV^2) \Rightarrow V \frac{dV}{dh} = -(g + kV^2), \text{ i.e. } V \frac{dV}{dh} + kV^2 + g = 0$$

SECTION 2

Question 1

a. i. $p(z) = z^3 - 2(1 - \sqrt{3}i)z^2 - 4(1 + \sqrt{3}i)z + 8$

$$p(2) = 8 - 8(1 - \sqrt{3}i) - 8(1 + \sqrt{3}i) + 8$$

$$\text{So } p(2) = 8 - 8 + 8\sqrt{3}i - 8 - 8\sqrt{3}i + 8 = 0 \quad \text{A1}$$

ii. Using CAS, $\frac{p(z)}{z-2} = z^2 - 4 + 2\sqrt{3}zi$

$$\text{Thus } b = 2\sqrt{3}i \text{ and } c = -4 \quad \text{A1}$$

iii. By completing the square, $z^2 + 2\sqrt{3}i - 4 = 0$ can be written as

$$z^2 + 2\sqrt{3}zi + (\sqrt{3}i)^2 = 4 + (\sqrt{3}i)^2 \quad \text{M1}$$

$$\text{Thus } (z + \sqrt{3}i)^2 = 4 + 3i^2 = 1, \text{ giving } h = \sqrt{3}i \text{ and } k = 1 \quad \text{A1}$$

$$\text{Solving } (z + \sqrt{3}i)^2 = 1 \Rightarrow z + \sqrt{3}i = \pm 1$$

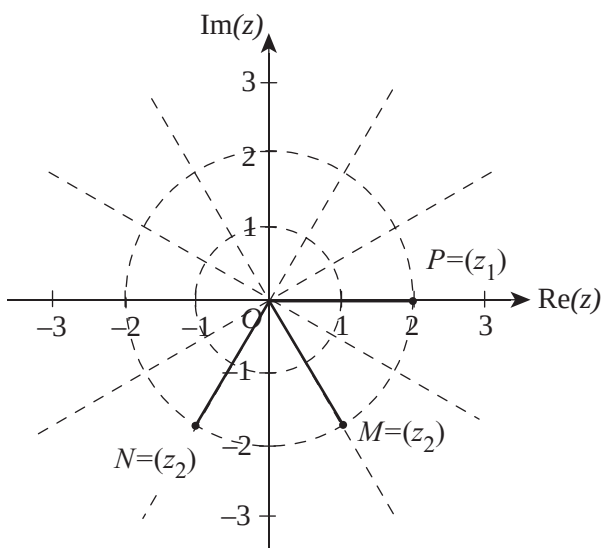
$$\text{Thus } z = \pm 1 - \sqrt{3}i \text{ which gives the 3 solutions to } p(z) = 0 \text{ as}$$

$$z_1 = 2, \quad z_2 = 1 - \sqrt{3}i, \quad z_3 = -1 - \sqrt{3}i \text{ as required.} \quad \text{A1}$$

iv. $|z_1| = |z_2| = |z_3| = 2 \quad \text{A1}$

b. i. $z_3 = 2\text{cis}\left(-\frac{2\pi}{3}\right) \quad \text{A1}$

ii.



points in correct position

A1

correct labelling

A1

c. $\overrightarrow{NP} = \overrightarrow{OP} - \overrightarrow{ON} = 2\mathbf{i} - (-\mathbf{i} - \sqrt{3}\mathbf{j}) = 3\mathbf{i} + \sqrt{3}\mathbf{j}$

$$\overrightarrow{OM} = \mathbf{i} - \sqrt{3}\mathbf{j} \quad \text{A1}$$

$$\overrightarrow{NP} \cdot \overrightarrow{OM} = (3\mathbf{i} + \sqrt{3}\mathbf{j}) \cdot (\mathbf{i} - \sqrt{3}\mathbf{j}) = 3 - 3 = 0 \quad \text{M1}$$

Thus the diagonals of the quadrilateral $ONMP$ are perpendicular.

Also, as $ONMP$ is a parallelogram, and its diagonals are at right angles, then it is a rhombus. A1

Question 2

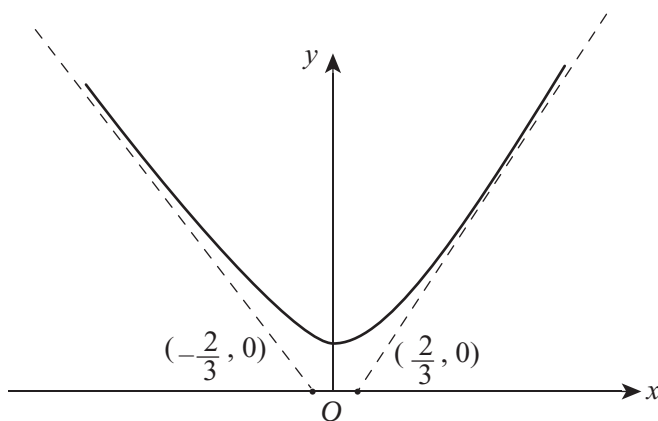
a. $(0, 2)$ A1

b. Asymptotes $(y - k) = \pm \frac{b}{a}(x - h)$

Here $(y + 1) = \pm \frac{3}{2}(x - 0)$

$$2(y + 1) = 3x \text{ or } 2(y + 1) = -3x \quad \text{M1}$$

$$3x - 2y = 2 \text{ or } 3x + 2y = -2 \quad \text{A1}$$



two asymptotes shown with $x = -\frac{2}{3}$ and $x = \frac{2}{3}$ as the x-intercepts A1

c. $V = \pi \int_{y_1}^{y_2} x^2 dy$ M1

$$= \pi \int_2^{10} \left(4 \left(\frac{(y+1)^2}{9} - 1 \right) \right) dy \quad \text{M1}$$

$$= 506 \text{ mL} \quad \text{A1}$$

d. mass of a glass = $2.5 \times$ volume of glass

$$= 2.5\pi \left(\int_0^{10} \left(\frac{2y+2}{3} \right)^2 dy - \int_2^{10} 4 \left(\frac{(y+1)^2}{9} - 1 \right) dy \right) \quad \text{M1 A1}$$

Question 3

a. i. $V(12) = 12\,960\pi$ A1

time to fill $= \frac{12\,960\pi}{6000} = \frac{54\pi}{25}$ hours A1

ii. Using the chain rule

$$\frac{dh}{dt} = \frac{dh}{dv} \times \frac{dv}{dt} \text{ and } V = \pi(6h^3 + 216h)$$

$$\frac{dh}{dt} = \frac{1}{\pi(18h^2 + 216)} \times 6000, \text{ since } \frac{dv}{dh} = \pi(18h^2 + 216)$$
 M1

$$= \frac{1000}{3\pi(h^2 + 12)}$$
 A1

At $h = 6$, $\frac{dh}{dt} = \frac{125}{18\pi}$ m/hr A1

b. i. $\frac{dh}{dt} = \frac{dh}{dv} \times \frac{dv}{dt}$ M1

$$= \frac{1}{\pi(18h^2 + 216)} \times \frac{-36\pi}{\sqrt{h}}$$

$$= \frac{-2}{\sqrt{h}(h^2 + 12)}$$
 A1

ii. Using $\frac{dt}{dh} = \frac{\sqrt{h}(h^2 + 12)}{-2}$

gives $t = \int_{12}^1 \frac{\sqrt{h}(h^2 + 12)}{-2} dh$ M1

$$= \frac{4128\sqrt{3} - 29}{7}$$

$$= 1017.27 \text{ hours}$$
 A1

Question 4

a. Terminal velocity is found by solving $0.3v^2 + 80 = 120 \times 9.8$ M1

Solving on CAS gives $v = 60.4$

So the terminal velocity of Melanie is 60.4 m/s, correct to one decimal place. A1

b. Applying Newton's second law we have

$$ma = mg - (0.3v^2 + 80) \quad \text{A1}$$

$$a = g - \frac{0.3v^2 + 80}{120}$$

$$\text{Thus } \frac{dv}{dt} = 9.8 - \frac{v^2}{400} - \frac{2}{3} \quad \text{M1}$$

$$= \left(\frac{49}{5} - \frac{2}{3} \right) - \frac{v^2}{400}$$

$$\text{As required, } \frac{dv}{dt} = \frac{137}{15} - \frac{v^2}{400} \quad \text{A1}$$

c. i. Using $\frac{dv}{dt} = \frac{137}{15} - \frac{v^2}{400}$. Using CAS we have $\frac{dv}{dt} = \frac{10\,960 - 3v^2}{1200}$

$$\text{Thus } \frac{dt}{dv} = \frac{1200}{10\,960 - 3v^2} \quad \text{M1}$$

$$v = 150 \text{ km/h} = 150 \times \frac{10}{36} \text{ m/s} = \frac{125}{3} \text{ m/s}$$

$$\text{The required definite integral is } t = \int_0^{\frac{125}{3}} \frac{1200}{10\,960 - 3v^2} dv \quad \text{A1}$$

ii. Evaluating the integral above using CAS $t = 5.604$

Melanie's panic set in after 5.6 seconds. A1

d. $\frac{dv}{dt} = \frac{137}{15} - \frac{v^2}{400} = \frac{10\,960 - 3v^2}{1200}$

$$\text{Thus we have } v \frac{dv}{dx} = \frac{10\,960 - 3v^2}{1200} \Rightarrow \frac{dv}{dx} = \frac{10\,960 - 3v^2}{1200v} \quad \text{M1}$$

$$\frac{dx}{dv} = \frac{1200v}{10\,960 - 3v^2} \Rightarrow x = \int_0^{60.4} \frac{1200v}{10\,960 - 3v^2} dv = 200 \log_e \left(\frac{68\,500}{97} \right) \quad \text{A1}$$

- e. We need to solve $\frac{dv}{dt} = -9.8 - 0.6v$ with $v(0) = -60.4$, the velocity of Melanie at the instant the parachute is opened.

$$\frac{dv}{dt} = -9.8 - 0.6v \Rightarrow \frac{dt}{dv} = -\frac{1}{0.6v + 9.8} = -\frac{1}{0.6\left(v + \frac{49}{3}\right)}$$

$$\text{This gives } (-0.6t) + c = \int \frac{1}{v + \frac{49}{3}} dv \quad \text{M1}$$

$$\text{i.e. } (-0.6t) + c = \log_e\left(v + \frac{49}{3}\right)$$

$$v + \frac{49}{3} = Ce^{-0.6t} \text{ and using } v(0) = -60.4 \text{ we have } C = -44.07 \quad \text{M1}$$

$$\text{This gives } v = -\left(44.07e^{-0.6t} + \frac{49}{3}\right) \text{ as the required approximate expression for velocity.} \quad \text{A1}$$

- f. We need to solve $\int_0^T v(t)dt = -3500 + 200\log_e\left(\frac{68\,500}{97}\right)$ M1

$$\text{Using CAS to solve } \int_0^T -\left(49.07e^{-0.6t} + \frac{49}{3}\right)dt = -2188 \text{ gives } T = 129.46.$$

Thus it will take Melanie approximately 129 seconds to reach the ground from when the parachute is opened. A1

g. $\frac{dv}{dt} = -kv - g \Rightarrow \frac{dt}{dv} = -\frac{1}{k\left(v + \frac{g}{k}\right)}$

$$t = -\frac{1}{k}\log_e\left(v + \frac{g}{k}\right) + c, \text{ giving } -kt + c = \log_e\left(v + \frac{g}{k}\right) \quad \text{M1}$$

$$\text{Thus } v = Ce^{-kt} - \frac{g}{k}$$

$$\text{As } v(0) = -50 \text{ we have } v = \left(\frac{g}{k} - 50\right)e^{-kt} - \frac{g}{k} \quad \text{A1}$$

$$\text{As } t \rightarrow \infty, v \rightarrow -\frac{g}{k}$$

This is the velocity with which the parachutist is descending. Now $\frac{g}{k} < 5.5 \Rightarrow k > \frac{98}{55}$ A1

Question 5

- a. $y = 2$ when $x = 0$

$$\text{This gives } y = \frac{-g \sec^2(\theta)}{2v^2} x^2 + \tan(\theta)x + 2$$

$$\text{At } v = 16, \theta = 45^\circ, g = -9.8$$

$$y = \frac{-9.8 \sec^2(45^\circ)}{2(16)^2} x^2 + \tan(45^\circ)x + 2$$

$$y = \frac{-49}{1445} x^2 + x + 2$$

A1

$$\text{Solving } \frac{-49}{1445} x^2 + x + 2 = 3$$

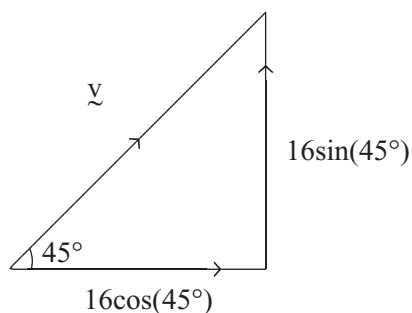
M1

$$\text{gives } x = 1.04 \text{ or } 25.0809$$

The point (25.08, 3) is inside the line segment $y = 3$ where $24.25 < x < 25.25$

A1

- b.



$$\underline{y} = 16 \cos(45^\circ) \underline{i} + 16 \sin(45^\circ) \underline{j}$$

$$= 8\sqrt{2} \underline{i} + 8\sqrt{2} \underline{j}$$

A1

c. i. $\mathbf{a} = 0\mathbf{i} - g\mathbf{j}$

$$\begin{aligned} \mathbf{v}(t) &= \int \mathbf{a} \, dt \\ &= c_1\mathbf{i} + (c_2 - gt)\mathbf{j} \end{aligned} \quad \text{M1}$$

given $\mathbf{v}(0) = 8\sqrt{2}\mathbf{i} + 8\sqrt{2}\mathbf{j}$, $c_1 = 8\sqrt{2}$ and $c_2 = 8\sqrt{2}$

$$\mathbf{v}(t) = 8\sqrt{2}\mathbf{i} + (8\sqrt{2} - gt)\mathbf{j} \quad \text{A1}$$

$$\begin{aligned} \mathbf{r}(t) &= \int \mathbf{v}(t) \, dt \\ &= d_1 + 8\sqrt{2}t\mathbf{i} + \left(d_2 + 8\sqrt{2}t - \frac{1}{2}gt^2\right)\mathbf{j} \end{aligned} \quad \text{M1}$$

given $\mathbf{r}(0) = 0\mathbf{i} + 2\mathbf{j}$ $d_1 = 0$ and $d_2 = 2$

$$\mathbf{r}(t) = 8\sqrt{2}t\mathbf{i} + \left(2 + 8\sqrt{2}t - \frac{1}{2}gt^2\right)\mathbf{j} \quad \text{A1}$$

ii. Solving $8\sqrt{2}t = 25.0809\dots$ M1

$$t = \frac{25.0809\dots}{8\sqrt{2}}$$

$$\therefore \mathbf{v}\left(\frac{25.0809}{8\sqrt{2}}\right) = 8\sqrt{2}\mathbf{i} + \left(8\sqrt{2} - g\left(\frac{25.0809}{8\sqrt{2}}\right)\right)\mathbf{j}$$

$$\begin{aligned} \therefore \text{speed} = |\mathbf{v}| &= \sqrt{(8\sqrt{2})^2 + \left(8\sqrt{2} - g\frac{25.0809}{8\sqrt{2}}\right)^2} \\ &= 15.375 \\ &= 15.4 \text{ m/s} \end{aligned} \quad \text{A1}$$