



Trial Examination 2012

VCE Specialist Mathematics Units 3 & 4

Written Examination 1

Suggested Solutions

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Question 1

Using the chain rule $\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$ where $\frac{dy}{dt} = 3\cos(3t)$ and $\frac{dx}{dt} = e^t$ we get $\frac{dy}{dt} = 3\cos(3t) \cdot e^{-t}$ M1

At $(1, 0)$, $1 = e^t \Rightarrow t = 0$

Thus the gradient of the tangent at $(1, 0)$ is $\frac{dy}{dx} = 3\cos(0) \cdot e^0 = 3$

$\therefore$ the equation of the tangent is $y - 0 = 3(x - 1) \Rightarrow y = 3x - 3$ A1

Question 2

Let $u = \sqrt{x} \Rightarrow \frac{du}{dx} = \frac{1}{2\sqrt{x}}$

Changing the terminals: $x = 1 \Rightarrow u = 1$, $x = 4 \Rightarrow u = 2$ M1

So $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$ is equivalent to $\int_1^2 2e^u du = [2e^u]_1^2 = 2(e^2 - e) = 2e(e - 1)$ A1

Question 3

a. $\frac{dh}{dt} = -\frac{\sqrt{h}}{10} \Rightarrow \frac{dt}{dh} = -\frac{10}{\sqrt{h}}$

Thus $t = \int -10h^{-\frac{1}{2}} dh \Rightarrow t = -20h^{\frac{1}{2}} + c$

or $t = \left[-20h^{\frac{1}{2}} \right]_4^h$ M1

As $t = 0$, $h = 4$, $0 = -20\sqrt{4} + c \Rightarrow c = 40$

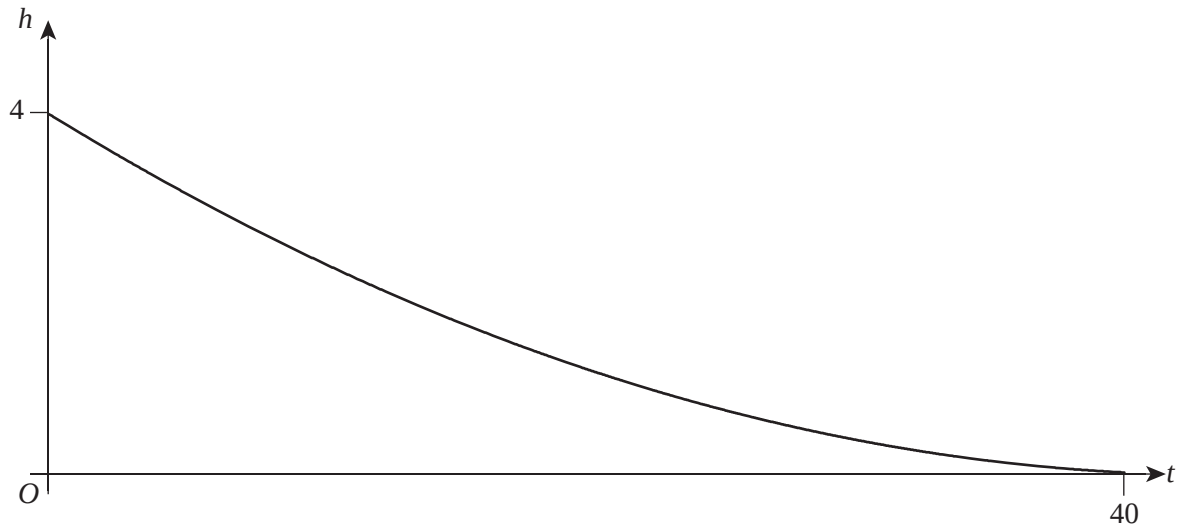
$= -20\sqrt{h} + 40$

Thus $t = 40 - 20\sqrt{h}$

So we have $20\sqrt{h} = 40 - t \Rightarrow \sqrt{h} = 2 - \frac{t}{20}$

Thus $h = \left(2 - \frac{t}{20}\right)^2$ A1

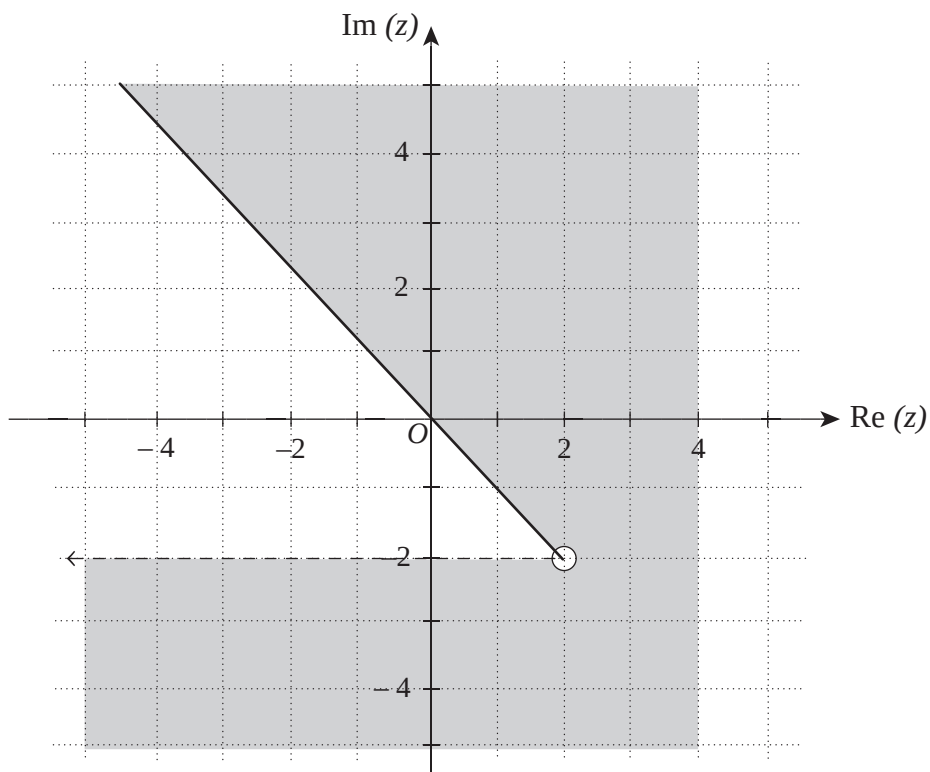
b.

parabola over domain $[0, 40]$

A1

Question 4

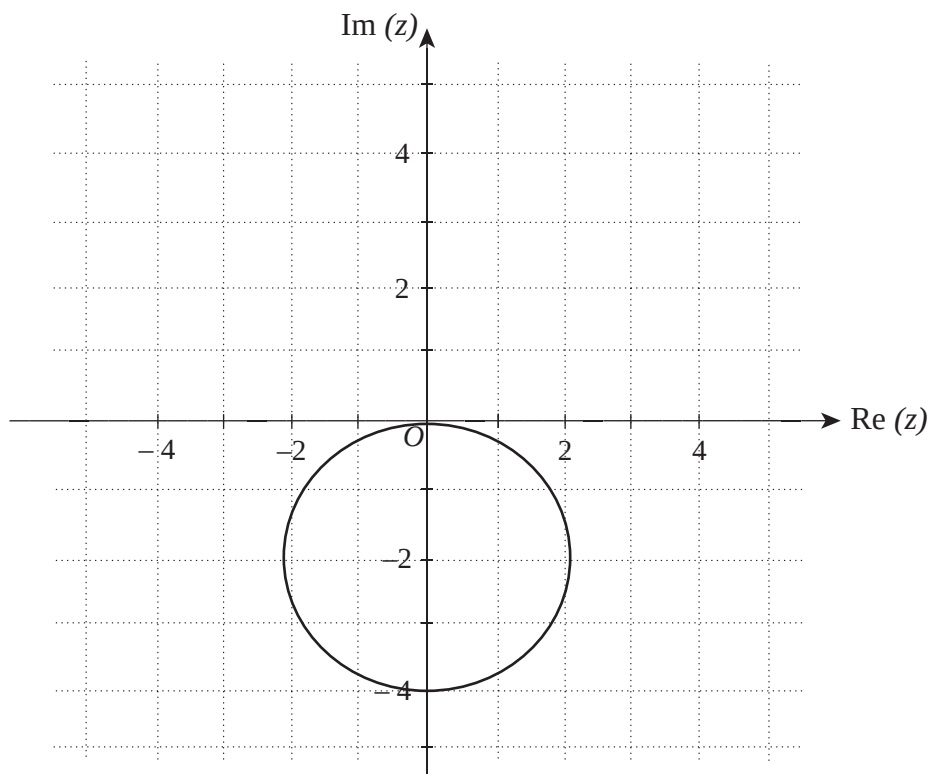
a.



$$\text{Arg}(z - 2 + 2i) \leq \frac{3\pi}{4}$$

Ray starting at $2 - 2i$, at angle of $\frac{3\pi}{4}$ A1Shaded region, with dotted ray starting at $(2, -2)$ extending horizontally A1

b.



Standard form of circle centre z_1 , radius r^2 is $(z - z_1)(\bar{z} - \bar{z}_1) = r^2$

so $(z + 2i)(\bar{z} - 2i) = 4$ is a circle with centre $(0, -2)$ and radius 2.

OR

$$(x + yi + 2i)(x - yi - 2i) = 4$$

$$x^2 - xyi - 2xi + xyi + y^2 + 2y + 2xi + 2y + 4 = 4$$

$$x^2 + y^2 + 4y + 4 = 4$$

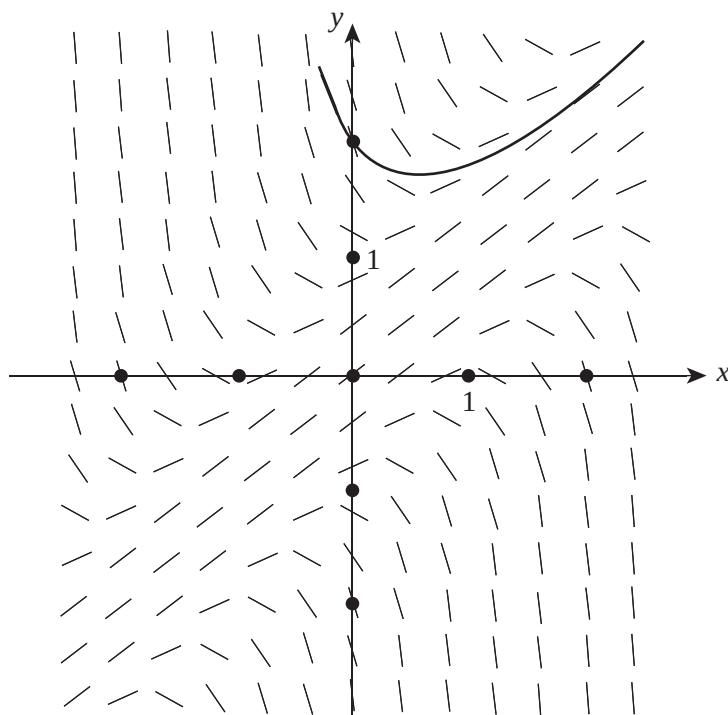
$$x^2 + (y + 2)^2 = 4$$

circle centre $(-2, 0)$

A1

radius = 2

A1

Question 5**a.**correct shape and passing through $(0, 2)$

A1

- b. i.** Equation (A) is not correct because it represents a differential equation, which does not depend on x . The slopes would need to be constant in the horizontal direction, for each value of y .
- ii.** Equation (B) is correct. It predicts that if $y = x$, $\frac{dy}{dx} = 1$, which is observed for all the slopes along the line $y = x$. Equation (C) does not give this result.

A1

A1

Question 6

$$\text{Let } y = x \arctan(x) - \frac{1}{2} \log_e(x^2 + 1)$$

$$\frac{dy}{dx} = x \cdot \frac{1}{x^2 + 1} + \arctan(x) - \frac{1}{2} \left(\frac{2x}{x^2 + 1} \right) = \frac{x}{x^2 + 1} + \arctan(x) - \frac{x}{x^2 + 1} = \arctan(x)$$

M1 A1

$$\text{Thus } \int_{-\sqrt{3}}^1 \arctan(x) dx = \left[x \arctan(x) - \frac{1}{2} \log_e(x^2 + 1) \right]_{-\sqrt{3}}^1$$

$$= \left(\frac{\pi}{4} - \frac{1}{2} \log_e(2) \right) - \left(-\sqrt{3} \left(-\frac{\pi}{3} \right) - \frac{1}{2} \log_e(4) \right)$$

M1

$$= \frac{\pi}{4} - \frac{\sqrt{3}}{3} \pi + \frac{1}{2} \log_e(2)$$

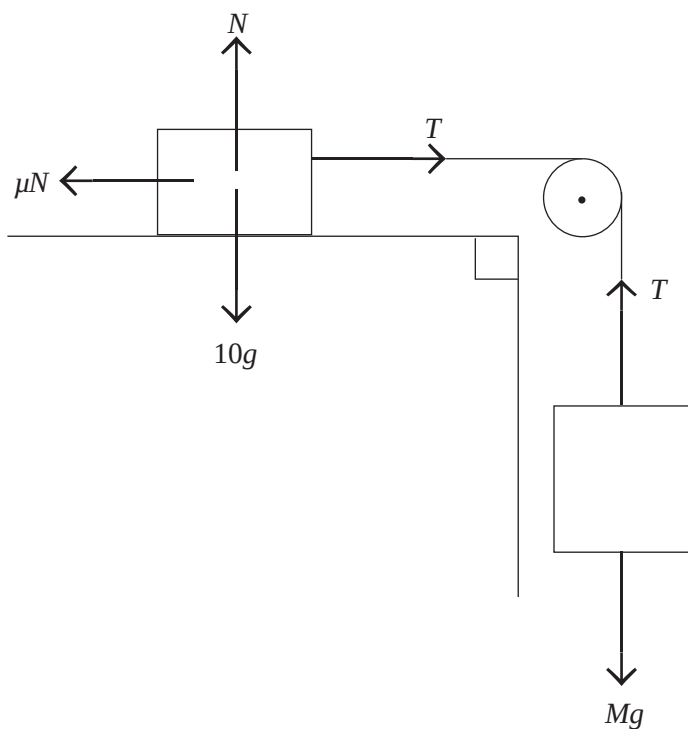
A1

OR

$$= \pi \left(\frac{1}{4} - \frac{\sqrt{3}}{3} \right) + \frac{1}{2} \log_e(2)$$

or
A1

Question 7



a. $N = 10g$, $Fr = \mu N$
 $= 0.2 \times 10g$
 $= 2g$

M1

$$T = 2g$$

$$Mg = 2g$$

$$M = 2$$

A1

b. If $M = 4$, the resultant force $= 4g - 2g$
 $= 2g$

M1

Using $R_F = ma$

$$a = \frac{R_F}{M}$$

$$= \frac{2g}{14}$$

$$= \frac{g}{7}$$

A1

OR

At 10 kg, $T - 2g = 10a$

At 4 kg, $4g - T = 4a$

$$2g = 14a$$

$$a = \frac{g}{7}$$

A1

Question 8

$$\cos(2x) = \cos^2(x)$$

$$2\cos^2(x) - 1 = \cos^2(x)$$

M1

$$\cos^2(x) = 1$$

$$\cos(x) = \pm 1$$

A1

$$x = 0, \pi, 2\pi$$

$\therefore$ the points of intersection are $(0, 1)$, $(\pi, 1)$ and $(2\pi, 1)$.

A1

Question 9

a. $\overrightarrow{QR} = \overrightarrow{OR} - \overrightarrow{OQ}$

$$= (3\mathbf{i} - 4\mathbf{j} + m\mathbf{k}) - (\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$$

$$= 2\mathbf{i} - 2\mathbf{j} + (m - 3)\mathbf{k}$$

A1

b. $\overrightarrow{QP} = \overrightarrow{OP} - \overrightarrow{OQ}$

$$= (-\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}) - (\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$$

$$= -2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$$

M1

We require $\overrightarrow{QP} \cdot \overrightarrow{QR} = 0$

$$(-2\mathbf{i} - \mathbf{j} + 2\mathbf{k}) \cdot (2\mathbf{i} - 2\mathbf{j} + (m - 3)\mathbf{k}) = 0$$

M1

$$-4 + 2 + 2(m - 3) = 0$$

$$m = 4$$

A1

c. $|\overrightarrow{QP}| = |-2\mathbf{i} - \mathbf{j} + 2\mathbf{k}| = \sqrt{(-2)^2 + (-1)^2 + (2)^2} = 3$

$$|\overrightarrow{QR}| = |2\mathbf{i} - 2\mathbf{j} + \mathbf{k}| = \sqrt{(2)^2 + (-2)^2 + (1)^2} = 3$$

$\therefore PQR$ is an isosceles triangle.

A1

Question 10

a. The area A of the region is given by $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{e^{\tan(x)}}{1 - \sin^2 x} dx$

Notice that $\frac{e^{\tan(x)}}{1 - \sin^2 x} = \frac{e^{\tan(x)}}{\cos^2(x)} = e^{\tan(x)} \sec^2(x)$ M1

Thus $A = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} e^{\tan(x)} \sec^2(x) dx = [e^{\tan(x)}]_{-\frac{\pi}{4}}^{\frac{\pi}{4}}$ A1

$A = \left[e^{\tan\left(\frac{\pi}{4}\right)} - e^{\tan\left(-\frac{\pi}{4}\right)} \right] = e - e^{-1} = e - \frac{1}{e}$ A1

OR

$$u = \tan(x)$$

$$\frac{dy}{dx} = \frac{1}{\cos^2(x)}$$

$A = \int_{-1}^1 e^u du$ M1

$= [e^u]_{-1}^1$ A1

$= e - \frac{1}{e}$ A1

b. $V\pi \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left(\frac{e^{\tan(x)}}{1 - \sin^2(x)} \right)^2 dx = \pi \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left(\frac{e^{\tan(x)}}{\cos^2(x)} \right)^2 dx = \pi \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (e^{\tan(x)} \sec^2(x))^2 dx$ M1

Thus $V = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} e^{2\tan(x)} \sec^4(x) dx$, giving $k = 2$, $m = 4$. A1

Question 11

- a. Substituting $x = \frac{\pi}{2}$ into $\cos(x) + e^{xy} = 2$ gives $\cos\left(\frac{\pi}{2}\right) + e^{\frac{\pi y}{2}} = 2 \Rightarrow \frac{\pi y}{2} = \log_e(2)$

$$\text{Thus } y = \frac{2\log_e(2)}{\pi} \quad \text{A1}$$

- b. Using implicit differentiation,

$$-\sin(x) + \frac{d}{dx}(e^{xy}) = 0$$

$$-\sin(x) + e^{xy}\left(x\frac{dy}{dx} + y\right) = 0$$

$$x\frac{dy}{dx} + y = \frac{\sin(x)}{e^{xy}} \Rightarrow \frac{dy}{dx} = \frac{1}{x}\left(\frac{\sin(x)}{e^{xy}} - y\right) \quad \text{M1 A1}$$

- c. At $x = \frac{\pi}{2}$, $y = \frac{2\log_e(2)}{\pi}$,

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{\frac{\pi}{2}} \left(\frac{\sin\left(\frac{\pi}{2}\right)}{e^{\frac{\pi}{2} \times \frac{2\log_e(2)}{\pi}}} - \frac{2\log_e(2)}{\pi} \right) \\ &= \frac{2}{\pi} \left(\frac{1}{e^{\log_e(2)}} - \frac{2\log_e(2)}{\pi} \right) \\ &= \frac{2}{\pi} \left(\frac{1}{2} - \frac{2\log_e(2)}{\pi} \right) \\ &= \frac{1}{\pi} - \frac{4\log_e(2)}{\pi^2} \quad \text{M1} \end{aligned}$$

Thus the gradient of the tangent is $\frac{\pi - 4\log_e(2)}{\pi^2}$, giving the value for the gradient of the normal as

$$\frac{\pi^2}{4\log_e(2) - \pi} \text{ as required.} \quad \text{A1}$$