The Mathematical Association of Victoria

Trial Exam 2012

SPECIALIST MATHEMATICS

Written Examination 2

STUDENT NAME ____

Reading time: 15 minutes
Writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of Book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	5	5	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of **21** pages with a detachable sheet of miscellaneous formulas at the back.
- Answer sheet for multiple-choice questions.

Instructions
- Detach the formula sheet from the back of this book during reading time.
- Write your name in the space provided above on this page.
- All written responses must be in English.

At the end of the examination
- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where g = 9.8.

Question 1

The range of an ellipse is $[-2, 6]$. The equation of the ellipse could be

A. $4(x+3)^2 + 16(y-2)^2 = 64$

B. $4(x-3)^2 + 16(y+2)^2 = 64$

C. $16(x-3)^2 + 4(y+2)^2 = 64$

D. $16(x+3)^2 + 4(y-2)^2 = 64$

E. $4(x+2)^2 + 16(y-3)^2 = 64$

Question 2

The graph of $y = \dfrac{x^2+6}{x^2-5x+4}$ has exactly

A. one asymptote with equation $x = 6$

B. two asymptotes with equations $x = 4$ and $x = 1$

C. two asymptotes with equations $x = 4$ and $y = 6$

D. three asymptotes with equations $y = 0$, $x = 4$ and $x = 1$

E. three asymptotes with equations $y = 1$, $x = 4$ and $x = 1$

Question 3

The path of a particle traces out a curve that is given by the equations $x = -2\cos(2t)$ and $y = 2\cos(t)$. The cartesian equation of the curve is

A. $y = x^2 - 2$

B. $x^2 + y^2 = 4$

C. $x^2 + y + 2 = 0$

D. $y^2 + x - 2 = 0$

E. $x^2 - y^2 = 4$

Question 4

If $\sec\left(\sin^{-1}(a)\right) = \frac{13}{5}$, then a could equal

A. $\frac{5}{12}$

B. $\frac{-5}{13}$

C. $\frac{12}{13}$

D. $\frac{-13}{12}$

E. $\frac{12}{5}$

Question 5

Consider the function $h: D \to R,\ h(x) = 3\cos^{-1}\left(\frac{x+a}{2}\right)$, where a is a real constant and D is the maximal domain of h. If the circle with equation $(x-6)^2 + y^2 = 1$ intersects the graph of h at an x-axis intercept, then a can equal

A. 3 only
B. −3 only
C. 5 only
D. 3 or −3
E. −3 or −5

Question 6

Let $f: R \to R$ be a continuous twice differentiable function. Given that $f'(b) = 0$, which one of the following statements is **not necessarily** true?

A. f has a point of inflection at $x = b$, if $f''(b) = 0$

B. f has a local minimum at $x = b$, if $f''(b) > 0$

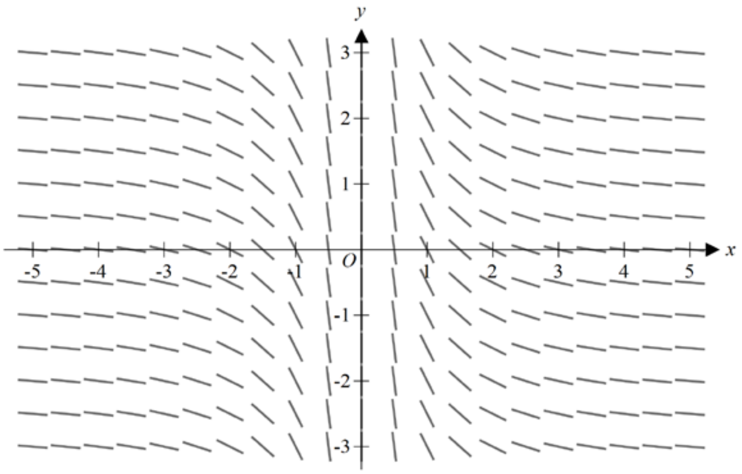
C. f has a local maximum at $x = b$, if $f''(b) < 0$

D. f has a point of inflection at $x = b$, if, for $a < b < c$, $f''(a) < 0$, $f''(b) = 0$ and $f''(c) > 0$

E. f has a point of inflection at $x = b$, if, for $a < b < c$, $f''(a) > 0$, $f''(b) = 0$ and $f''(c) < 0$

Question 7

The direction field of a certain first order differential equation is shown.



If a is a positive real constant, the differential equation could be

A. $\dfrac{dy}{dx} = \dfrac{a}{x}$

B. $\dfrac{dy}{dx} = -\dfrac{a}{x^2}$

C. $\dfrac{dy}{dx} = -\dfrac{ax}{y}$

D. $\dfrac{dy}{dx} = \dfrac{a}{x^2}$

E. $\dfrac{dy}{dx} = \dfrac{a}{y^2}$

Question 8

Euler's method is used to solve the differential equation $\dfrac{dy}{dx} = x\log_e(x)$, with a step size of $\dfrac{1}{5}$ and initial values $x = 1$ and $y = -3$. The value of y when $x = \dfrac{7}{5}$ is given by

A. $\dfrac{1}{5}\log_e(1) - 3$

B. $\dfrac{1}{5}\log_e\left(\dfrac{1}{5}\right) - 3$

C. $\dfrac{6}{25}\log_e\left(\dfrac{6}{5}\right) - 3$

D. $\dfrac{6}{5}\log_e\left(\dfrac{1}{5}\right) - 3$

E. $-\dfrac{14}{5} - \dfrac{6}{25}\log_e\left(\dfrac{6}{5}\right)$

Question 9

Let $z = a + bi$, where a and b are non-zero real numbers. Which one of the following is **not** a real number?

A. $\overline{z} + z$

B. $z^{-1}\overline{z}$

C. $z^{-1}z$

D. $\operatorname{Im}(z)$

E. $\overline{z}\,z$

Question 10

If $u = \sqrt{3} - 3i$, then $\operatorname{Arg}(u^5)$ is equal to

A. $\dfrac{\pi}{3}$

B. $-\dfrac{\pi}{3}$

C. $\dfrac{\pi}{6}$

D. $-\dfrac{5\pi}{3}$

E. $\dfrac{5\pi}{6}$

Question 11

If one of the sixth roots of a complex number, w, is $\sqrt{3}\operatorname{cis}\left(\dfrac{\pi}{15}\right)$, then $\overline{w}$ is equal to

A. $9\operatorname{cis}\left(\dfrac{2\pi}{5}\right)$

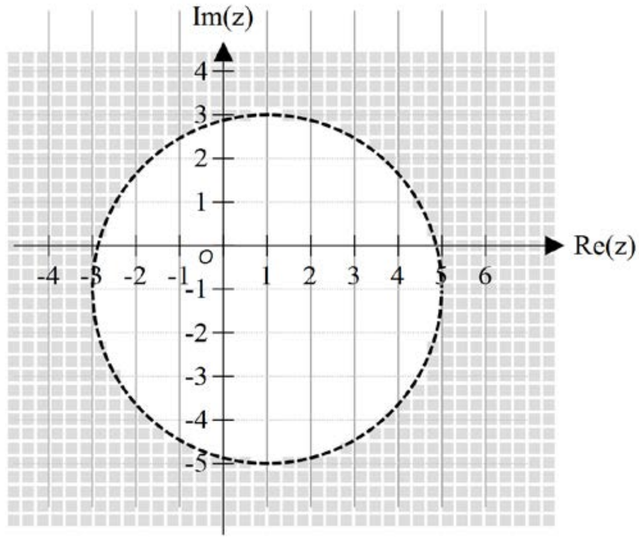
B. $9\operatorname{cis}\left(-\dfrac{2\pi}{5}\right)$

C. $\sqrt{3}\operatorname{cis}\left(-\dfrac{2\pi}{15}\right)$

D. $-27\operatorname{cis}\left(\dfrac{\pi}{15}\right)$

E. $27\operatorname{cis}\left(-\dfrac{2\pi}{5}\right)$

Question 12



The set of points defined by the shaded region of the argand diagram shown above could be

A. $\{z \in C : |z| > |-1+i|\}$

B. $\{z \in C : |z| > 4|-1+i|\}$

C. $\{z \in C : |z-1+i| > 4\}$

D. $\{z \in C : |z-(1+i)| > 4\}$

E. $\{z \in C : |z-1+i| > 16\}$

Question 13

The angle between the vectors $\underset{\sim}{u} = \underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}$ and $\underset{\sim}{v} = -2\underset{\sim}{i} - \underset{\sim}{j} - \underset{\sim}{k}$ is

A. $\dfrac{\pi}{3}$

B. $\dfrac{\pi}{6}$

C. $-\dfrac{\pi}{3}$

D. $\dfrac{2\pi}{3}$

E. $-\dfrac{2\pi}{3}$

Question 14

The scalar resolute of the vector $\underset{\sim}{m} = 4\underset{\sim}{i} + 5\underset{\sim}{j} - 3\underset{\sim}{k}$ in the direction of the vector $\underset{\sim}{n} = 2\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$ is

A. $\frac{5}{3}$

B. $-\frac{5}{3}$

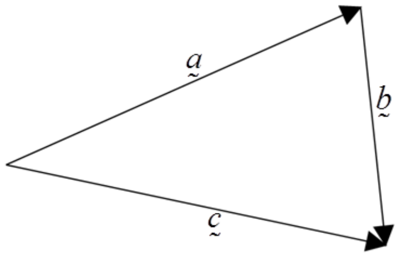
C. $\frac{5}{9}$

D. $-\frac{5}{9}$

E. $\frac{3}{5}$

Question 15

Vectors $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ are shown in the following diagram.



If $\underset{\sim}{a} \cdot \underset{\sim}{b} = |\underset{\sim}{a}||\underset{\sim}{b}|\cos(\theta)$, then θ is given by

A. $\cos^{-1}\left(\frac{|\underset{\sim}{c}|^2 - |\underset{\sim}{a}|^2 - |\underset{\sim}{b}|^2}{2|\underset{\sim}{a}||\underset{\sim}{b}|}\right)$

B. $\cos^{-1}\left(\frac{|\underset{\sim}{a}|^2 + |\underset{\sim}{b}|^2 - |\underset{\sim}{c}|^2}{2|\underset{\sim}{a}||\underset{\sim}{b}|}\right)$

C. $\cos^{-1}\left(\frac{|\underset{\sim}{a}||\underset{\sim}{b}|}{|\underset{\sim}{c}|^2 - |\underset{\sim}{a}|^2 - |\underset{\sim}{b}|^2}\right)$

D. $\cos^{-1}\left(\frac{|\underset{\sim}{a}||\underset{\sim}{b}|}{|\underset{\sim}{a}|^2 + |\underset{\sim}{b}|^2 - |\underset{\sim}{c}|^2}\right)$

E. $\cos^{-1}\left(\frac{|\underset{\sim}{c}|^2 + |\underset{\sim}{a}|^2 - |\underset{\sim}{b}|^2}{2|\underset{\sim}{a}||\underset{\sim}{c}|}\right)$

Question 16

Let $2x^2y-4y+x^3-7=0$. When $x=1$, then $\frac{dy}{dx}$ equals

A. 0
B. -3
C. -8
D. $\frac{7}{2}$
E. $-\frac{9}{2}$

Question 17

Using the substitution $u=\log_e(x)$, $\int_1^e \left(\frac{\left(\log_e\left(x^3\right)\right)^2}{x}\right)dx$ is equivalent to

A. $\int_1^e 6u\,du$
B. $\int_1^e 3u^2\,du$
C. $\int_0^1 3u^2\,du$
D. $\int_0^1 9u^2\,du$
E. $\int_1^3 9u^2\,du$

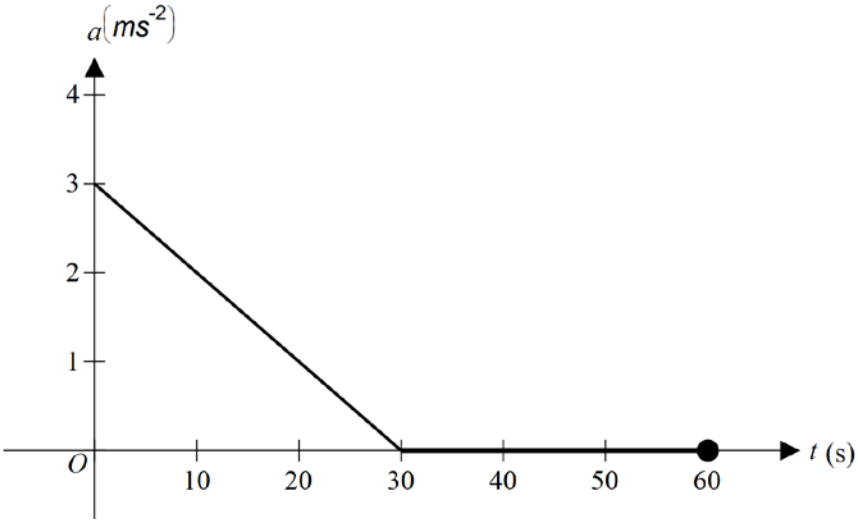
Question 18

During a New Year's Eve fireworks display, a rocket is launched vertically into the air with an initial velocity of u ms^{-1} . The rocket continues to be propelled at constant velocity until it runs out of fuel at a height of *h* metres. The maximum height reached by the rocket is given by

A. $\frac{u^2}{g}$
B. $\frac{u^2}{2g}$
C. $\frac{u^2+2gh}{2g}$
D. $\frac{u+h}{g}$
E. $uh+\frac{1}{2}gh^2$

Question 19

The acceleration-time graph for a particle moving in a straight line, from rest, is shown.



The graph shows that at the end of 60 seconds

A. the speed of the particle is zero

B. the speed of the particle is $45\,\text{ms}^{-1}$

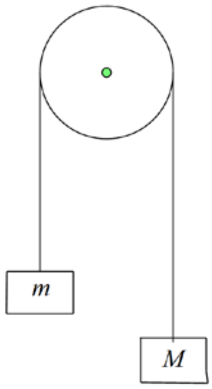
C. the displacement of the particle is zero

D. the distance travelled by the particle is 45 metres

E. the distance travelled by the particle is 10 metres

Question 20

Two particles of mass m and M, where $M > m$, are attached to each end of a light inextensible string that passes over a smooth pulley of negligible mass.



The upwards acceleration of the particle of mass m is given by

A. $\dfrac{(M-m)g}{M+m}$

B. $\dfrac{(M+m)g}{M-m}$

C. $\dfrac{mg}{M-m}$

D. $\dfrac{Mg}{m}$

E. $\dfrac{mg}{M}$

Question 21

The velocity $v\,\text{ms}^{-1}$ of a particle moving in a straight line is given by the equation $v = \cos^2(x)$, where x metres is the displacement of the particle from the origin at time t seconds, and $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. If the initial position of the particle is $x = \frac{\pi}{4}$, then t is equal to

A. $-2\sin(x)\cos(x)$

B. $\dfrac{\sin(x)\cos(x)+x}{2}$

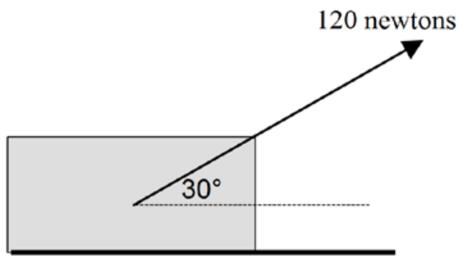
C. $\tan(x)$

D. $\tan(x)-1$

E. $\dfrac{4\sin(x)\cos(x)+4x-\pi-2}{8}$

Question 22

Grainger is dragging a 60 kg chest along a rough level surface by applying an upward force of magnitude 120 newtons, at an angle of 30° to the horizontal, as shown.



If the velocity of the chest is constant, the value of the coefficient of friction between the chest and the surface is

A. $\dfrac{1}{2g}$

B. $\dfrac{g-1}{2}$

C. $\dfrac{\sqrt{3}}{g-1}$

D. $\dfrac{g\sqrt{3}}{2}$

E. $\dfrac{g-1}{\sqrt{3}}$

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.
Unless otherwise specified an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

At time t, where $t \geq 0$,the position vector of a particle is defined by
$\underset{\sim}{r} = m\sin(2t)\underset{\sim}{i} + m\cos(2t)\underset{\sim}{j} + nt\underset{\sim}{k}$, where m and n are non-zero real constants.

a. Let $\dot{\underset{\sim}{r}}$ and $\ddot{\underset{\sim}{r}}$ represent the velocity and acceleration vectors of the particle.

i. Show that $\ddot{\underset{\sim}{r}}$ is perpendicular to $\dot{\underset{\sim}{r}}$.

ii. Find the magnitudes of $\dot{\underset{\sim}{r}}$ and $\ddot{\underset{\sim}{r}}$, and hence show that they are constant.

3 + 2 = 5 marks

b.

i. Find a unit vector parallel to $\dot{\underset{\sim}{r}}$.

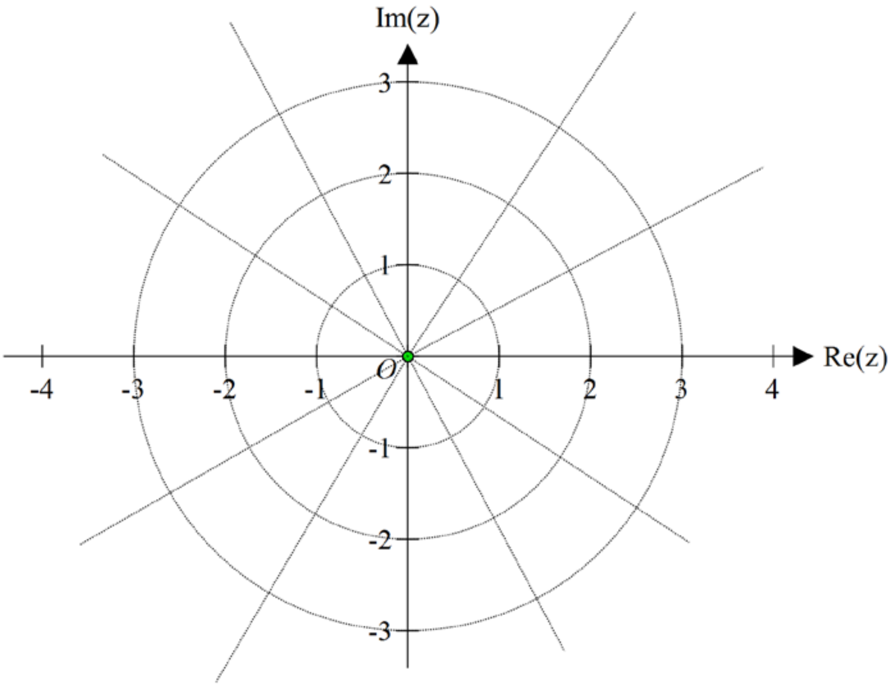
ii. Using the result in **part b.i.** above, or otherwise, show that $\dot{\underset{\sim}{r}}$ makes a fixed angle with the $\underset{\sim}{k}$ direction. Hence find the magnitude of that angle when $m = 3$ and $n = -5$, correct to the nearest degree.

1 + 3 = 4 marks
Total 9 marks

Question 2

Consider the two graphs with rules $\text{Im}(z)=2$ and $\text{Re}(z)=-3$, where $z \in C$.

a. On the argand diagram below, sketch and label the graphs with rules $\text{Im}(z)=2$ and $\text{Re}(z)=-3$. Hence shade the region $S_1=\{z:\text{Im}(z)\ge 2, z\in C\}\cap\{z:\text{Re}(z)\le -3, z\in C\}$.



2 marks

b. The straight line that passes through the point with coordinates $(-3,2)$ and the origin can be expressed as $13z = w\overline{z}$,where $w \in C$. Find the value of w.

2 marks

c. Let S_2 be the region defined by $\{z:|z-2|\le 1, z\in C\}$.

i. Write the rule for S_2 as a cartesian inequation.

ii. On the argand diagram in **part a.** above, shade the region S_2.

2 + 1 = 3 marks

d.

i. On the argand diagram in **part a.** above, sketch a line passing through the points with coordinates $(-3, 2)$ and $(2, 0)$, and find the cartesian equation of this line.

ii. Hence find the **minimum distance** between regions S_1 and S_2.

2 + 2 = 4 marks

Total 11 marks

Question 3

Let $f_k : [0, \infty) \to R$ be a family of functions with rule $f_k(x) = x^k e^{-x}$, where $k \in Z^+$.

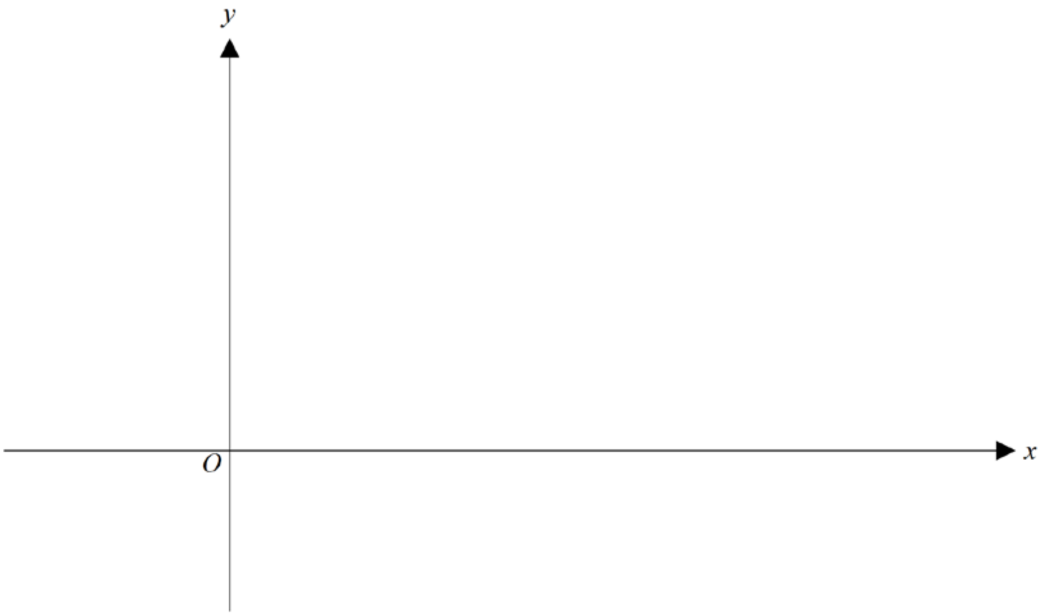
Consider the case where $k = 4$. The maximum value of f_4 is a.

a. Find the value of a, correct to three decimal places.
(You are **not** required to formally show that the point is a maximum)

2 marks

b. On the set of axes below, sketch the graph of $y = f_4(x)$. Label any stationary points and endpoints with their coordinates and label any asymptote with its equation. Provide answers to non-exact values correct to three decimal places.

3 marks



c. The region bounded by the graph of $y = f_4(x)$, the y-axis and the line $y = a$ is rotated about the x-axis to form a volume of revolution, which models a solid object. The values on the coordinate axes represent centimetres.

i. Shade the required region on the graph from **part b.** above.

ii. Write down a definite integral which will give the volume of the solid.

iii. Evaluate the integral in **part c.ii.** above to find the volume of the solid, correct to three decimal places.

1 + 1 + 1 = 3 marks

d. For another value of *k*, the graph of f_k has points of inflection at the points *P* and *Q*, with coordinates $\left(p, f_k\left(p\right)\right)$ and $\left(q, f_k\left(q\right)\right)$, respectively, where $q > p > 0$.

i. Find *p* and *q* in terms of *k*.

ii. Find x_m, the *x*-coordinate of the midpoint of the line segment *PQ*.

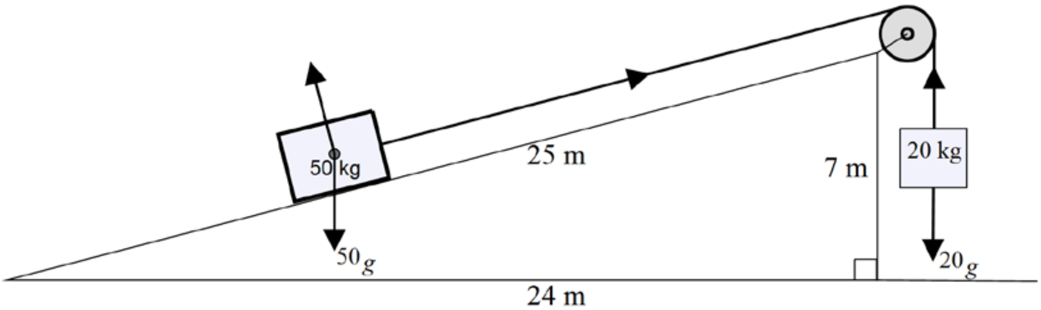
iii. Hence show that the maximum stationary point of the graph has coordinates $\left(x_m, f_k\left(x_m\right)\right)$.

2 + 2 + 2 = 6 marks
Total 14 marks

Question 4

A block of mass 50 kg is placed on a lubricated frictionless inclined plane. The cross section of the plane forms a right-angled triangle of height 7 m and hypotenuse 25 m. The block is connected by a light inextensible string which passes over a smooth pulley of negligible mass, to a particle of mass of 20 kg which is hanging freely, as shown.

The 20 kg mass is 7 m above the ground when the system is released from rest.



- Let $a\,\text{ms}^{-2}$ be the magnitude of the acceleration of the 50 kg block up the plane when the system is released.
- Let T newtons be the tension in the string.

a. Show that $a = \dfrac{3g}{35}$.

2 marks

b. Find, in terms of g, the value of T.

2 mark

c. Find the time taken for the 20 kg mass to reach the ground, in seconds, correct to two decimal places.

2 marks

d. Consider the instant when the 20 kg mass hits the ground and the string becomes slack.

i. Find the speed of the 50 kg block at this instant, in ms^{-1}, correct to two decimal places.

ii. Find the acceleration of the 50 kg block just after this instant, in ms^{-2}, correct to two decimal places.

2 + 2 = 4 marks
Total 10 marks

Question 5

The differential equation $m\frac{dv}{dt} = mg - kv$ models the equation of motion of an object of mass m kg falling vertically through the air, where v ms^{-1} is the speed of the object at time t seconds, and k is a positive real constant.

a.

If an object dropped vertically from rest at time $t = 0$ is falling through the air, show that at time t seconds the speed of the object is given by $v = \frac{mg}{k}\left(1 - e^{-\frac{k}{m}t}\right)$.

4 marks

Yun, a skydiver, drops vertically from a hot air balloon so that at $t = 0$, $v = 0$, and falls through the air for 10 seconds before opening her parachute. The combined mass of Yun and her skydiving gear is 60 kg.

b. If just before Yun opens the parachute her speed is $47.5\,\text{ms}^{-1}$, show that, before the parachute opens, the value of k ,correct to the nearest integer, is equal to 10. (Assume that her parachute opens instantly and that there is no upthrust when the parachute opens.)

1 mark

As $t \to \infty$, $v \to v_t$, where v_t is the terminal speed of the skydiver.

c. Just before the parachute opened, what percentage of the terminal speed had Yun reached? Assume that k = 10 and give the answer correct to the nearest integer.

2 marks

d. Assuming that k = 10, find the distance that Yun falls **before** the parachute is opened. Give the answer correct to the nearest metre.

2 marks

e. The parachute opened successfully and Yun landed on the ground 2 minutes after dropping from the balloon, having reached a terminal speed of $6\,\mathrm{m\,s^{-1}}$.

i. Show that **after the parachute opens** the value of k is equal to $10g$.

ii. Given that t seconds **after the parachute opens**, $v = Ae^{-\frac{k}{m}t} + \frac{mg}{k}$, where A is a real constant, show the value of A is equal to 41.5.

iii. Hence find the height of the balloon above the ground when Yun dropped from the balloon. Give the answer correct to the nearest metre.

1 + 1 + 3 = 5 marks

Total 14 marks

END OF QUESTION AND ANSWER BOOKLET

SPECIALIST MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of a triangle:	$\frac{1}{2}bc\sin A$
sine rule:	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
cosine rule:	$c^2 = a^2 + b^2 - 2ab\cos C$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ hyperbola: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Circular (trigonometric) functions

$\cos^2(x) + \sin^2(x) = 1$

$1 + \tan^2(x) = \sec^2(x)$ $\cot^2(x) + 1 = \mathrm{cosec}^2(x)$

$\sin(x + y) = \sin(x)\cos(y) + \cos(x)\sin(y)$ $\sin(x - y) = \sin(x)\cos(y) - \cos(x)\sin(y)$

$\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$ $\cos(x - y) = \cos(x)\cos(y) + \sin(x)\sin(y)$

$\tan(x+y) = \frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$ $\tan(x-y) = \frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$

$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$

$\sin(2x) = 2\sin(x)\cos(x)$ $\tan(2x) = \frac{2\tan(x)}{1-\tan^2(x)}$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (complex numbers)

$z = x + yi = r(\cos\theta + i\sin\theta) = r\,\text{cis}\,\theta$

$|z| = \sqrt{x^2 + y^2} = r$ $\quad$ $-\pi < \text{Arg}\,z \le \pi$

$z_1 z_2 = r_1 r_2\,\text{cis}(\theta_1 + \theta_2)$ $\quad$ $\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}\text{cis}(\theta_1 - \theta_2)$

$z^n = r^n\,\text{cis}(n\theta)$ (de Moivre's theorem)

Calculus

$\dfrac{d}{dx}\left(x^n\right) = nx^{n-1}$ $\quad$ $\displaystyle\int x^n\,dx = \frac{1}{n+1}x^{n+1} + c, n \ne -1$

$\dfrac{d}{dx}\left(e^{ax}\right) = ae^{ax}$ $\quad$ $\displaystyle\int e^{ax}\,dx = \frac{1}{a}e^{ax} + c$

$\dfrac{d}{dx}\left(\log_e(x)\right) = \dfrac{1}{x}$ $\quad$ $\displaystyle\int \frac{1}{x}\,dx = \log_e|x| + c$

$\dfrac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$ $\quad$ $\displaystyle\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$

$\dfrac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$ $\quad$ $\displaystyle\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$

$\dfrac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$ $\quad$ $\displaystyle\int \sec^2(ax)\,dx = \frac{1}{a}\tan(ax) + c$

$\dfrac{d}{dx}\left(\sin^{-1}(x)\right) = \dfrac{1}{\sqrt{1-x^2}}$ $\quad$ $\displaystyle\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c, a > 0$

$\dfrac{d}{dx}\left(\cos^{-1}(x)\right) = \dfrac{-1}{\sqrt{1-x^2}}$ $\quad$ $\displaystyle\int \frac{-1}{\sqrt{a^2-x^2}}\,dx = \cos^{-1}\left(\frac{x}{a}\right) + c, a > 0$

$\dfrac{d}{dx}\left(\tan^{-1}(x)\right) = \dfrac{1}{1+x^2}$ $\quad$ $\displaystyle\int \frac{a}{a^2+x^2}\,dx = \tan^{-1}\left(\frac{x}{a}\right) + c$

product rule: $\dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

quotient rule: $\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

chain rule: $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx}$

Euler's method: If $\dfrac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$

acceleration: $a = \dfrac{d^2x}{dt^2} = \dfrac{dv}{dt} = v\dfrac{dv}{dx} = \dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right)$

constant (uniform) acceleration: $v = u + at$ $\quad$ $s = ut + \dfrac{1}{2}at^2$ $\quad$ $v^2 = u^2 + 2as$ $\quad$ $s = \dfrac{1}{2}(u + v)t$

TURN OVER

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r$

$\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos\theta = x_1 x_2 + y_1 y_2 + z_1 z_2$

$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$

Mechanics

momentum: $\underset{\sim}{p} = m\underset{\sim}{v}$

equation of motion: $\underset{\sim}{R} = m\underset{\sim}{a}$

friction: $F \le \mu N$

END OF FORMULA SHEET

MULTIPLE CHOICE ANSWER SHEET

STUDENT NAME: ……………………………………………………….

Circle the letter that corresponds to each correct answer.

1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E
13	A	B	C	D	E
14	A	B	C	D	E
15	A	B	C	D	E
16	A	B	C	D	E
17	A	B	C	D	E
18	A	B	C	D	E
19	A	B	C	D	E
20	A	B	C	D	E
21	A	B	C	D	E
22	A	B	C	D	E