

The Mathematical Association of Victoria

Trial Exam 2012

SPECIALIST MATHEMATICS

Written Examination 1

STUDENT NAME _____

Reading time: 15 minutes

Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of Book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers,
- Students are NOT permitted to bring into the examination room: notes of any kind, a calculator of any type, blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 13 pages with a detachable sheet of miscellaneous formulas at the back.

Instructions

- Detach the formula sheet from the back of this book during reading time.
- Write your name in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ m/s}^2$, where $g = 9.8$

Question 1

A helicopter is rising with a constant velocity of 20 m/s. When the helicopter is 60 m above the ground a parcel is dropped out of it. Find an equation in the form

$$gt^2 + bt + c = 0, \text{ where } g \text{ is the magnitude of the acceleration due to gravity and } b \text{ and } c \text{ are integers,}$$

which, when solved will give the exact time it takes for the parcel to hit the ground.

2 marks

Question 2

An object of mass 11 kg **at rest** on a rough horizontal surface is being pulled by a force of magnitude 20 newtons acting in a direction 30 degrees to the horizontal. The coefficient of friction between the object and the surface is 0.2.

- a. In the space below draw a diagram that shows all the forces acting on the object. You must clearly label these forces.

1 mark

- b. Find the exact size of the friction force acting on the object.

2 marks

Total 3 marks

Consider the function $f : (-1, +\infty) \rightarrow \mathbb{R}$, $f(x) = \tan^{-1}\left(\frac{2}{\sqrt{x+1}}\right)$.

- [illegible]

b. Find the exact value of x when $f(x) = \frac{\pi}{3}$.

Total 3 marks

Question 4

- a.** Find in polar form all numbers $w \in \mathbb{C}$ such that $w^3 + 4 - i4\sqrt{3} = 0$.

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

3 marks

- b.** Hence state in polar form all numbers $u \in \mathbb{C}$ such that $u^3 + 4 + i4\sqrt{3} = 0$.

1 mark

Total 4 marks

Question 5

Relative to an origin O , a toy train is at the point with coordinates $(-1, -1, 1)$ at time $t = \pi$ and has velocity

$$\underline{v} = 2 \sin\left(\frac{t}{2}\right) \underline{\hat{i}} + \cos(t) \underline{\hat{j}} + (2t) \underline{\hat{k}} \text{ at time } t.$$

Find the position vector of the toy train when $t = \frac{3\pi}{2}$.

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page. There are no margins, text, or other markings on the paper.

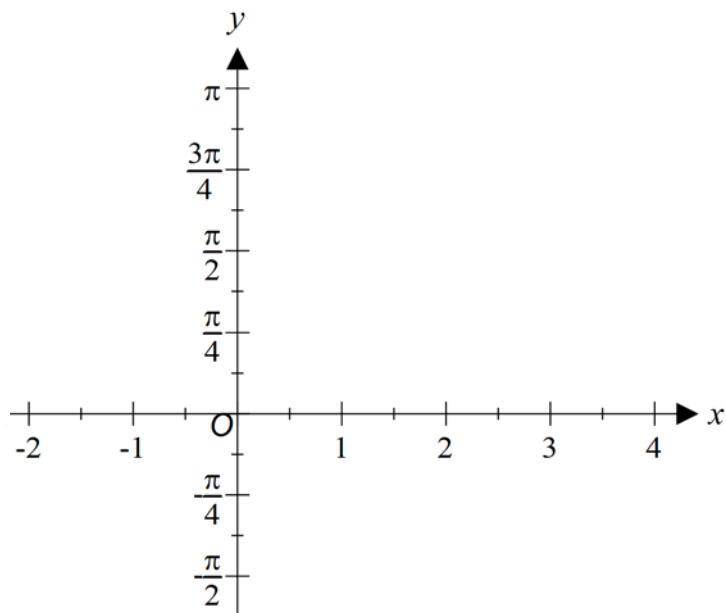
3 marks

Question 6

Consider the function f with rule $f(x) = \frac{3\pi}{4} - \cos^{-1}\left(\frac{x-1}{2}\right)$.

- a. Sketch the graph of $y = f(x)$ on the axes below.

Label the endpoints and the point of inflection with their **exact coordinates**, and label all axes intercepts with their exact values.



3 marks

The part of the graph drawn above over the interval $1 \leq x \leq 1 + \sqrt{2}$ is rotated about the y -axis to form a solid of revolution.

- b. i.** Write down a **definite integral** in terms of y , which when evaluated will give the volume of this solid.

- ii.** Find the exact value of the integral in **part i**.

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

2 + 4 = 6 marks

Total 9 marks

Question 7

The graph of $y = \frac{a}{x^2 + 2ax + b}$, where $a, b \in R$, has a range of $(-\infty, 0) \cup \left[\frac{1}{4}, +\infty\right)$ and one of its vertical asymptotes has the equation $x = -1$.

Find the exact values of a and b .

[illegible]

4 marks

Question 9

A mass has acceleration $a \text{ m/s}^2$ given by $a = \frac{1}{v+3}$ where $v \text{ m/s}$ is the velocity of the mass after t seconds. The displacement of the mass from the origin after t seconds is $x \text{ m}$. Given that $v=0$ and $x=0$ when $t=0$:

- a.** Find the exact position of the mass when $v=1$.

[illegible]

3 marks

- b.** Show that $v = -3 + \sqrt{2t + b}$ where b is an integer.

[illegible]

3 marks
Total 6 marks

Question 10

Consider the function $f : [-1, +\infty) \rightarrow \mathbb{R}$, $f(x) = xe^x$.

Find, in simplest form, the gradient of the normal to the **inverse function** f^{-1} at the point where $x = e$.

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

3 marks

END OF QUESTION AND ANSWER BOOK

SPECIALIST MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

curved surface area of a cylinder: $2\pi rh$

volume of a cylinder: $\pi r^2 h$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a pyramid: $\frac{1}{3}Ah$

volume of a sphere: $\frac{4}{3}\pi r^3$

area of a triangle: $\frac{1}{2}bc \sin A$

sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab \cos C$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

hyperbola: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Circular (trigonometric) functions

$$\cos^2(x) + \sin^2(x) = 1$$

$$1 + \tan^2(x) = \sec^2(x)$$

$$\sin(x+y) = \sin(x) \cos(y) + \cos(x) \sin(y)$$

$$\cos(x+y) = \cos(x) \cos(y) - \sin(x) \sin(y)$$

$$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x) \tan(y)}$$

$$\cos(2x) = \cos^2(x) - \sin^2(x) = 2 \cos^2(x) - 1 = 1 - 2 \sin^2(x)$$

$$\sin(2x) = 2 \sin(x) \cos(x)$$

$$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$$

$$\sin(x-y) = \sin(x) \cos(y) - \cos(x) \sin(y)$$

$$\cos(x-y) = \cos(x) \cos(y) + \sin(x) \sin(y)$$

$$\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x) \tan(y)}$$

$$\tan(2x) = \frac{2 \tan(x)}{1 - \tan^2(x)}$$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (complex numbers)

$$z = x + yi = r(\cos \theta + i \sin \theta) = r \operatorname{cis} \theta$$

$$|z| = \sqrt{x^2 + y^2} = r$$

$$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$$

$$z^n = r^n \operatorname{cis}(n\theta) \quad (\text{de Moivre's theorem})$$

$$-\pi < \operatorname{Arg} z \leq \pi$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$$

Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$$

$$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$$

$$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$$

$$\frac{d}{dx}(\tan(ax)) = a \sec^2(ax)$$

$$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$$

$$\int \frac{1}{x} dx = \log_e |x| + c$$

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$$

$$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$$

$$\int \sec^2(ax) dx = \frac{1}{a} \tan(ax) + c$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c, a > 0$$

$$\int \frac{-1}{\sqrt{a^2 - x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, a > 0$$

$$\int \frac{a}{a^2 + x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$$

product rule:

$$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

quotient rule:

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

chain rule:

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

Euler's method:

$$\text{If } \frac{dy}{dx} = f(x), \quad x_0 = a \text{ and } y_0 = b, \text{ then } x_{n+1} = x_n + h \text{ and } y_{n+1} = y_n + hf(x_n)$$

acceleration:

$$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$\text{constant (uniform) acceleration: } \quad v = u + at \quad s = ut + \frac{1}{2}at^2 \quad v^2 = u^2 + 2as \quad s = \frac{1}{2}(u+v)t$$

TURN OVER

Vectors in two and three dimensions

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$|\vec{r}| = \sqrt{x^2 + y^2 + z^2} = r$$

$$\dot{\vec{r}} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\vec{i} + \frac{dy}{dt}\vec{j} + \frac{dz}{dt}\vec{k}$$

$$\vec{r}_1 \cdot \vec{r}_2 = r_1 r_2 \cos \theta = x_1 x_2 + y_1 y_2 + z_1 z_2$$

Mechanics

momentum:

$$\vec{p} = m\vec{v}$$

equation of motion:

$$\vec{R} = m\vec{a}$$

friction:

$$F \leq \mu N$$

END OF FORMULA SHEET