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Specialist Mathematics

2012

Trial Examination 1

Instructions

Answer **all** questions. Do **not** use calculators.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working or explanation **must** be shown.

Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ ms}^{-2}$, where $g = 9.8$.

Question 1

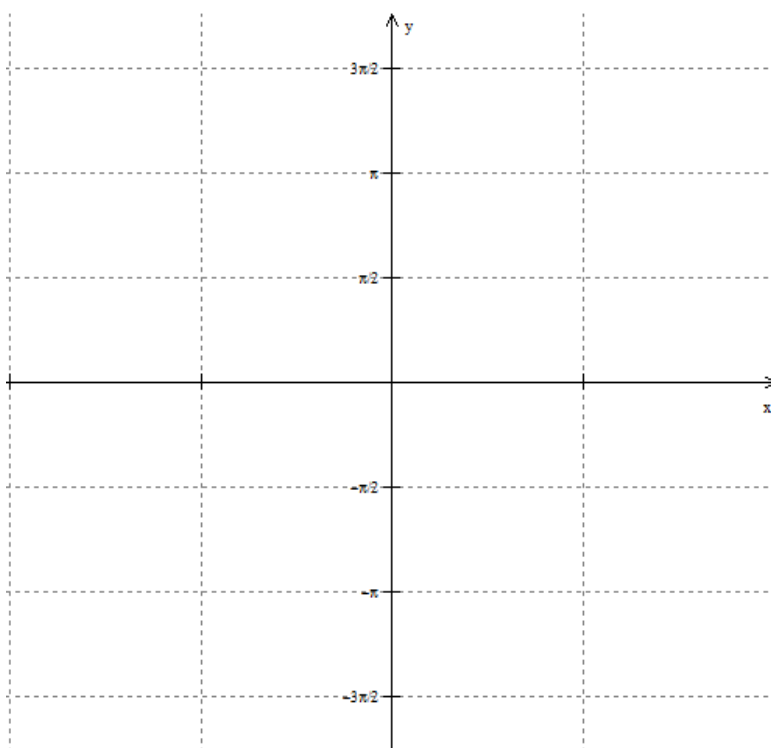
Let $y = \tan^{-1}(x) + \tan^{-1}(2x) + \tan^{-1}(3x)$.

a. Find the exact value of $\tan^{-1}(1) + \tan^{-1}(2) + \tan^{-1}(3)$.

2 marks

b. Sketch the graph of $y = \tan^{-1}(x) + \tan^{-1}(2x) + \tan^{-1}(3x)$. Clearly draw and label the asymptotes.

2 marks



Question 2

Given the relation $(x + 2)^2 + 2(y - \sqrt{2})^2 = 4$.

a. Find the maximal domain and range of the given relation.

2 marks

b. The relation is dilated from the x -axis by a factor of $\sqrt{2}$. Find the equation of the relation after the dilation.

2 marks

c. Find the exact area of the region enclosed by the relation in part **b**.

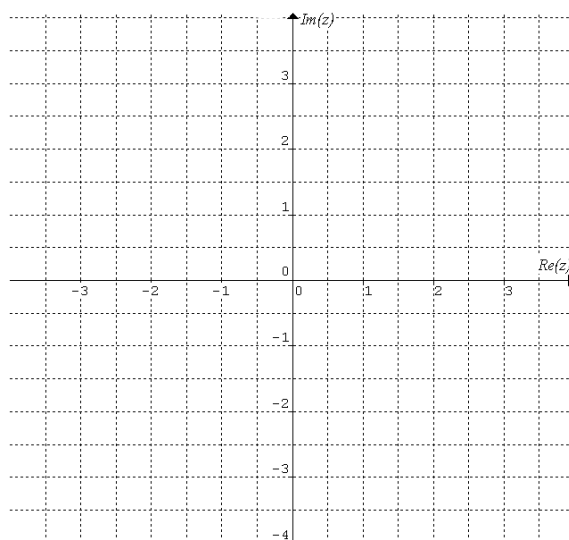
1 mark

Question 3

Let $z \in \{z : |z - 2i| \leq 1\}$.

a. Shade clearly the region in the complex plane for $\{z : |z - 2i| \leq 1\}$.

2 marks



b. Find the maximum value of $\text{Arg}(z)$.

2 marks

Question 4

Solve $z^5 = \bar{z}$ for z .

3 marks

Question 5

Given 3-dimensional position vector $\tilde{r} = 2\tilde{i} - \tilde{j} + 3\tilde{k}$.

a. Find a 3-dimensional vector perpendicular to $\tilde{r} = 2\tilde{i} - \tilde{j} + 3\tilde{k}$.

2 marks

b. Hence or otherwise, find a 3-dimensional *unit* vector perpendicular to $\tilde{r} = 2\tilde{i} - \tilde{j} + 3\tilde{k}$.

1 mark

c. Hence or otherwise, find the exact coordinates of a point which is 1 unit from $\tilde{r} = 2\tilde{i} - \tilde{j} + 3\tilde{k}$.

1 mark

Question 6

The position of a particle is given by $\tilde{r} = (\cos t)\tilde{i} - (2 \sin t)\tilde{j} + 3\tilde{k}$.

a. State the position of the centre of the motion.

1 mark

b. Show that the particle's acceleration always points towards the centre of the motion.

3 marks

Question 7

Given $f(x) = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$ where $-1 < x < 1$.

a. Find the exact value of $f\left(\frac{\sqrt{2}}{2}\right)$.

2 marks

b. Find the exact area of the region bounded by the x -axis, $y = f(x)$, $x = -1$ and $x = 1$.

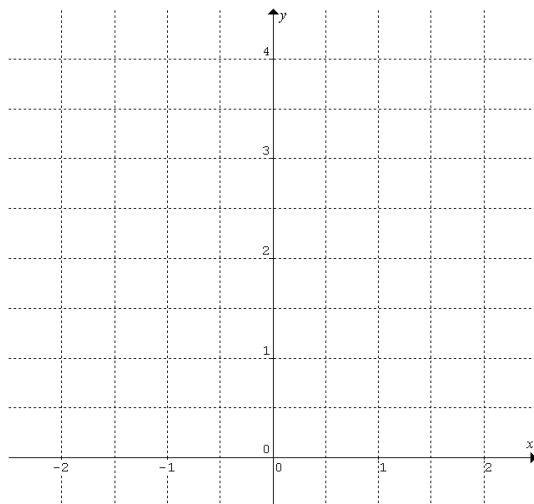
2 marks

Question 8

Given $\frac{dy}{dx} = xy$ where $-2 \leq x \leq 2$ and $0 \leq y \leq 4$.

- a. Construct a slope field for the differential equation using 1 unit interval for both x and y . Draw each tangent line about 0.5 unit long.

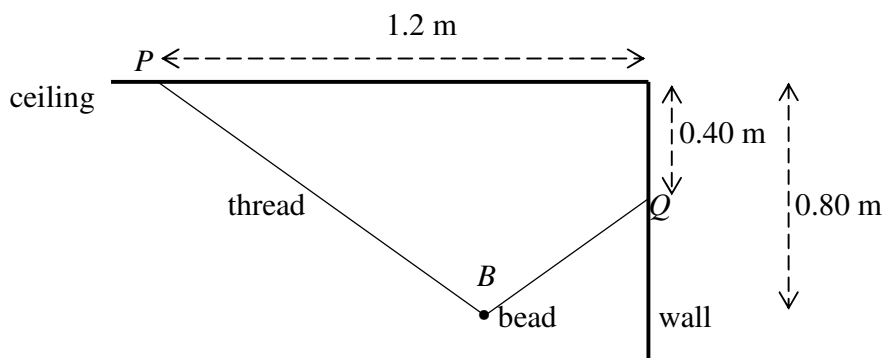
3 marks



- b. Sketch the solution curve to the differential equation $\frac{dy}{dx} = xy$ through $(0,1)$.

1 mark

Question 9



A 0.010 kg bead is free to slide along a weightless and frictionless thread fixed to the ceiling and wall as shown above. The bead is in equilibrium when it is 0.80 m below the ceiling.

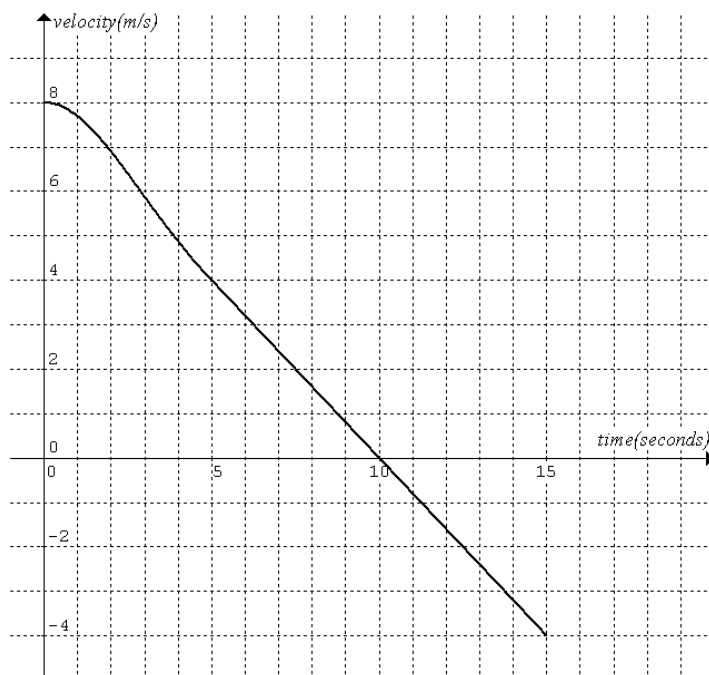
Points P , B and Q are on the same vertical plane.

Calculate the tension (2 significant figures) in the thread.

3 marks

Question 10

The velocity-time graph of a 0.50 kg particle for the first 15 seconds is shown below. Time is measured in seconds and distance in metres.



The equation of the graph is $v(t) = \begin{cases} \frac{8}{1+0.04t^2} & 0 \leq t < 5 \\ -0.8t + 8 & 5 \leq t \leq 15 \end{cases}$

- a. Find the magnitude of the change in momentum of the particle in the first 15 seconds. 1 mark
- b. Find the magnitude of the resultant force on the particle at $t = 5$. 1 mark
- c. Find the exact distance travelled by the particle in the first 15 seconds. 3 marks

End of Exam 1