

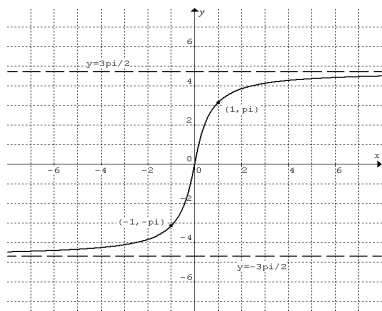
Q1a $\tan^{-1}(1) = \frac{\pi}{4}$, $0 < \tan^{-1}(2) < \frac{\pi}{2}$, $0 < \tan^{-1}(3) < \frac{\pi}{2}$

$$\tan(\tan^{-1}(2) + \tan^{-1}(3)) = \frac{\tan(\tan^{-1}(2)) + \tan(\tan^{-1}(3))}{1 - \tan(\tan^{-1}(2)) \times \tan(\tan^{-1}(3))}$$

$$= \frac{2+3}{1-2 \times 3} = -1, \therefore \tan^{-1}(2) + \tan^{-1}(3) = \frac{3\pi}{4}$$

$$\therefore \tan^{-1}(1) + \tan^{-1}(2) + \tan^{-1}(3) = \pi$$

Q1b Sketch $y = \tan^{-1}(x)$, $y = \tan^{-1}(2x)$ and $y = \tan^{-1}(3x)$ on the same set of axes. By addition of ordinates:



Q2a $\frac{(x+2)^2}{4} + \frac{(y-\sqrt{2})^2}{2} = 1$, an ellipse

Domain: $[-4, 0]$, range: $[0, 2\sqrt{2}]$

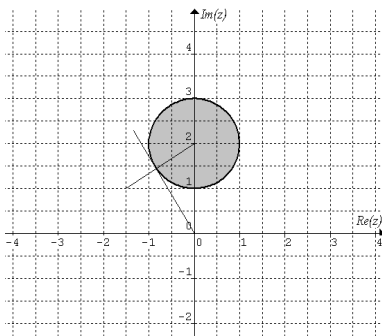
Q2b $\frac{(x+2)^2}{4} + \frac{(y-\sqrt{2})^2}{2} = 1 \rightarrow \frac{(x+2)^2}{4} + \frac{\left(\frac{y}{\sqrt{2}} - \sqrt{2}\right)^2}{2} = 1$

Simplify $\frac{(x+2)^2}{4} + \frac{\left(\frac{y}{\sqrt{2}} - \sqrt{2}\right)^2}{2} = 1$ to $(x+2)^2 + (y-2)^2 = 4$

Q2c $(x+2)^2 + (y-2)^2 = 4$ is a circle of radius 2.

Area = $\pi \cdot 2^2 = 4\pi$

Q3a



Q3b $\text{Max Arg}(z) = \frac{\pi}{2} + \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2} + \frac{\pi}{6} = \frac{2\pi}{3}$

Q4 By inspection, $z = 0$ is a solution.

For $z \neq 0$, $z^5 = \bar{z}$, $zz^5 = z\bar{z}$, $z^6 = |z|^2$

Let $z = r\text{cis}\theta$, $r^6\text{cis}(6\theta) = r^2$, $r^4\text{cis}(6\theta) = 1$

$\therefore r = 1$ and $6\theta = 2k\pi$ where $k = 0, 1, 2, 3, 4, 5$

$\therefore z = \text{cis}0, \text{cis}\frac{\pi}{3}, \text{cis}\frac{2\pi}{3}, \text{cis}\pi, \text{cis}\frac{4\pi}{3}$ and $\text{cis}\frac{5\pi}{3}$

$\therefore z = 1, \frac{1}{2} + \frac{\sqrt{3}}{2}i, -\frac{1}{2} + \frac{\sqrt{3}}{2}i, -1, -\frac{1}{2} - \frac{\sqrt{3}}{2}i$ and $\frac{1}{2} - \frac{\sqrt{3}}{2}i$

Q5a Let $\tilde{p} = a\tilde{i} + b\tilde{j} + c\tilde{k}$ be a vector perpendicular to $\tilde{r} = 2\tilde{i} - \tilde{j} + 3\tilde{k}$ where a, b and c are non-zero scalars.

$\tilde{p} \cdot \tilde{r} = 0$, $\therefore 2a - b + 3c = 0$

Let $a = -1$ and $b = 1$, $\therefore c = 1$, $\therefore \tilde{p} = -\tilde{i} + \tilde{j} + \tilde{k}$

Note: Infinitely many such vectors can be obtained by choosing values for a and b , and then find c .

Q5b $|\tilde{p}| = \sqrt{(-1)^2 + 1^2 + 1^2} = \sqrt{3}$, $\hat{p} = \frac{1}{\sqrt{3}}(-\tilde{i} + \tilde{j} + \tilde{k})$

Q5c A possible point is $\left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$.

Q6a The position of the centre of motion is given by $\tilde{r}_c = 3\tilde{k}$.

Q6b The position of the particle from the centre of motion is $\tilde{R} = \tilde{r} - \tilde{r}_c = (\cos t)\tilde{i} - (2\sin t)\tilde{j}$

$\dot{\tilde{R}} = -(\sin t)\tilde{i} - (2\cos t)\tilde{j}$, $\ddot{\tilde{R}} = -(\cos t)\tilde{i} + (2\sin t)\tilde{j}$, $\therefore \ddot{\tilde{R}} = -\tilde{R}$
 i.e. the particle's acceleration always towards the centre of motion.

Q7a $f(x) = \frac{\sin^{-1}x}{\sqrt{1-x^2}}$, $-1 < x < 1$

$$f'(x) = \frac{\sqrt{1-x^2} \times \frac{1}{\sqrt{1-x^2}} - (\sin^{-1}x) \frac{-x}{\sqrt{1-x^2}}}{\left(\sqrt{1-x^2}\right)^2}$$

$$= \frac{\sqrt{1-x^2} + x(\sin^{-1}x)}{\left(\sqrt{1-x^2}\right)^3}$$

$$f'\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{1-\frac{1}{2}} + \frac{\sqrt{2}}{2} \left(\sin^{-1}\frac{\sqrt{2}}{2}\right)}{\left(\sqrt{1-\frac{1}{2}}\right)^3} = \frac{\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \left(\frac{\pi}{4}\right)}{\left(\frac{\sqrt{2}}{2}\right)^3}$$

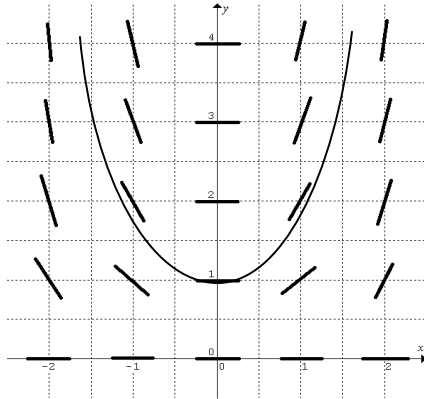
$$= \frac{1 + \frac{\pi}{4}}{\frac{1}{2}} = 2 + \frac{\pi}{2}$$

Q7b Note that $f(-a) = -f(a)$ for $-1 < a < 1$.

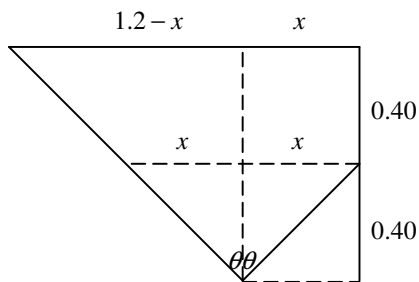
$$\text{Area} = 2 \times \int_0^1 \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx, \text{ let } u = \sin^{-1} x, \frac{du}{dx} = \frac{1}{\sqrt{1-x^2}}$$

$$\therefore \text{area} = 2 \times \int_0^{\frac{\pi}{2}} u du = 2 \times \left[\frac{u^2}{2} \right]_0^{\frac{\pi}{2}} = \frac{\pi^2}{4}$$

Q8a, b



Q9



$$\frac{0.40}{x} = \frac{0.80}{1.2-x}, \therefore x = 0.40, \therefore \theta = 45^\circ$$

Let T be the tension in the thread.

$$2T \cos 45^\circ - 0.010 \times 9.8 = 0, \sqrt{2}T = 0.098, 1.414T = 0.098, T \approx 0.069 \text{ N}$$

$$\text{Q10a } \Delta v = -4 - 8 = -12, |p| = m|\Delta v| = 0.50 \times 12 = 6 \text{ kg m s}^{-1}$$

$$\text{Q10b Gradient at } t = 5 \text{ is } -0.8, \text{ i.e. } a = -0.8 \text{ m s}^{-2}$$

$$|F| = m|a| = 0.50 \times 0.8 = 0.4 \text{ N}$$

$$\begin{aligned} \text{Q10c Distance} &= \int_0^5 \frac{8}{1+0.04t^2} dt + 2 \left(\frac{1}{2} \times 4 \times 5 \right) \\ &= \int_0^5 \frac{8}{1+(0.2t)^2} dt + 20 = \left[\frac{8 \tan^{-1}(0.2t)}{0.2} \right]_0^5 + 20 = 40 \tan^{-1} 1 + 20 \\ &= 10\pi + 20 \text{ metres} \end{aligned}$$

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