INSIGHT

YEAR 12 *Trial Exam Paper*

2012

SPECIALIST MATHEMATICS

Written examination 2

STUDENT NAME:

QUESTION AND ANSWER BOOK

Reading time: 15 minutes
Writing time: 2 hours

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	5	5	58
			Total 80

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT have to be cleared.
- Students are NOT permitted to bring sheets of paper or white-out liquid/tape into the examination.

Materials provided

- The question and answer book of 27 pages, a formula sheet, and an answer sheet for the multiple-choice questions.

Instructions

- Write your **name** in the box provided and on the multiple-choice answer sheet.
- Remove the formula sheet during reading time.
- You must answer the questions in English.

At the end of the examination

- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones or any other electronic devices into the examination.

SECTION 1

Instructions for Section 1
Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

The ellipse with equation $9x^2 + 16y^2 - 18x + 64y - 71 = 0$ has

A. centre $(1, -2)$ with major axis of length 4 units.

B. centre $(-1, 2)$ with major axis of length 8 units.

C. centre $(1, -2)$ with minor axis of length 8 units.

D. centre $(1, -2)$ with major axis of length 8 units.

E. centre $(-1, 2)$ with major axis of length 16 units.

Question 2

The curve $y = \left(x - 2 + \frac{1}{\sqrt{x-2}}\right)\left(x - 2 - \frac{1}{\sqrt{x-2}}\right) + 4$ has a

A. curved asymptote $y = x^2 - 4x - 8$ and vertical asymptote $x = 2$.

B. curved asymptote $y = (x - 2)^2$ and vertical asymptote $x = 2$.

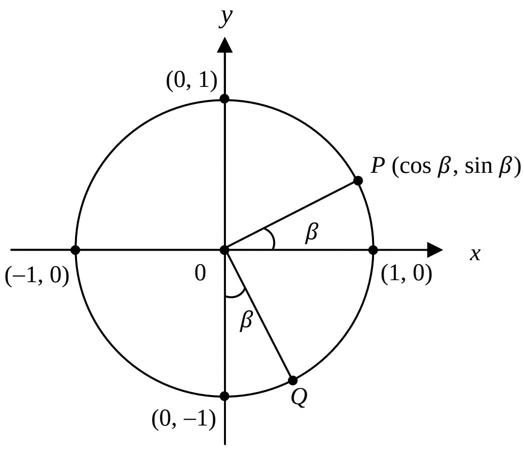
C. curved asymptote $y = x^2 - 4x + 8$ and vertical asymptote $x = 2$.

D. curved asymptote $y = x^2 - 4x + 8$ and vertical asymptote $x = -2$.

E. horizontal asymptote $y = 4$ and vertical asymptote $x = 2$.

Question 3

In the diagram below, P has coordinates $(\cos\beta, \sin\beta)$.
Hence, Q would be the point with coordinates



A. $(\sin\beta, -\cos\beta)$

B. $(\cos\beta, \sin\beta)$

C. $(\sin\beta, \cos\beta)$

D. $(\cos\beta, -\sin\beta)$

E. $(-\sin\beta, -\cos\beta)$

Question 4

If $\sin(x) = -\frac{3}{4}$ and $\pi < x < \frac{3\pi}{2}$ then $\sec(x)$ equals

A. $-\frac{4}{3}$

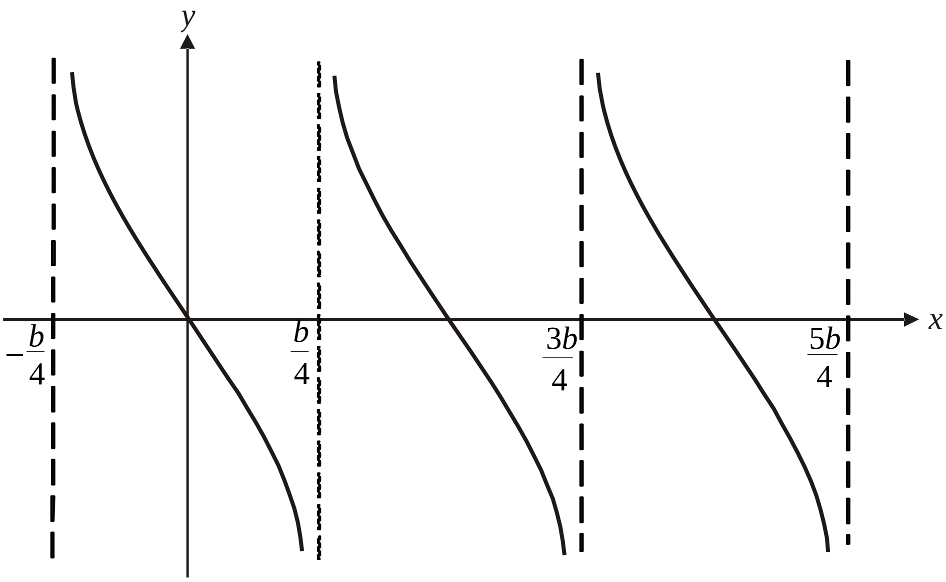
B. $-\frac{4\sqrt{7}}{7}$

C. $-\frac{\sqrt{7}}{4}$

D. $\frac{4}{\sqrt{7}}$

E. $\frac{3\sqrt{7}}{7}$

Question 5



The function whose graph is shown above, where $b > 0$, could have the rule given by

A. $y = -\tan\left(\frac{2bx}{\pi}\right)$

B. $y = \cot\left(\frac{2\pi x}{b}\right)$

C. $y = -\tan\left(\frac{\pi x}{b}\right)$

D. $y = \cot\left[\frac{2\pi}{b}\left(x - \frac{b}{4}\right)\right]$

E. $y = \cot\left[\frac{2\pi}{b}\left(x - \frac{b}{2}\right)\right]$

Question 6

For $y = k - a\sin^{-1}(2x - b)$, $a > 0$, $b > 0$, the maximal domain and range are

A. domain $\left[\frac{-b}{2}, \frac{b}{2}\right]$, range $\left[\frac{a-k\pi}{2}, \frac{a+k\pi}{2}\right]$

B. domain $\left[\frac{b-1}{2}, \frac{b+1}{2}\right]$, range $\left[\frac{a-k\pi}{2}, \frac{a+k\pi}{2}\right]$

C. domain $\left[\frac{b-1}{2}, \frac{b+1}{2}\right]$, range $\left[\frac{2k-a\pi}{2}, \frac{2k+a\pi}{2}\right]$

D. domain $\left[\frac{-b}{2}, \frac{b}{2}\right]$, range $\left[\frac{k-2\pi}{2}, \frac{k+2\pi}{2}\right]$

E. domain $\left[\frac{-b}{2}-1, \frac{b}{2}+1\right]$, range $\left[k-\frac{a\pi}{2}, k+\frac{a\pi}{2}\right]$

Question 7

If $z = 1 + i$, then $\text{Arg}(i^3 z)$ is

A. $\frac{7\pi}{4}$

B. $\frac{5\pi}{4}$

C. $-\frac{3\pi}{4}$

D. $\frac{3\pi}{4}$

E. $-\frac{\pi}{4}$

Question 8

The complex relation $\text{Re}(z - 1) \times \text{Im}(z - 1) = 1$ can be represented on a Cartesian plane as a

A. hyperbola with asymptotes $x = 1$, $y = 0$.

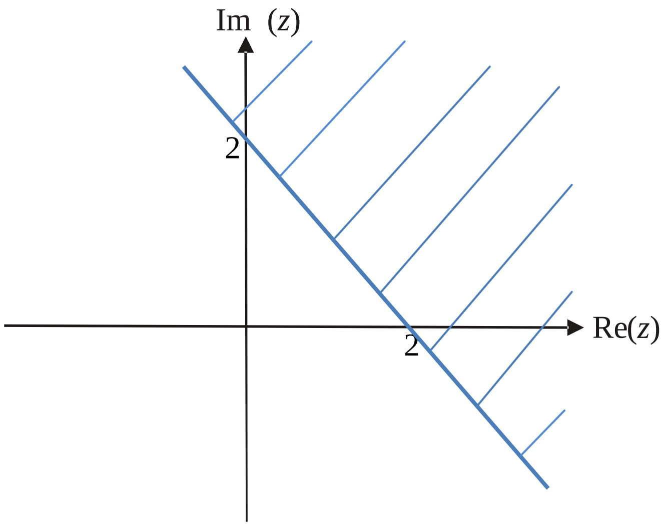
B. hyperbola with asymptotes $y = \pm x$.

C. hyperbola with asymptotes $x = 0$, $y = 1$.

D. straight line with equation $y = 1 - x$.

E. hyperbola with equation $y = \frac{1}{x-1} + 1$.

Question 9



The shaded region on the Argand diagram above represents

A. $|z + 2 + 2i| \geq |z|$

B. $|z - 2 - 2i| \leq |z - 2|$

C. $|z - 2| \geq |z - 2i|$

D. $|z - 2i| \leq |z - 2|$

E. $|z| \geq |z - 2 - 2i|$

Question 10

Given that $z + i$ is a factor of $P(z) = z^3 - z^2 + z - 1$, which one of the following statements is **not** true?

A. $P(z) = 0$ has three solutions.

B. $P(-i) = 0$.

C. $P(z) = 0$ has two real solutions.

D. $P(z)$ can be factorised over C.

E. $P(i) = 0$.

Question 11

If $\dfrac{da}{dy} = 9 + a^2$ and $y = 0$ when $a = 0$, then y is equal to

A. $\tan^{-1}\left(\dfrac{a}{3}\right)$

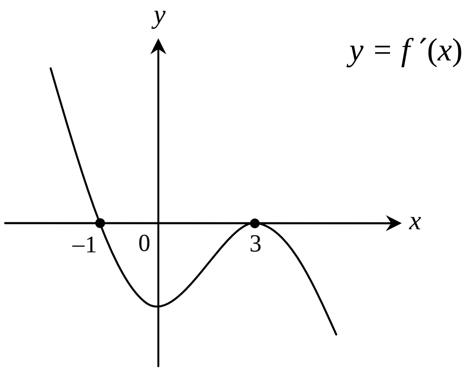
B. $\dfrac{1}{3}\tan^{-1}(3a)$

C. $3\tan^{-1} a$

D. $\dfrac{1}{3}\tan^{-1}\left(\dfrac{a}{3}\right)$

E. $\dfrac{1}{3}\tan\left(\dfrac{a}{3}\right)$

Question 12



If $f(x)$ is an antiderivative of $f'(x)$, the graph of $y = f(x)$ has

A. a stationary point of inflexion at $x = 0$ and a local maximum at $x = -1$

B. stationary points of inflexion at $x = 0$ and $x = 3$ and a local maximum at $x = -1$

C. a stationary point of inflexion at $x = 3$ and a local maximum at $x = -1$

D. a local minimum at $x = 0$ and a local maximum at $x = 3$

E. a stationary point of inflexion at $x = 3$ and a local minimum at $x = -1$

Question 13

For the curve $xy^3 - 2x^2 = 7$

A. $\dfrac{dy}{dx} = \dfrac{3x - 4y^3}{3xy^2}$

B. $\dfrac{dy}{dx} = \dfrac{7 + 4x}{6x}$

C. $\dfrac{dy}{dx} = \dfrac{4x - y^3}{6xy^2}$

D. $\dfrac{dy}{dx} = \dfrac{y^3}{3xy^2}$

E. $\dfrac{dy}{dx} = \dfrac{4x - y^3}{3xy^2}$

Question 14

Using an appropriate substitution, $\int \dfrac{e^{2x}}{1 + e^x}\, dx$ can be written as

A. $\int \left(1 - \dfrac{1}{u}\right) du$

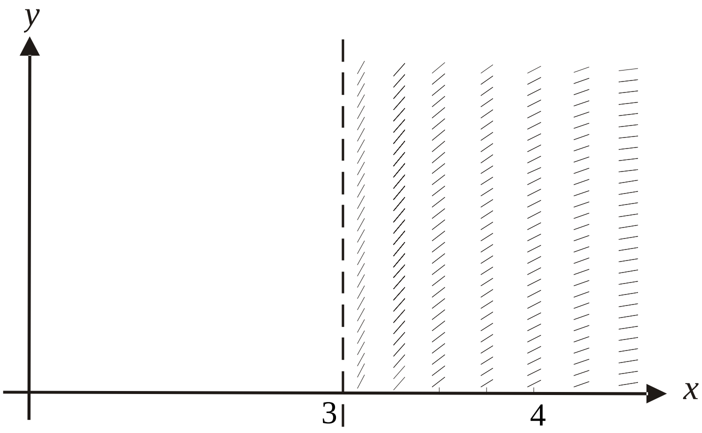
B. $\int \left(\dfrac{u}{1 + u^2}\right) du$

C. $\int \left(\dfrac{u^2}{1 + u}\right) du$

D. $\int \left(\dfrac{2u}{1 + u}\right) du$

E. $\int \left(1 + \dfrac{1}{u}\right) du$

Question 15



The direction or slope field for a particular first-order differential equation is shown above. The differential equation could be

A. $\dfrac{dy}{dx} = \dfrac{1}{x+3}$

B. $\dfrac{dy}{dx} = \dfrac{1}{x-3}$

C. $\dfrac{dy}{dx} = \log_e(x-3)$

D. $\dfrac{dy}{dx} = \log_e(x+3)$

E. $\dfrac{dy}{dx} = \sqrt{x-3}$

Question 16

If $\underset{\sim}{u} = 3\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{v} = \underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k}$, the vector resolute of $\underset{\sim}{u}$ in the direction of $\underset{\sim}{v}$ is

A. $\frac{9}{7}\underset{\sim}{i} - \frac{6}{7}\underset{\sim}{j} + \frac{3}{7}\underset{\sim}{k}$

B. $2\underset{\sim}{i} - 2\underset{\sim}{j} + 2\underset{\sim}{k}$

C. $2\underset{\sim}{i} + 2\underset{\sim}{j} - 2\underset{\sim}{k}$

D. $2\underset{\sim}{i} - 2\underset{\sim}{j} - 2\underset{\sim}{k}$

E. $\frac{9}{7}\underset{\sim}{i} + \frac{6}{7}\underset{\sim}{j} + \frac{3}{7}\underset{\sim}{k}$

Question 17

A particle moves along a straight line such that its acceleration at time *t* (seconds) is given by $\ddot{x} = 3t^2 - 4t + 5$ m/s^2.

Initially, the particle is at a fixed point, *O*, ($x = 0$) with a velocity of –2 m/s^2.

The position of the particle from *O* at time *t* is

A. 6

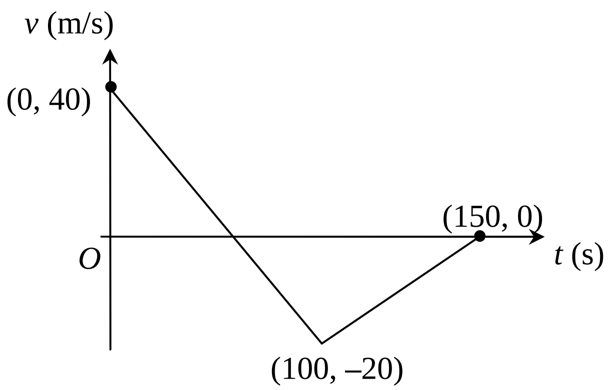
B. $\frac{1}{4}t^4 - \frac{2}{3}t^3 + \frac{5}{2}t^2 - 2t$

C. –6

D. $3t^4 - \frac{2}{3}t^3 + \frac{5}{2}t^2 - 2t$

E. $\frac{1}{4}t^4 - t^3 + \frac{5}{2}t^2 - 2t$

Question 18



The velocity–time graph of a particle moving in a straight line starting from a fixed position, *O*, is shown above. If the initial velocity is 40 m/s in an easterly direction, where is the particle located 150 seconds later?

A. 1000 m west of *O*

B. $\frac{6500}{3}$ m east of *O*

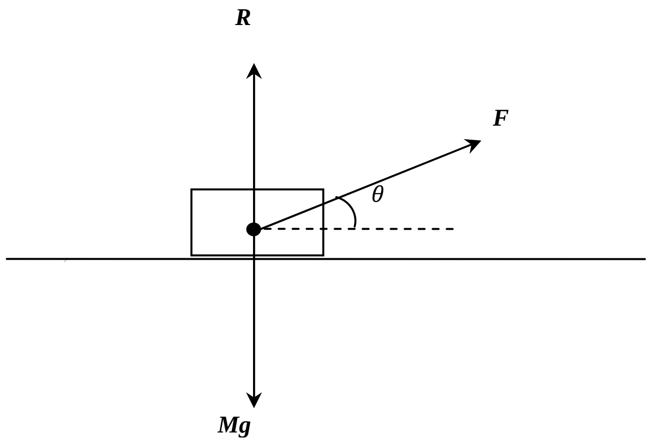
C. 2000 m east of *O*

D. 500 m east of *O*

E. 1000 m east of *O*

Question 19

A body of mass M kg is being pulled along a smooth horizontal table by means of a force, F, acting at an angle θ to the horizontal.
The diagram below indicates the forces acting on the body.



Which statement regarding the magnitude of the forces is true?

A. $R - Mg = 0$

B. $R - F\sin\theta - Mg = 0$

C. $R + F\sin\theta - Mg = 0$

D. $R + F\cos\theta - Mg = 0$

E. $R - F\cos\theta - Mg = 0$

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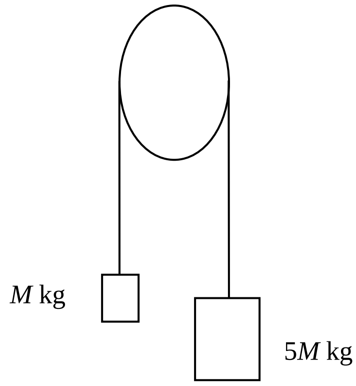
SECTION 1 – continued

Question 20

The diagram below shows a smooth pulley with masses of *M* kg and 5*M* kg attached to each end of an inextensible string.



The magnitude of the acceleration of the 5*M* kg mass is

A. $\frac{2}{3}$ m/s^2

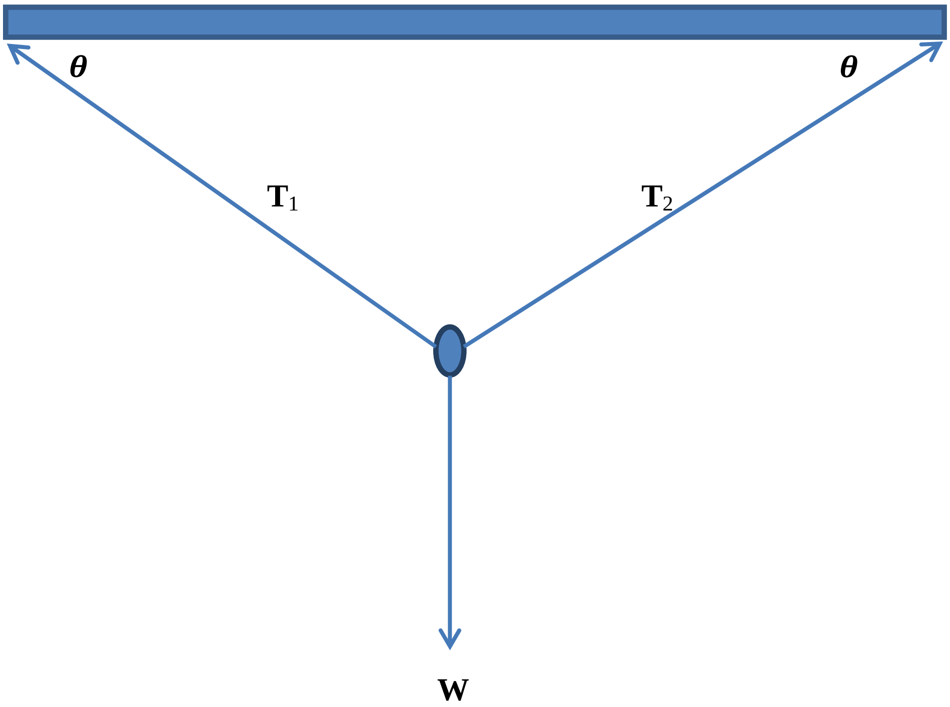
B. $\frac{4g}{3}$ m/s^2

C. $\frac{2g}{3}$ m/s^2

D. $\frac{4}{3}$ m/s^2

E. $3g$ m/s^2

Question 21



An object in **equilibrium** is suspended from a beam by two ropes, each making an angle of θ with the horizontal. The tension forces acting on the object are $\mathbf{T}_1$ and $\mathbf{T}_2$ and **W** is the weight force. Which one of the following statements is true?

A. $\mathbf{T}_1 = \mathbf{T}_2$

B. $\mathbf{W} = \mathbf{T}_1 \cos \theta° + \mathbf{T}_2 \cos \theta°$

C. $\mathbf{T}_1 + \mathbf{T}_2 = -\mathbf{W}$

D. $\mathbf{T}_1 \cos \theta° = \mathbf{T}_2 \sin \theta°$

E. $\mathbf{T}_1 + \mathbf{T}_2 = 2\mathbf{W}$

Question 22

An object of mass 3 kg comes to a complete stop from 20 m/s over a horizontal distance of 60 m. If the acceleration acting on the object is constant, then the **magnitude** of the resultant force acting on the object is

A. 10 N

B. $\frac{10}{3}$ N

C. 19.6 N

D. –10 N

E. $\frac{10}{3}$ N

END OF SECTION 1

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

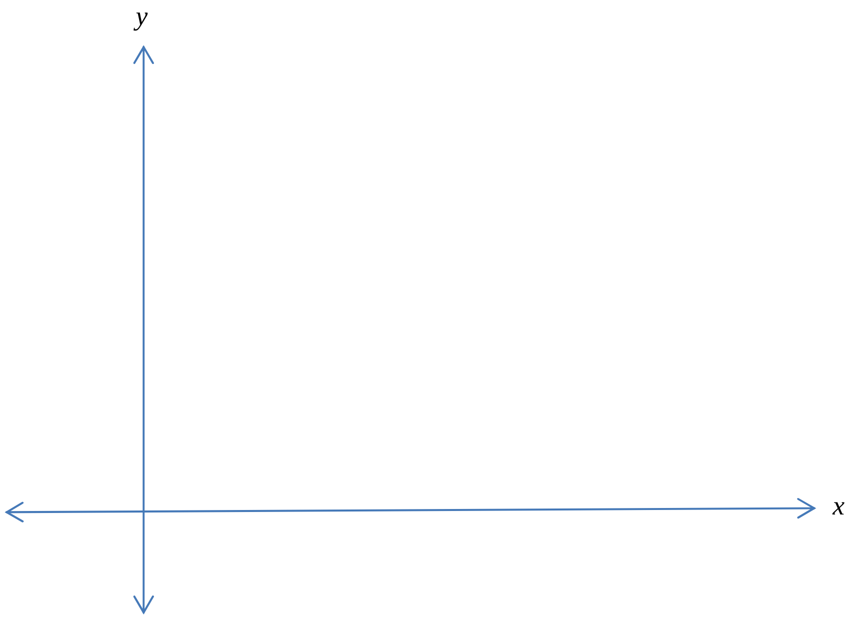
Question 1

Let $f : [a, b] \to R$, where $f(x) = (a - x)^2$.

a. If $f(1) = 0$ and $f(b) = 9$, show that $a = 1$ and $b = 4$.

2 marks

b. Sketch the graph of the function, clearly labelling the endpoints with their co-ordinates.



1 mark

c. Find the area enclosed between the graph of the function and the y-axis. (All units are in centimetres.)

2 marks

d. The function f is rotated about the y-axis to generate a vessel. If the volume of the vessel is $\frac{m\pi}{n}$ cm^3 find the values of m and n where m and n are both integers.

2 marks

e. Water now enters the vessel at a constant rate of 2 cm^3/s. How long does it take for the vessel to be filled to 20% of its capacity?
(Write the answer in minutes, to 2 decimal places.)

2 marks

f. Write an expression that will determine the height (h cm) of water in the vessel when it is filled to 20% of its capacity and determine the value of h, to 2 decimal places.

2 marks

Total 11 marks

Question 2

The position of a tugboat relative to an island, *O*, at any time (*t* hours) is given by

$$\underset{\sim}{r}(t) = \left(t + \frac{1}{t}\right)\underset{\sim}{i} + \left(t - \frac{1}{t}\right)\underset{\sim}{j},\ t > 0,$$

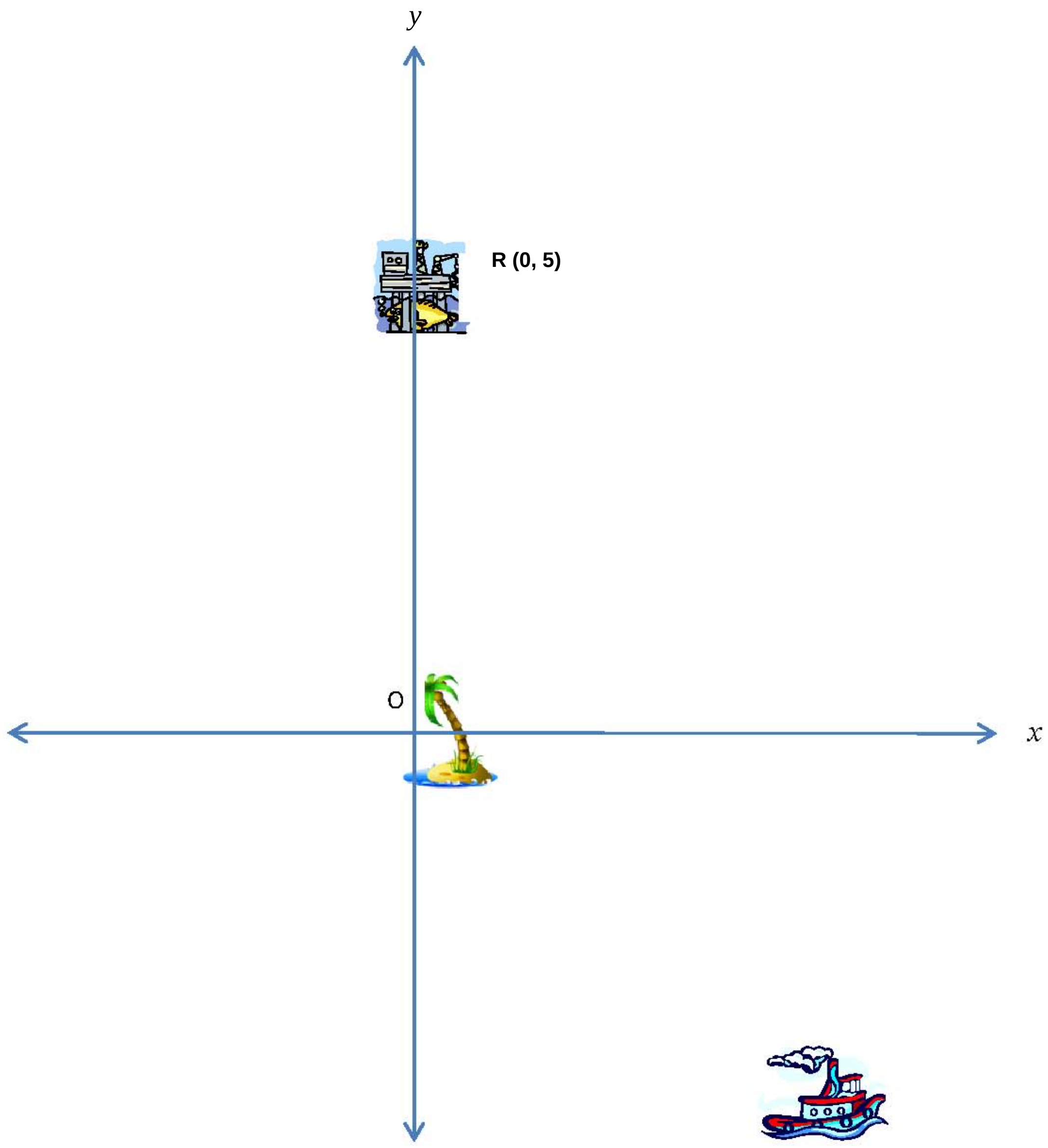
where $\underset{\sim}{i}$ and $\underset{\sim}{j}$ are unit vectors east and north of the island, respectively. An oil rig, *R*, is located 5 km north of the island.

(All distances are in km and the island can be considered as the origin of a Cartesian plane.)

a. Show that the path of the tugboat can be described by the Cartesian equation $\frac{x^2}{4} - \frac{y^2}{4} = 1$ and state the domain and range of the path.

3 marks

b. Sketch a graph of the path travelled by the tugboat, clearly indicating the direction of motion and label any intercepts.



2 marks

c. When will the tugboat be due east of the island?

2 marks

d. Find the velocity vector of the tugboat at any time *t*.

1 mark

e. If P is any point on the path of the tugboat, find the vector $\underset{\sim}{RP}$ in terms of t.

2 marks

f. Find the time, to the nearest minute, when the tugboat is **closest** to the oil rig.

3 marks

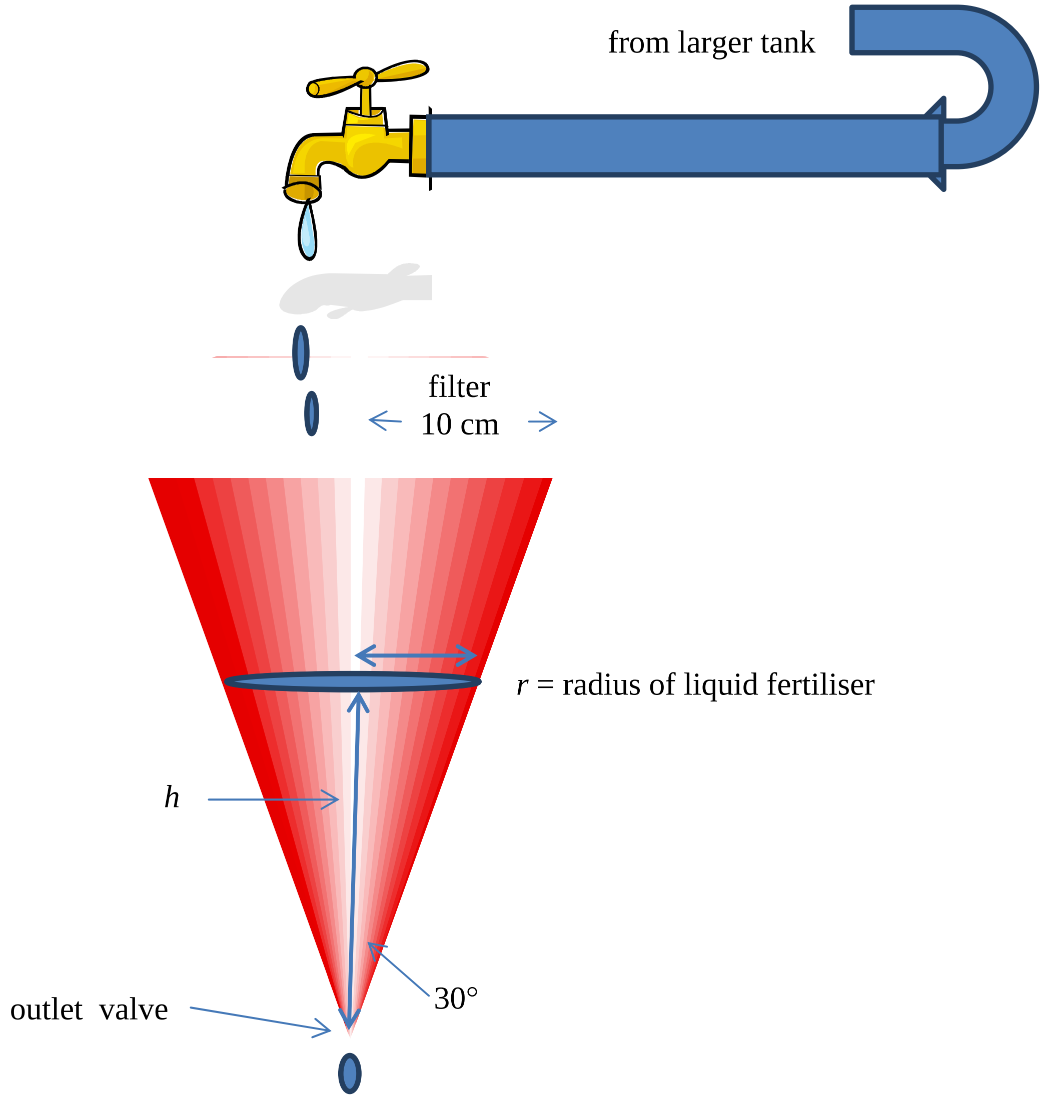
g. Find the times, to the nearest minute, when the island, tugboat and oil rig form the vertices of an isosceles triangle with equal sides of 5 km.

4 marks

Total 17 marks

Question 3

A fertiliser drip system contains a filter in the form of an inverted cone with a semi-vertical angle of 30°, as shown in the diagram below. The radius of the filter is 10 cm.



a. Liquid fertiliser from a larger tank pours from a tap into the filter at a constant rate of 2 cm^3/s and drips from an outlet valve in the bottom at $\frac{\sqrt{h}}{5}$ cm^3/s, where h cm is the depth of the water in the filter at any time t seconds.

i. Show that $\frac{dV}{dt} = \frac{10 - \sqrt{h}}{5}$.

1 mark

ii. Find the volume of the liquid fertiliser (V) in terms of its depth (h).
Hence, find $\frac{dh}{dt}$.

4 marks

1 + 4 = 5 marks

b. The outlet valve is now closed and the filter is completely filled from the larger tank. Find the **exact** height of liquid fertiliser in the filter when it is completely full.

1 mark

c. When the filter is completely full, the tap is closed and the outlet valve opened. The liquid fertiliser drips out from the outlet valve at the same rate as before;

i.e. $\dfrac{\sqrt{h}}{5}$ cm^3/s.

i. Find $\dfrac{dh}{dt}$ and **use calculus to find *t* exactly in terms of *h*.**

5 marks

ii. How long, to the nearest minute, does it take for the filter to empty?

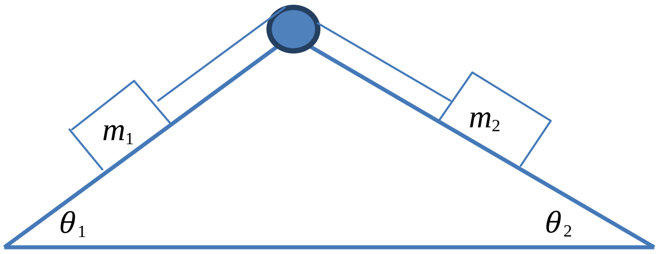
1 mark

5 + 1 = 6 marks

Total 12 marks

Question 4

Two objects of mass m_1 and m_2 are connected by a light string passing over a smooth pulley, which is located at the apex of two inclined planes, as shown in the diagram below. The coefficients of friction of the two surfaces are μ_1 and μ_2 and the angles of inclination are θ_1 and θ_2, respectively.



Note: The system is on the verge of moving to the left.

a. Label all the forces acting on the two objects using the symbols **T**, $\mathbf{N}_1$, $\mathbf{N}_2$, $\boldsymbol{\mu}_1\mathbf{N}_1$, $\boldsymbol{\mu}_2\mathbf{N}_2$, $\boldsymbol{m_1 g}$ and $\boldsymbol{m_2 g}$.

2 marks

b. The planes are now lubricated and can be considered to be smooth.

i. If the system is in equilibrium, find $\dfrac{m_1}{m_2}$ in terms of θ_1 and θ_2.

3 marks

ii. The object m_1 is now replaced by an object with a mass that is double that of m_2 and both angles of inclination are the same (θ).
Find the acceleration of the system if it is moving to the left.

3 marks

3 + 3 = 5 marks

Total 8 marks

Question 5

a. Consider $u = a - a\sqrt{3}\,i$, where $a < 0$, and $w = b\,\text{cis}\left(\frac{\pi}{4}\right)$.

i. Express u in polar form.

2 marks

ii. Express w in Cartesian form.

1 mark

2 + 1 = 3 marks

b. Express each of the following as a complex number.

i. $u + w$ in Cartesian form

1 mark

ii. uw in polar form

1 mark

iii. u^6 in Cartesian form

1 mark

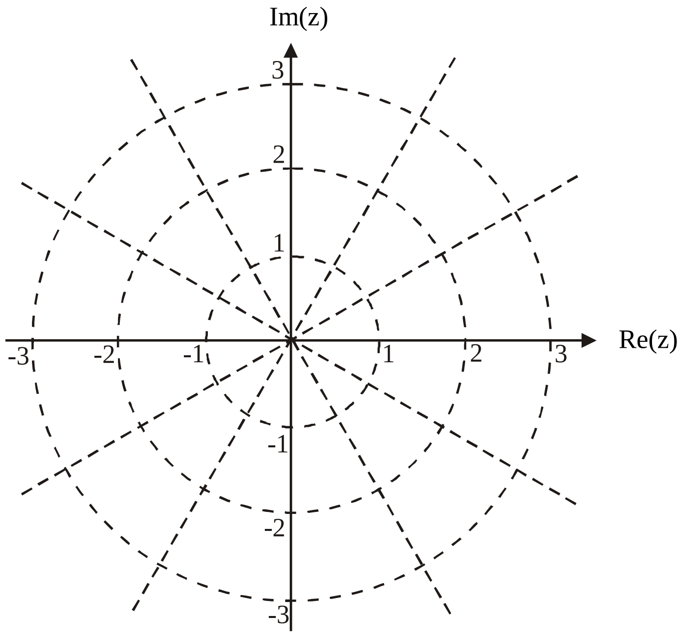
1 + 1 + 1 = 3 marks

SECTION 2 – Question 5 – continued

c. Find the square roots of w in polar form.

2 marks

d. If $a = -\frac{1}{2}$ and $b = \sqrt{2}$, plot u^{12} and w^{-2} on the Argand diagram below.



2 marks
Total: 3 + 3 + 2 + 2 = 10 marks

END OF SECTION 2

END OF QUESTION AND ANSWER BOOK