INSIGHT

Year 12 *Trial Exam Paper*

2012

SPECIALIST MATHEMATICS

Written examination 1

STUDENT NAME:

QUESTION AND ANSWER BOOK

Reading time: 15 minutes
Writing time: 1 hour

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring sheets of paper, notes of any kind or white out liquid/tape into the examination.
- Calculators are not permitted in this examination.

Materials provided

- The question and answer book of 15 pages with a separate sheet of miscellaneous formulas.
- Working space is provided throughout this book.

Instructions

- Write your **name** in the box provided.
- Remove the formula sheet during reading time.
- You must answer the questions in English.

Students are NOT permitted to bring mobile phones and/or any other electronic devices into the examination.

Question 1

a. Show that for $0 < x < 1$, $\frac{d}{dx}\left(\arcsin\left(2x-1\right)\right) = \frac{1}{\sqrt{x-x^2}}$.

2 marks

b. Hence, find the exact value of $\int_{\frac{1}{2}}^{\frac{3}{4}} \frac{6}{\sqrt{x-x^2}}\,dx$

2 marks

END OF QUESTION 1

Question 2

a. Solve the following equation over *C*.

$z^2 - 2iz + 5 = 0$

2 marks

b. Let $z_1 = \sqrt{3} - i$.

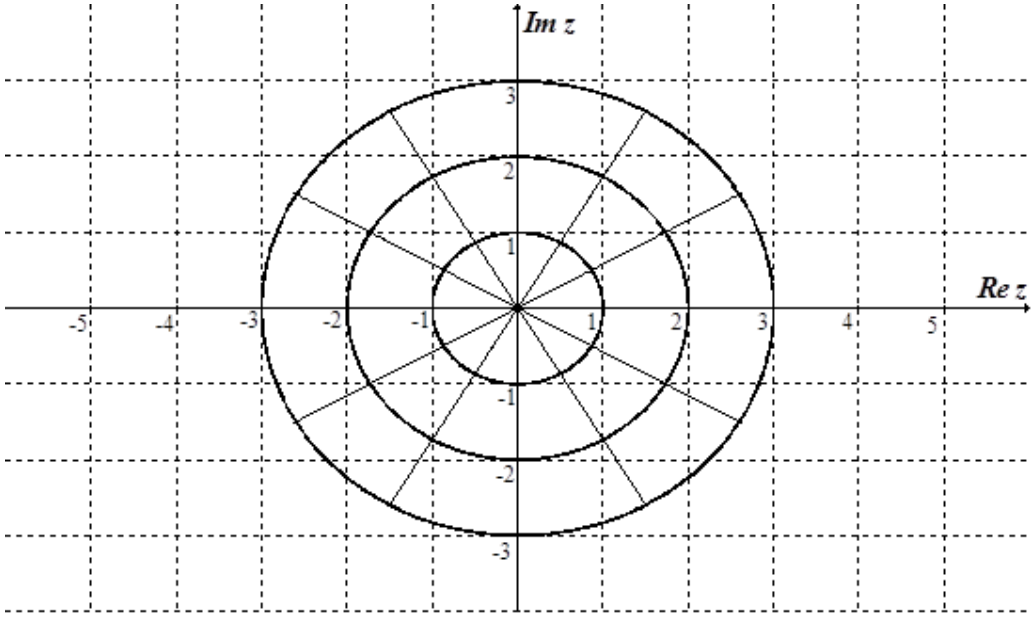
Express z_1 in polar form, $rcis\,\theta$ where $\theta = Arg(z_1)$.

1 mark

c. On the argand diagram below, plot and clearly label

i. z_1

ii. $z_2 = \left|\overline{z_1}\right| i$



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2 marks

END OF QUESTION 2

Question 3

A particle moves in a straight line with an acceleration of a m/s^2, as given by

$$a = \frac{v^2 + v}{1 + \log_e(v+1)} \quad , \quad v > 0 .$$

At time t seconds, its displacement is x metres from a fixed point and its velocity is v m/s. What is the displacement of the particle as it moves from its position where $v = (e$ ï 1) m/s to its position where $v = (e^2$ ï 1) m/s?

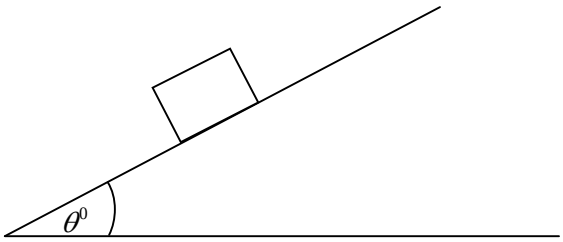
3 marks

END OF QUESTION 3
TURN OVER

Question 4

A container of mass 400 kg rests on the rough surface of an inclined tray truck. The tray is inclined at an angle of $\theta°$ to the horizontal.

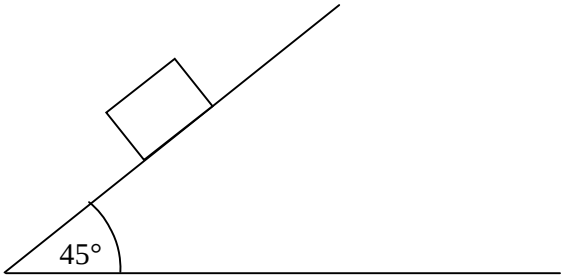
a. On the diagram below, clearly label the three forces, including the normal force, *N*, and the friction force, *F*, acting on the container. All forces are in Newtons.



1 mark

When the tray is raised to an angle of 45° to the horizontal, the container accelerates down the tray at $\frac{g\sqrt{2}}{20}$ m/s^2.

b. What is the coefficient of friction between the container and the surface of the tray?



3 marks

END OF QUESTION 4

Question 5

a. Use a compound angle formula to show that $\cos\left(\frac{\pi}{12}\right) = \frac{\sqrt{6}+\sqrt{2}}{4}$.

1 mark

b. Hence, evaluate $\int_{\frac{\pi}{12}}^{\frac{\pi}{2}} 4\sin x \cos^3 x.dx$

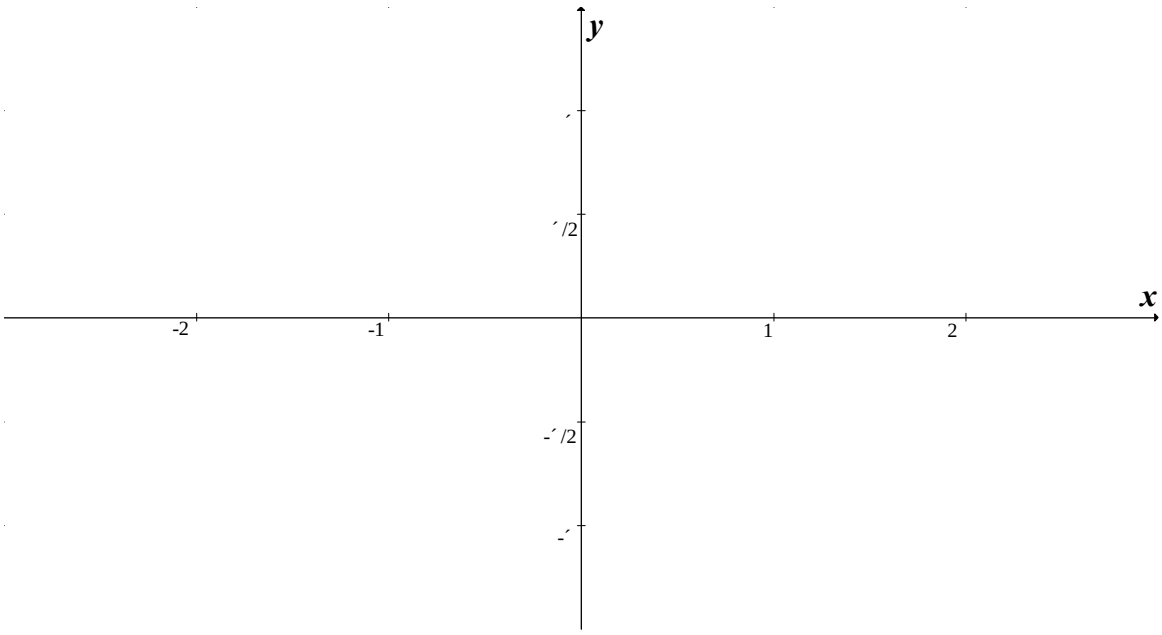
Express the answer in the form $\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right)^n$, where n is an integer.

2 marks

END OF QUESTION 5
TURN OVER

Question 6

a. Sketch the graph of the curve with equation $y = \cos^{-1}(x) - \frac{\pi}{2}$ on the set of axes below.



1 mark

Question 6 ï continued

b. Find the volume generated when the region enclosed by the curve with equation $y = \cos^{-1}(x) - \frac{\pi}{2}$, the y-axis and the lines $y = 0$ and $y = \frac{\pi}{2}$ is rotated about the y-axis to form a solid of revolution.

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4 marks

END OF QUESTION 6
TURN OVER

Question 7

For the relation $\log_e(xy) = x^2y^2$, show that $\frac{dy}{dx} = \frac{-y}{x}$.

3 marks

END OF QUESTION 7

Question 8

The position vector of a moving particle, $\underset{\sim}{r}(t)$ metres, at any time, t seconds, is given by

$$\underset{\sim}{r}(t) = 2\tan(t)\underset{\sim}{i} + \sec^2(t)\underset{\sim}{j}, \quad t \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right).$$

a. Determine the Cartesian equation for the path of the particle. State the domain and range.

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3 marks

b. Find the minimum speed of the particle.

3 marks

END OF QUESTION 8

Question 9

Three points, A, B and O, are given by $A(2,1,2)$, $B(2,2,0)$ and $O(0,0,0)$.

a. Find the vector $\overrightarrow{AB}$ expressed in the form $x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

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1 mark

b. A point, C, on vector $\overrightarrow{AB}$ is closest to O. Find the coordinates of point C.

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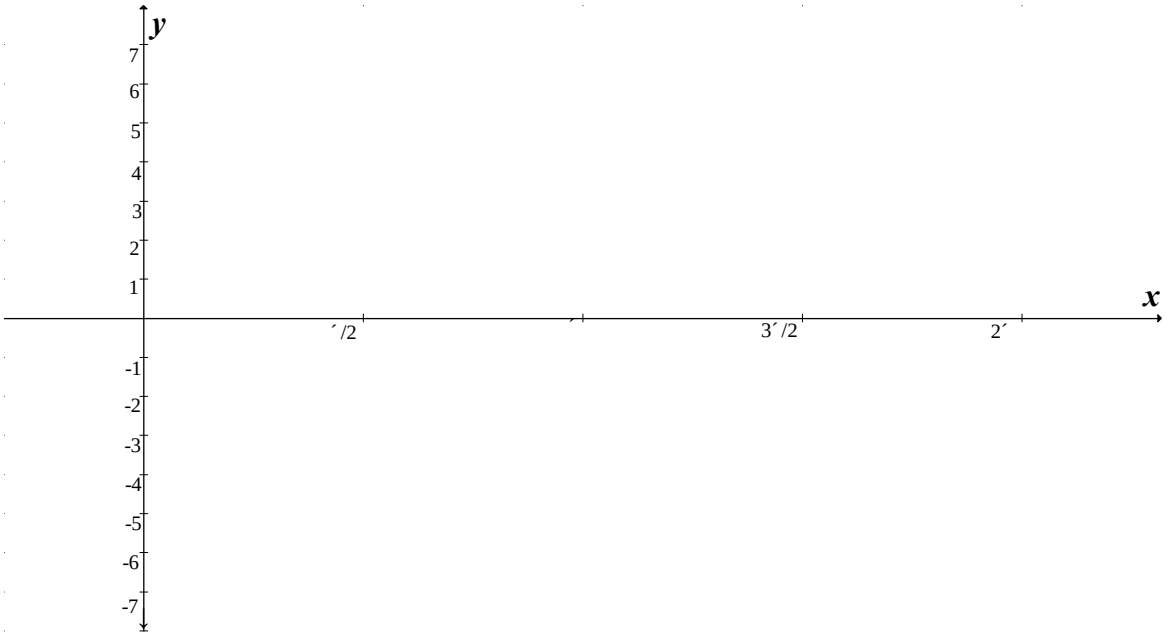
3 marks

END OF QUESTION 9
TURN OVER

Question 10

On the axes supplied, sketch the graph of $f:[0, 2\pi] \to R, f(x) = \cot\left(\frac{x}{2}\right) - 1$, clearly indicating the location of any asymptotes and intercepts with the axes.



Question 10 ï continued

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3 marks

END OF QUESTION AND ANSWER BOOK