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Unit 3 and 4 Specialist Mathematics: Exam 1

Practice Exam Question and Answer Booklet

Duration: 15 minutes reading time, 1 hour writing time

Structure of book:

Number of questions	Number of questions to be answered	Number of marks
8	8	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers and rulers.
- Students are not permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.
- No calculator is allowed in this examination.

Materials supplied:

- This question and answer booklet of 11 pages, including a sheet of miscellaneous formulas.

Instructions:

- You must complete all questions of the examination.
- Write all your answers in the spaces provided in this booklet.

Instructions

Answer all questions in the spaces provided.

Unless otherwise specified, an exact answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this book are not drawn to scale.

Take the acceleration due to gravity to have magnitude $g \text{ m/s}^2$, where $g = 9.8$.

Questions

Question 1

Consider the relation $3xy^2 + 4y = 12 - 2x$.

- a. Find an expression for $\frac{dy}{dx}$ in terms of x and y .

2 marks

- b. Hence, find the exact value of $\frac{dy}{dx}$ when $y = x$.

1 mark

Question 2

Solve the differential equation $\frac{dy}{dx} = x\sqrt{x^2 + 9}$, $x \geq 3$ for y, given that $y = 8$ when $x = 0$.

[illegible]

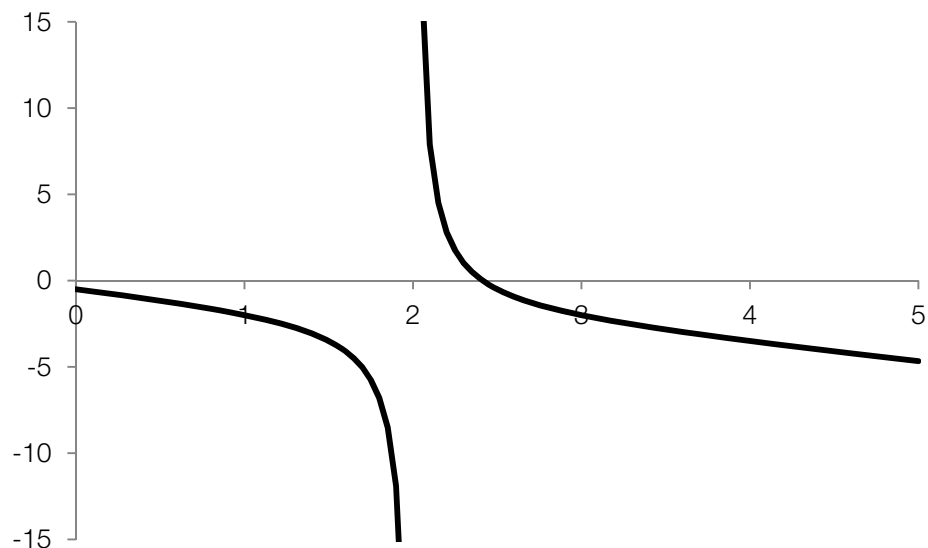
4 marks

Question 3

- a. If $\frac{dy}{dx} = -x + \frac{1}{x-2}$, find y .

2 marks

- b. The graph $y = -x + \frac{1}{x-2}$ is shown below.



Find the exact area bounded by y , the x -axis and the line $x = 3$.

4 marks

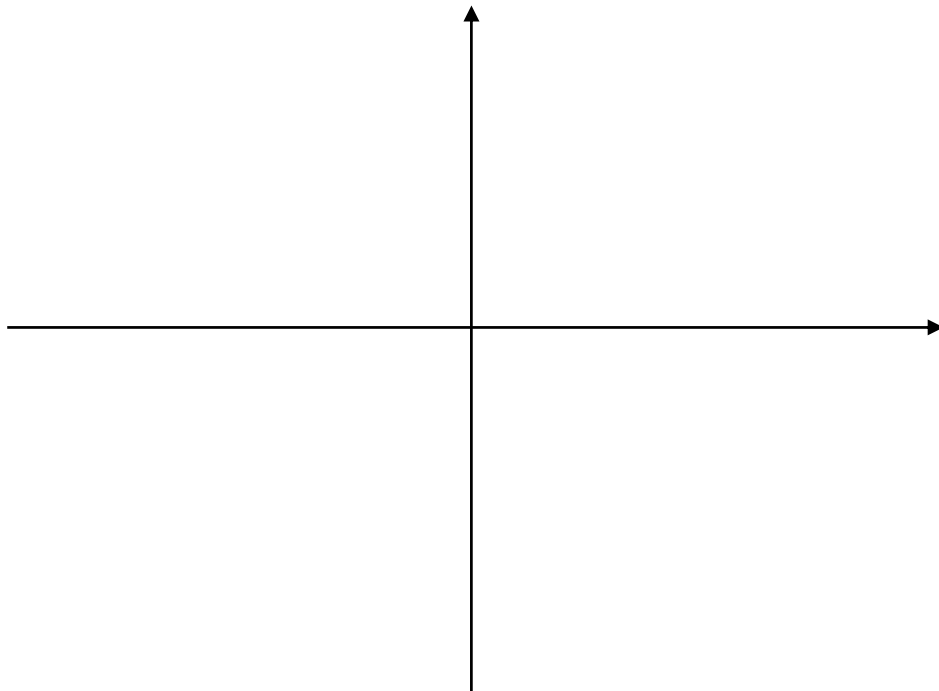
Question 4

The position vector of a moving particle is given by $\mathbf{r}(t) = \cos(2t)\mathbf{i} + \sin^2(t)\mathbf{j}$, for $t > 0$.

- a. Find the Cartesian equation of the path followed by the particle.

3 marks

- b. On the axes below, sketch the path of the particle, from $0 > t > \pi$.



3 marks

Question 5

Find the roots of $z^2 + 2z + 5$ over $\mathbb{C}$, expressing your answers in Cartesian form.

3 marks

Question 6

Find all real values of m such that $y = \sin(mx)$ is always a solution of $y = -3\frac{d^2y}{dx^2}$.

4 marks

Formula sheet

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder	$2\pi rh$
volume of a cylinder	$\pi r^2 h$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a pyramid	$\frac{1}{3}Ah$
volume of a sphere	$\frac{4}{3}\pi r^3$
area of a triangle	$\frac{1}{2}bc \sin A$
sine rule	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
cosine rule	$c^2 = a^2 + b^2 - 2ab \cos C$

Coordinate geometry

ellipse	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	hyperbola	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$
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Circular (trigonometric) functions

$$\cos^2(x) + \sin^2(x) = 1$$

$$1 + \tan^2(x) = \sec^2(x)$$

$$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$$

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$

$$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$$

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

$$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$$

$$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$$

$$\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$$

$$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	$\mathbb{R}$
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (complex numbers)

$$z = x + yi = r(\cos \theta + i \sin \theta) = r \operatorname{cis} \theta$$

$$z^n = r^n \operatorname{cis}(n\theta) \text{ (de Moivre's theorem)}$$

$$|z| = \sqrt{x^2 + y^2} = r$$

$$-\pi < \operatorname{Arg} z \leq \pi$$

$$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$$

Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$$

$$\frac{d}{dx}(\log_e x) = \frac{1}{x}$$

$$\int \frac{1}{x} dx = \log_e |x| + c$$

$$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$$

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$$

$$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$$

$$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$$

$$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a \sec^2(ax)$$

$$\int \sec^2(ax) dx = \frac{1}{a} \tan(ax) + c$$

$$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$$

$$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c, a > 0$$

$$\frac{d}{dx}(\cos^{-1}(x)) = -\frac{1}{\sqrt{1-x^2}}$$

$$\int \frac{-1}{\sqrt{a^2-x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, a > 0$$

$$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$$

$$\int \frac{a}{a^2+x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$$

product rule

$$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

quotient rule

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{\left(v \frac{du}{dx} - u \frac{dv}{dx}\right)}{v^2}$$

chain rule

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

Euler's method

$$\text{If } \frac{dy}{dx} = f(x), x_0 = a \text{ and } y_0 = a, \text{ then } y_{n+1} = y_n + hf(x_n)$$

acceleration

$$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

constant (uniform) acceleration

$$v = u + at, s = ut + \frac{1}{2}at^2, v^2 = u^2 + 2as, s = \frac{1}{2}(u+v)t$$

Vectors in two and three dimensions

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

$$|\mathbf{r}| = \sqrt{x^2 + y^2 + z^2} = r$$

$$\mathbf{r} = \frac{d\mathbf{r}}{dt} = \frac{dx}{dt}\mathbf{i} + \frac{dy}{dt}\mathbf{j} + \frac{dz}{dt}\mathbf{k}$$

$$\mathbf{r}_1 \cdot \mathbf{r}_2 = r_1 r_2 \cos \theta = x_1 x_2 + y_1 y_2 + z_1 z_2$$

Mechanics

momentum	$\mathbf{p} = m\mathbf{v}$
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equation of motion	$\mathbf{R} = m\mathbf{a}$
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friction	$F \leq \mu N$
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